SYNOPSIS.

In many communication and control systems, the derived signal is either quantized in time or in amplitude or in both. This quantized signal is related to the original analogue signal by Whittaker's classical "cardinal function"[10] used as an interpolation formula for approximating an arbitrary function $f(t)$, given by:

$$\widetilde{f(t)} = \sum_{n=-\infty}^{\infty} C_n \, r_n(t - t_n) \qquad \ldots (1)$$

A simple time-quantizer, i.e. a sampler used in all synchronous modulation (PAM, PIM, PPM, etc.) and sampled-data control systems, is obtained by making $C_n = f(nT_o)$ and $r_n(t-t_n) = \frac{\sin \pi\left(\frac{t}{T_o} - n\right)}{\pi\left(\frac{t}{T_o} - n\right)}$, where T_o is the sampling period. On the other hand, a simple limiter quantizes C_n into ± 1 or 1/0 and by synchronizing t_n with the zero crossings of $f(t)$, the output signal of an "on-off" servo system may be expressed as:

$$\widetilde{f(t)} = \sum_{n} (-1)^n C_o \, u(t - t_n) \qquad \ldots (2)$$

where C_o is a constant and $U(t - t_n)$ = unit steps at the zero crossings t_n. More complex systems are derived by making $C_n = 2^m$ and $r_n = \frac{\sin x_n}{x_n}$, as in the binary coded multidigit Pulse Code Modulation (PCM), or by making $C_n = \pm 1$ and $r_n = U(t - nT_o)$ as in Delta Modulation (ΔM)[7] and Slope-Quantized Pulse Code Modulation (SQ-PCM)[9] systems. The outputs of these systems are quantized both in time and in amplitude, and the corresponding approximation is rather poor unless m and n are large.

Although theoretically, the signals quantized in time only may be recovered exactly by using suitable interpolating functions such as, $\frac{\sin x_n}{x_n}$, certain amount of distortion and noise are produced in the reconstructed $\widetilde{f(t)}$ due to the inadequacies of instrumentation. The errors generally are[11]: (a) time jitter error, (b) amplitude sampling error, (c) aliasing error, and (d) non-ideal filter error. The amplitude quantization also generates a noise which is due to the uncertainty involved in adopting a certain quantum level and this "quantizing noise" limits the performances of amplitude-limited systems. In PCM and ΔM/Sq-PCM systems, the coding efficiencies have been limited mainly by the quantizing noise and partly by the circuit complexity. Although PCM has so far proved to be the most efficient digital modulation system, it uses fairly complicated coder-decoder for a satisfactory grade of performance. On the other hand, ΔM has the defect of having a frequency distortion and a smaller coding efficiency than that of PCM. Various methods have been suggested to reduce the different errors associated with sampling and quantization, but most of them either involve nonrealizable pre-and post-filters[11], or require complicated circuits. Spang and others[26] have suggested a delayed digital feedback technique for PCM, where they use an idealized quantizer. But the instrumentation of such an arrangement is quite elaborate. The optimization of ΔM has also been attempted by various authors[20,23].

The purpose of the investigations reported in the thesis has been to obtain an improvement in the coding efficiencies of the present-day PCM and ΔM/Sq-PCM systems (by decreasing the quantizing noise) through simpler circuits. Attempts have also been made to develop certain hybrid and derived systems where the overall performances have improved considerably without much increase in the complexity of their circuitry. The reduction of the sampling errors and the quantizing noise has been achieved by the use of analogue feedback, which requires simple circuits but gives a similar order of improvement as obtained in the digital methods suggested by others. These investigations have led to the development of a Feedback-Sampler, a Partially-linearized two-level quantizer, a multi-stage ΔM- called here as Δ-ΔM, PCM with analogue feedback - Feedback-PCM[32,33] and Δ-PCM systems. Detailed studies on the effect of negative feedback on such systems and their stability problems have been made, often taking the help of linear and nonlinear feedback control theory.

The role of negative feedback in reducing noise and distortion in linear continuous systems is well known. The negative feedback applied to the quantized modulation systems may also play essentially a similar role, but the optimization of such closed-loop systems is no longer straight forward as the systems are discrete both in time and amplitude. It has been possible, however, to reduce the systems to partially linearized models and the necessary theoretical analysis carried out. Considering the two important errors in a sampler, due to aliasing and the use of non-ideal filters, the linear model shows that considerable improvement in the reconstructed analogue signal may be obtained by the use of negative feedback in the usual way. When the sampling frequency $f_r \gg 2 f_{mx}$, f_{mx} being the highest message frequency, the feedback network may be the interpolating filter itself or a simple integrator, whose break point is approximately f_{mx}. But when $f_r \simeq 2 f_{mx}$, it is not possible to provide sufficient negative feedback at higher frequency if a simple filter is used in the feedback path, as the large phase lag around the higher end of the message band makes the system unstable with little feedback. The remedy to this problem is to eliminate the lowpass network in the feedback path, thus giving a direct pulse feedback and reducing the closed-loop system to a unity feedback loop. Though the feedback is apparently of the pulses only, the reduction in error is actually due to the feedback of the reconstructed message signal. This reconstruction takes place in the amplifiers which can be considered as integrators with the break points determined by the upper cut-off frequencies of the amplifiers. Since there is negligible phase-lag inside the message band, large reduction in error is obtained here. Such a pulse feedback has been successfully used to reduce aliasing and non-ideal filter errors, the reduction being approximately 20 db in each case. The system is largely stable and the limit in the improvement is either due to saturation or due to low frequency oscillations. Both the time jitter and amplitude sampling errors can also be reduced in a similar way.

