S Y N O P S I S.

For transmission and storage of analogue message signals in a quantized form, either synchronous or asynchronous, it is necessary to approximate the message f(t) with a time series, steps or linear segments.

Shannon's classical sampling theorem, given by:

$$f(t) = \sum_n f\left(\frac{n}{2f_{mx}}\right)\frac{\sin \pi (2 f_{mx} t - n)}{\pi (2 f_{mx} t - n)} \qquad \ldots (1)$$

where f_{mx} is the highest frequency in the message band, has been used for the time series representation of f(t). Many pulse modulation systems, e.g., PAM, PLM, PPM and PCM, have been developed on the basis of this form of signal approximation. Alternatively, message slopes f'(t) may be represented by discrete values, a technique extensively, used in network theory and the representation will be[12]

$$\left.\begin{aligned} f(t) &\cong \sum_n \Delta b_n u(t-t_n) \\ \text{and } f'(t) &\cong \sum_n \Delta b_n \delta(t-t_n) \end{aligned}\right\} \qquad \ldots (2)$$

where $\Delta b_n = (f_n - f_{n-1})$,

$u(t - t_n)$ = unit steps and $\delta(t - t_n)$ = unit impulses at $t = t_n$. By suitably quantising Δb_n to the values of (± 1), $(\pm 1, 0)$ and $(1, 0)$ and sampling the signal at uniform intervals unidigit PCM systems[5,14,6] e.g., $\dot{\Delta}$-M, Δ-ΣM, Ternary and Binary SQ-PCM, have been developed. All the above systems, however, have been basically used for synchronized transmission only.

Asynchronous pulse modulation systems, having (±1) or (1,0) outputs, may also be obtained by approximating the message either with steps of equal height or with linear segments having equal positive and negative slopes only. The message f(t) may then, be represented either by Eqn.(2) in the case of equal steps or by ramp functions as follows:

$$\left.\begin{aligned} f(t) &\cong \sum_n \Delta r_n R(t-t_n) \\ \text{and} \quad f'(t) &\cong \sum_n \Delta r_n U(t-t_n) \end{aligned}\right\} \qquad \ldots (3)$$

where $R(t - t_n)$ is a ramp with unit slope at $t = t_n$ and $\Delta r_n = f'_n - f'_{n-1}$. By quantizing Δr_n with (±1), the signal may be further approximated as:

$$f'(t) \cong \sum_n (-1)^n U(t-t_n) \qquad \ldots (4)$$

The above representations of f(t) show that various asynchronous modulation systems may be evolved by using different quantizations for Δb_n and Δr_n in the Eqns.(2) and (3). Briefly the possible systems are:

(i) Asynchronous Δ-M system, using ($\pm$1) values for Δb_n in Eqn.(2).

(ii) Asynchronous Binary Sq-PCM system, using (1,0) values for Δb_n in Eqn.(2).

(iii) Asynchronous PLM, using ($\pm$1) values for $\Delta \Upsilon_n$. in Eqn.(3).

(iv) Asynchronous Bipolar PLM, using ($\pm$1,0) values of $\Delta \Upsilon_n$ in Eqn.(3).

Other hybrid systems may also be developed by using more complex quantization of the coefficients in Eqns.(2) and (3).

Although the coded systems, viz., PCM, Δ-M, Sq-PCM etc. which use simultaneous quantization in time and amplitude, have been found to be very efficient in their noise combating properties, wide applications, e.g. in storage of analogue signals, single channel message transmission, signal processing, and non-linear controls, have been found also for systems, such as, FM and PM, where signals have quantized amplitudes only (and not synchronised in time). It was, thus, felt that along with the synchronous communication systems, the asynchronous systems should also be developed. Only recently some authors have shown interest in such systems and no detailed studies on these have been reported so far.

