Abstract

This thesis presents a variety of new results regarding the local and global bifurcations in two- and three-dimensional piecewise smooth discrete dynamical systems. Particular emphasis is placed on the creation and destruction of torus in such maps, resulting in the appearance of chaos. These results are relevant for a detailed analysis of a large number of practical systems which can be modeled by discrete time piecewise smooth maps.

In this thesis we generalize the map replacement approach for general two-dimensional piecewise smooth maps, and obtain interesting inferences regarding the existence of periodic orbits. We show that the map replacement approach is applicable also for the calculation of the border collision bifurcations occurring at infinity and degenerate period-doubling bifurcations. For some periodic orbits of the first and second levels of complexity, we calculate the border collision bifurcation curves and stability boundaries using the map replacement approach.

A two-dimensional piecewise smooth map can exhibit mode-locked periodic orbit, quasiperiodicity and transition to chaos via torus breakdown. Here we present three different routes to chaos through resonance torus destruction in a two-dimensional invertible piecewise smooth map. These routes are qualitatively different from those reported for smooth maps in literature.

Most of the past studies on nonsmooth maps have considered maps which are invertible. However, it has also been found that some switching dynamical systems give rise to nonsmooth noninvertible map. We report a mechanism of appearance of chaos via torus destruction in a two-dimensional piecewise smooth noninvertible map. Specifically we identify the transition to chaos through loop torus destruction via homoclinic bifurcation.

While there are practical dynamical systems which can be modeled by three dimensional piecewise smooth maps, very little investigation has been carried out on such maps. In this thesis, by investigating the bifurcations in a three-dimensional piecewise smooth map, we derive the conditions of the occurrence of nonsmooth local bifurcations like period doubling, Neimark-Sacker, and saddle-node bifurcations. We show that atypical phenomena like dangerous border collision bifurcation can occur in this map. Moreover, several torus breakdown routes to chaos are reported in detail.

Keywords: piecewise smooth map, discontinuous map, normal form map, border collision bifurcation, map replacement approach, torus destruction, homoclinic bifurcation.