Abstract

This thesis addresses the generalized inverses, their representations, computations and applications for finding solutions of given systems of linear equations. These are important and challenging areas in the field of applied sciences and numerical analysis. They generalize the notion of ordinary matrix inverse and exist for a much bigger set of singular or rectangular matrices of real or complex elements and are not unique. However, they are not easily obtainable, especially for large dimensions which usually arise in practical examples. A large number of real world applications use them extensively. Some of the most important applications involving generalized inverses are Statistics, Optimization, Prediction theory, Curve fitting, Control theory, Multibody dynamic system and Cryptography. These results are later extended to operators in Banach spaces. The major contribution of the thesis can be summarized as follows.

The Chapter 1 is the introduction. The Chapter 2 gives a brief review of the literature survey on the generalized inverses, their representations, computations and applications for finding solutions of given systems of linear equations.

The Chapter 3 is concerned with the higher-order iterative methods for computing Moore-Penrose inverse $A^\dagger$ and its associated orthogonal projections $AA^\dagger$ and $A^\dagger A$ for arbitrary matrix $A \in \mathbb{C}^{m\times n}$. First, a third-order iterative method is developed by extending an existing second-order iterative method. The convergence analysis is established and the estimation of error bounds are derived. Two numerical examples involving randomly generated rectangular matrices and a set of singular matrices obtained from the Matrix Computation Toolbox (mctoolbox) with the large condition numbers are worked out. The performance in terms of the number of iterations (NN), the mean CPU time (MCT) in seconds and the error bounds are measured. On comparing them with the existing similar works, it is observed that proposed method gives improved performance. Next, the second-order iterative method is extended for computing the orthogonal projections of arbitrary matrices. The convergence analysis is established along with the first and second-order error estimates. The efficacy of this work is demonstrated by working out two numerical examples. The performance measure in terms of the MCT in seconds and the error bounds are listed in tables. The results obtained are also compared with an existing method. Finally, the *pth*-order iterative method for $p \geq 2$ along with its convergence analysis and estimation of error bounds for computing $A^\dagger$ is described. It is observed that as the order of the method increases the number of iterations decreases as expected and the mean CPU time first decreases gradually and then increases.

In Chapter 4, a new representation of generalized inverse $A^{(2,3)}_{T,S}$ for every $A \in \mathbb{C}^{m\times n}$ with the prescribed range T, a subspace of $\mathbb{C}^n$ and null space S, a subspace of $\mathbb{C}^m$ is described. This representation is then used to give representations and computations of Moore-Penrose

inverse $A^{\dagger}$, Drazin inverse A^d, group inverse A_g, Bott-Duffin inverse $A^{(-1)}_{(L)}$, and the generalized Bott-Duffin inverse $A^{(\dagger)}_{(L)}$. Three numerical examples are worked out to demonstrate the applicability of our work.

In Chapter 5, a new second-order iterative method and its higher-order extension is proposed for computing $A^{(2)}_{T,S}$. An accelerated hyperpower iterative method is also described. The necessary and sufficient conditions for convergence of all these methods along with the estimation of error bounds are established. All these iterative methods are used for computing $A^{(2)}_{T,S}$, $A^{\dagger}$ and A^d for singular square, rectangular, randomly generated rank deficient and full rank matrices and the results obtained are compared with some of the existing methods. The performance of these methods is measured. It is observed that our methods give improved performance.

In Chapter 6, a second-order iterative method for approximating $A^{(2)}_{T,S}$ of the bounded linear operator A between Banach spaces is described. This method is further extended to approximate the generalized Drazin inverse a^d of Banach algebra element a. Next, a higher-order iterative method for computing $A^{(2)}_{T,S}$ of bounded linear operator A between Banach spaces is developed which gives the second-order method as one of its particular case. The convergence theorem for the existence of $A^{(2)}_{T,S}$ along with the error bounds is established for both of them. Numerical examples on singular square and rectangular matrices are worked out and the results obtained are compared with other existing methods. The performance of these methods is measured. It is found that our method gives improved results.

In Chapter 7, new second-order iterative methods for solving singular linear systems with index one is developed. Next, a second-order and a family of iterative methods with accelerated convergence for solving general restricted linear system of equation is developed. Necessary and sufficient conditions for the convergence along with the estimation of error bounds are derived. The applications of these iterative methods for solving some special linear systems are also discussed. Numerical examples including singular square, rectangular, randomly generated rank deficient matrices, full rank matrices and a set of matrices given in mctoolbox with large condition numbers, singular square M- matrix, sparse symmetric and nonsymmetric singular matrices obtained from discretization of the special partial differential equations are worked out. The performance of these methods is measured. On comparing proposed methods with existing methods, the improved performance for our methods is observed in terms of computational speed and accuracy.

Finally, the conclusions and the scope of future work are listed.

Keywords: Generalized inverse, Moore-Penrose inverse, Drazin inverse, Group inverse, Outer inverse, Singular matrices, Banach space, Representation, Restricted linear system, Cramer rule, Iterative method, Convergence analysis.