SYNOPSIS

This thesis is concerned with the solution of certain two-dimensional elastostatic problems. With the application of the method of complex function theory developed mainly by Muskhelishvili, exact solutions are obtained for some important class of boundary-value problems in plane strain and generalised plane stress. The whole work is divided into six chapters.

The first chapter gives a brief outline of the different methods of solution of the plane boundary-value problems of elasticity. The method of complex representation discussed in detail here is based on expressing the stresses and displacements in terms of two complex functions of a single variable. The boundary value problems are reduced to the solution of certain functional equations in the complex domain. The application of the theory of conformal mapping, Cauchy integrals, analytic continuation and Plemelj formulae is found to give neat and simple solutions of the special problems. The chapter ends with the statements of the problems solved in the present work.

The second chapter is devoted to the solution of the problem of steady rotation of a plate in the form of an epitrochoid. It is found for the case of a two-cusped epicycloid that the hoop stress on the boundary is minimum at the cusps and maximum at the ends of the symmetric line of the plate, perpendicular to the line joining the cusps, the maximum stress being double the minimum stress. Also the hoop stress remains positive throughout the boundary.

In the third chapter is considered the problem for the region bounded by a cardioid when the body is subject to edge

thrusts over two equal and opposite arcs symmetrically situated on the boundary. The particular case when two equal and opposite forces act at the points on the boundary is also deduced from the general solution.

The fourth chapter analyses the stress distribution set up in the region bounded by a curvilinear polygon when concentrated forces and a couple operate inside the medium. Among the several cases of strain nuclei considered in detail are (i) the effect of a double force acting along the symmetric line of the plate when the curvilinear polygon is of a dumbbell shape or cogwheel with shallow teeth and (ii) the effect of two equal and opposite forces acting at some points of a line of symmetry of the dumbbell shape disc. When two equal double forces act at the centre of the dumb-bell shape disc and have their axes along two symmetric lines, the expressions for the stress components show that shear stress $\hat{r\theta}$ vanishes along these lines and the hoop stress vanishes on the boundary of the two-cusped epicycloid.

The fifth chapter deals with the solution of a boundary-value problem for semi-infinite region bounded by a parabola under the effect of a concentrated force or a couple operating inside the elastic region. The problem is an obvious extension of the two-dimensional analogue of the so-called 'problem of Boussinesq' dealing with a semi-infinite elastic solid with a plane boundary. This problem and also those ones considered in the next chapter are tackled by adopting the solution of the Hilbert problem.

The last chapter contains the solution of the boundary-value problem when displacements are prescribed over some portion and the stresses over the remaining portion of the boundary of a

given region. Solutions of the mixed boundary-value problem are obtained for the finite region when the cross-section is in the form of (i) a cardioid (ii) an inverse of an ellipse, and for an infinite region with a triangular hole. If the given displacements $u+iv$ and given tractions $\hat{\rho}\hat{\rho}+i\hat{\rho}\hat{\theta}$ on the boundary of the cardioid be taken as polynomials in the complex variable z it is found that the equilibrium conditions are identically satisfied.

Numerical work has been presented in case of some of these problems and graphs are drawn which show the important features of the solution.

The problems worked out here are believed to be new. However the results of the particular cases of some of the problems have been compared with known results.