Theoretical analysis and preliminary experimental studies have shown that all the four asynchronous systems have SNR-Q's of 30-40 db with a channel bandwidth requirement of thirty times the message bandwidth. Comparing this with the requirements of the synchronous pulse modulation systems, such as PLM and PPM, it is seen that the asynchronous systems are basically inefficient. This is so, because the noise spectrum of the error in the coder is similar to that given by Bennett[15] for the step approximation of f(t), and is given by,

$$N_e(f) = \frac{0.05\Delta^2\mu}{f^2+\mu^2} \qquad \ldots(5)$$

where Δ = peak-to-peak error. $(\propto \frac{1}{B})$
and μ = no. of zero crossings per second in the pulse output of the coder. $(\propto B)$

Thus the noise power (in the message band) is inversely proportional to the cube of the channel bandwidth and the noise power density decreases monotonically with frequency outside the band. As has been used in some feedback quantized PCM systems, an auxiliary feedback loop may be used around the basic asynchronous coder to decrease the low frequency noise, thus producing a peaky characteristic outside the message band, while keeping the total noise power same. This redistribution of the spectral density of the noise improves the message SNR-Q very much, but at the same time tends to make the coder unstable. In fact, we now get an asynchronous pulse-modulation system which is self-oscillating (similar to the limit cycles present in on-off servo systems) but the system characteristics do not deteriorate by this, except that there will be some power transmission even when the input to the coder is zero. Of the four systems described above, considerable improvements in SNR-Q have been obtained in the oscillating types of A-PIM and AB-SQ-PCM only. These two have been called here as Rectangular wave modulation (RWM) and Pulse interval modulation (PIM) and have been studied in considerable detail.

The basic coders for RWM[10] and PIM have been developed on the analogy of on-off servomechanisms by using Eqns.(2) and (3). The coder for RWM consists of a difference circuit followed by a symmetrical comparator (with hysteresis), whose output is integrated and compared with the input, thus giving an equal slope approximation $\widetilde{f(t)}$ of the message waveform $f(t)$. At the receiver, the rectangular output of the comparator is integrated and filtered to obtain the approximated $\widetilde{f(t)}$. The modulation introduced on $\widetilde{f'(t)}$ results in large width variations of the pulses, but small variations of the time period, thus giving rise to a hybrid PIM-FM system. The coder for PIM is also similar to that of RWM, except that a pulse-type comparator is now used, and the coder output consists of constant width pulses having only the time period varying with $f'(t)$. In the receiver the pulse widths are further increased by using a single shot multivibrator, whose output is integrated and filtered to give $f(t)$. It is, however, seen that the modulation index for both

$$P(t)\,[RWM] = \frac{V(\rho_0 + f(t))}{2\rho_0} + \frac{2V}{\pi} \sum_{n=1}^{\infty} \left[\frac{1}{n} \sin n\pi \left(\frac{1}{2} + \frac{f(t)}{2\rho_0} \right) \right] \cos n\,\theta(t) \qquad \ldots (7)$$

$$P(t)\,[PIM] = \frac{d_0 V[\rho_0 + f'(t)]}{\Delta} + \frac{2V}{\pi} \sum_{n=1}^{\infty} \left[\frac{1}{n} \sin \left(\frac{n\pi d_0}{\Delta} \cdot \overline{\rho_0 + f'(t)} \right) \right] \cos n\,\theta(t) \qquad (8)$$

where V = p-p pulse height.

d_0 = mean pulse width.

$\frac{\omega_{i0}}{2\pi} = \frac{1}{T_{i0}}$ = zero-signal pulse repetition frequency.

$f'(t)$ = slope of the message waveform.

$\frac{\Delta}{2}$ = the comparator threshold

and $\theta(t)$ = the instantaneous phase angle of the pulse carrier.

The signal is now obtained by integrating $P(t)$, but side-band distortion occurs because of higher order terms. The side-band distortion falling within the message band has been calculated using these two equations. For complex message signals (e.g. bandlimited Gaussian noise, equalized speech, etc.) the signal power density (for the message band only) has been calculated[28] as,

$$S(\omega)_{RWM}\,[\text{Message band}] = \frac{V^2 (d_{m_2} - d_{m_1})^2}{12\, T_{i0}} \qquad \ldots (9)$$

$$S(\omega)_{PIM}\,[\text{Message band}] = \frac{V^2 d_0^2 \omega^2 (a_2 - a_1)^2}{12\, T_{i0}} \qquad \ldots (10)$$

where d_{mx} = maximum pulse width.

d_{mn} = minimum pulse width.

a_2 = maximum increase in pulse interval

a_1 = maximum decrease in pulse interval and is to be substituted with negative sign.

The power spectrum of the error (for the message band only) in the approximation [obtained from the input $[f(t) - \widetilde{f(t)}]$ to the comparator] is given by,

$$N(\omega)_{RWM}[\text{Message band}] = \frac{\omega^2 \Delta^2 T_{io} a^2}{12} \qquad \ldots (11)$$

$$N(\omega)_{PIM}[\text{Message band}] = \frac{\omega^2 \Delta^2 T_{io} (a_2 - a_1)^2}{12} \qquad \ldots (12)$$

where a is the maximum increase in time period.

Thus the SNR-Q's of the two systems are found to be,

$$(SNR\text{-}Q)_{RWM} = \frac{192 f_{io}^4}{\omega_{mn} \cdot \omega_{mx}^3} \qquad \ldots (13)$$

$$(SNR\text{-}Q)_{PIM} = \frac{3}{T_{io}^2 \omega_{mx}^3 \left[\omega_{mx} - \frac{2}{3T_{io}}\right]} \qquad \ldots (14)$$

where ω_{mn} is the minimum frequency in the message band (in rad/sec.). The value of $(SNR\text{-}Q)_{RWM}$ for f_{io} = 10.5 Kc/s and f_{mx} = 3.5 Kc/s and ω_{mn} = 1, as obtained from Eqn.(13) is about 53 db. Also the value of $(SNR\text{-}Q)_{PIM}$ for f_{io} = 24.5 Kc/s,

f_{mx} = 3.5 Kc/s, and a signal gain of 266 in the receiver is about 46 db.

The above quantizing noise characteristics and message distortion have been experimentally measured and the results agree well with the theoretical values. Total distortion and noise in RWM for $f_{io}/f_{mx} = 3$ is -40 db for optimum sinusoidal inputs, and this improves by 12 db for each octave increase in f_{io}. The total distortion and noise in PIM is only -28 db for $f_{io}/f_{mx} = 4$, but improves to about -50 db for $f_{io}/f_{mx} = 7$ and further improvement with the increase in f_{io} is about 6 db/octave. The input-output relation of both the systems is linear over an input volume range of 60 db, and the frequency response is flat in the message band. Speech tests with $f_{io}/f_{mx} = 3$ for RWM and $f_{io}/f_{mx} = 6$ for PIM show excellent reproduction and the SNR-T for random noise inputs is about 4 db lower than that for the sinusoidal inputs. As in the PCM systems, SNR-T falls almost linearly with the input level. However, by using instantaneous compandors, it has been found that the SNR-T becomes flat at a value of about 7-10 db below the optimum over an input volume range of 50 db. In order to reduce the effect of channel noise in the received message, a slicer has to be used at the input to the receiver. This slicer while reducing the noise in the message, also introduces some distortion in the output for channel bandwidths less than [illegible] f_{mx}. To eliminate this distortion, the feedback circuit has been modified to include the channel filter and the slicer in the loop, and the overall distortion is found to be -40 db for channel bandwidths greater than or equal to 4 f_{lo}.

RWM and PIM are both asynchronous systems, where the signal amplitudes only are in the quantized form (±1 and 1/0) and attempts have been made to convert these signals to a synchronized digital form by sampling the coder outputs with suitable clock pulses. Because of the narrow widths of the signal pulses, PIM was not amenable to such simultaneous quantization in time and amplitude, but sampled RWM was found to have reasonable SNR-η characteristics. The SNR-η of this digital RWM coder may be improved considerably by interchanging the sampler and the RWM coder, the performance of this modified coder being similar to that of Binary Sη-PCM in all respects. Important defects of this modulator such as, non-uniform output Vs. f_m, SNR Vs. f_m and maximum input Vs. f_m characteristics, may be easily eliminated by incorporating an integrating equalizer prior to the sampler, but the SNR now comes down by about 6 db as compared to the optimum SNR in the absence of the integrating equalizer. The SNR of the integrating coder may now be improved further by using a secondary feedback loop containing a 12 db/octave low pass filter from the coder output to the input of the integrating equalizer and the two loop system, thus generated, has been called here as Hybrid Unidigit PCM (HU-PCM)[19] system, since it combines useful features of both $\Delta - \Sigma$ M and Binary $S\eta$-PCM. The SNR-η of this coder is given by

$$\left(SNR\text{-}Q\right)_{HU\text{-}PCM} \cong 0.5\,(1+k)\,(f_r/f_{mx})^{3/2} \qquad \ldots (15)$$

where K is the loop gain, and f_r is the sampling frequency. The results obtained from the experimental circuits agree well with the Eqn.(15), giving an SNR-η of 45 db for f_r = 60 Kc/s and f_{mx} = 3.5 Kc/s. This SNR increases by 9 db for each octave increase in f_r and is practically independent of the signal frequency. The frequency response of the system is flat over the message band, the linearity and overload characteristics being similar to those of other PCM systems. Because of the message slope approximation and the two-loop feedback circuit used in HU-PCM, its SNR-Q characteristic is better than those of other unidigit systems and is also superior to that of the conventional n-digit PCM using amplitude approximation, at least for the practical values of the transmission rates (upto 60 K bits/sec.). As in the case of PCM signals, the sampled HWM signals may now be multiplexed on a time-division basis. An alternative technique for time-division multiplexing of messages through HWM, may be the one similar to that used in PAM-FM or PAM-PM. Messages are first multiplexed on the basis of TDM-PAM and then passed through either the ideal ([illegible]) filter or through a box-car circuit, which is more practical. The desired analogue signal is now processed by the RWM modulator and the receiver uses the complimentary circuits with synchronised clock pulses. A two channel PAM-RWM system was designed and fabricated and the experimental results corroborate the theoretical expectations of a crosstalk ratio of about 50 db between the channels for a channel bandwidth of 75 Kc/s. Further improvements in this crosstalk ratio may be obtained if $\left(\frac{\sin x}{x}\right)$ filters are used to mix the messages and the channel bandwidth increased further.

An important consideration in the utility of asynchronous systems is their effectiveness in combating channel noise. The coder outputs may be transmitted as video signals through cables or may be further converted to RWM-AM, RWM-VSB, RWM-PSK, etc. in the RF band to match the transmission media. To evaluate the usefulness of RWM for transmission of messages in video and RF channels, theoretical analysis as well as tests with the random channel noise and different channel bandwidths have been made, the output message SNR at the threshold being given by, $(SNR\text{-}CN) \cong \frac{4 B^3}{2 f_{io}^2 f_{mx}}$, (for RWM-Video)

where B is the channel bandwidth, f_{io} the zero-signal PRF and f_{mx} the message bandwidth. In all cases, the experimental results were within ±2 db of the theoretical values. Since RWM behaves almost in the same way as the familiar FM and synchronous-PIM systems, comparison of the noise reducing properties of these has also been made and the results shown in the Table I. The effect of the channel-noise, as indicated in the Table, is seen to be similar in RWM-Video and FM systems, but the threshold is better by 3 db in RWM-Video. This threshold improvement has also been maintained in the VSB transmission of RWM using the same total bandwidth and the output SNR-CN is only 4 db less, because of the lesser rise time in the received pulses as compared to the case of DSB-AM transmission. The SNR-CN in all systems improves by about 9 db for each octave increase in the channel bandwidth, and PIM gives poorer SNR characteristics for equivalent bandwidths. Although the conventional RWM-PSK has lesser values of output SNR-CN, an optimized PSK system (Exalted-carrier PSK system) has been developed, where the threshold has improved by 10 db over that of FM and the output SNR-CN is approximately equal to that of FM.

The detailed study of the asynchronous pulse modulation systems has thus shown that the Pulse Interval Modulation and Rectangular Wave Modulation systems, would be quite useful in the processing, storage, and transmission of analogue signals, and would be simpler in circuitry than the familiar FM and PIM systems. RWM has, however, better approximating properties and also gives better SNR characteristics for equivalent channel bandwidths. Using the optimized RWM-PSK system, it is possible to obtain an improvement in (S_o/N_o) [illegible] which is obtainable only in the complex and sophisticated

TABLE I.

SNR-CN values (db) for bandlimited (0 - 3.5 Kc/s) Gaussian inputs with channel bandwidths of 45 and 90 Kc/s, and an input SNR of 15 db (Random channel noise). For RWM, m = 0.5, f_{io} = 10.5 Kc/s, and SNR-D = -40 db. For PLM, m = 0.5, PRF = 20 Kc/s, d_o = 25 μsecs. and SNR-D = -40 db.

Transmission system.	Bandwidth = 45 Kc/s.	Bandwidth = 90 Kc/s.	$(S_i/N_i)_{Th}$
RWM-Video.	35	44	12
RWM-VSB.	31	40	12
RWM-PSK.	29	38	9
Exalted-carrier RWM-PSK.	33	42	5
FM.	33.5	43.5	15
PLM-Video*	24.5	33.5	12

* Single channel balanced transmission.

FM-FM systems. For synchronised transmission, the HU-PCM, derived from RWM and PIM, has the best SNR characteristics among the unidigit systems and it will be particularly useful in digital conversion of wideband signals such as those from television and FDM systems.