In many communication systems, as well as in control systems, the message signal often undergoes severe limiting through a nonlinear circuit which usually would be an "on-off" switch (or a Schmitt trigger) giving an output of h or zero volts. Unlike the quantizer with a large number of levels, the output spectrum of a two-level quantizer depends on the nature of the nonlinear function. Two types of signals are generally encountered with in communication systems, viz., lowpass or RC-shaped and bandpass (SSB) signals. For the lowpass signal, the noise spectrum is usually flat and stretches upto many times the signal bandwidth. In the case of a bandpass signal, a large noise at d.c. and low frequencies is obtained and peaks of noise occur inside the signal band and its harmonics. The SNR's at the output for both types of input signals have been obtained from the approximate theoretical analysis and these results verified experimentally, for an unsymmetrical onesided switch with a dead zone (given by a Schmitt trigger).

The signal power spectrum $G_0(\omega)$ at the output of the switch has been determined by following the methods of Booton[17] and West[18], and for an RC-shaped input, $G_0(\omega)$ is found in the form of a power series as,

$$G_o(\omega) = \sum_r \frac{2a_r}{r\omega_n} \cdot \frac{1}{1 + \left(\frac{\omega}{r\omega_n}\right)^2} \qquad \ldots (3)$$

where a_r's are constants which depend upon the r.m.s. input and the nonlinear characteristic. For the bandpass signal, the input spectrum is approximated as the sum of a number of tuned spectra, each of which has the auto-correlation function $R(\tau)$ and the power spectral density $G(\omega)$ given by:

$$\left.\begin{aligned} R(\tau) &= \frac{\pi K \omega_o Q}{2} e^{-\frac{\omega_o}{2Q}|\tau|} \cos \omega_o \tau \\ G(\omega) &= \frac{K \cdot \omega_o^4}{(\omega_o^2 - \omega^2)^2 + \left(\frac{\omega_o^2}{Q^2}\right) \cdot \omega^2} \end{aligned}\right\} \qquad \ldots (4)$$

where ω_o is the tuned frequency, Q is the quality factor and K is a constant. The system output power spectrum is now given by:

$$G_o(\omega) = a_1^2 \, m \, G_i(\omega) + a_2^2 \left[\sum_{K,\omega_o,Q}^{n} \frac{K_n \, \omega_{on}^4}{(\omega_{on}^2 - \omega^2)^2 + \left(\frac{\omega_{on}^2}{Q_n^2}\right)\omega^2} + \sum_{k,\omega_o}^{n} \frac{k_n}{1 + \left(\frac{\omega}{\omega_{on}}\right)^2} \right]$$

$$+ a_3^2 \left[\sum_{K,\omega_o,Q}^{n} \frac{K_n \, \omega_{on}^4}{(\omega_{on}^2 - \omega^2)^2 + \left(\frac{\omega_{on}^2}{Q_n^2}\right)\omega^2} \right] \ldots \quad (5)$$

where $G_i(\omega) = \sum_{K,\omega_o,Q}^{n} \frac{K_n \, \omega_{on}^4}{(\omega_{on}^2 - \omega^2)^2 + \left(\frac{\omega_{on}^2}{Q_n^2}\right)\omega^2}$ is the input spectral density, and m, K_n and k_n are constants less than unity. The accuracy of the calculation of the power spectrum will be more if a large number of such tuned spectra are considered to approximate the bandpass signal. The theoretical optimum SNR, from Eqs.(4) and (5), is found to be 18 db for both types of input and the SNR decreases almost proportionately with the decrease of the input level. The experimental results were however poorer, because of the uncertainty of the dead zone in the Schmitt trigger used. The circuits were also tested with speech signals as input and the intelligibility and quality were poor for the lowpass case, but were better for the bandpass case. For low level speech, serious deterioration in the quality was observed which was mainly due to the "burst" type background noise caused by the switch.

This "burst" noise can be eliminated to a large extent through what is known as "Vibrational linearization" of the nonlinear circuit[19]. This is achieved by injecting a high frequency external periodic signal to the nonlinear circuit along with the low frequency input. The frequency of this periodic input should be many times the message signal and its effect is to partially linearize the nonlinear function by removing the dead zone from the characteristic. The nonlinear characteristic is now modified as:

$$\left.\begin{aligned} x_o &= \frac{2h}{\pi}\sin^{-1}\frac{x_i}{\Delta} && \text{when} \quad |x_i| < \Delta \\ x_o &= h && \text{when} \quad |x_i| > \Delta \end{aligned}\right\} \qquad \ldots (6)$$

where x_i is the input and Δ is the dead zone. When the low frequency input is less than Δ , the output consists of symmetrically modulated PLM pulses but for $|x_i| > \Delta$, the output is a rectangular wave having the zero crossings same as those of the input. A speech test of the system now shows a complete absence of the "burst" noise. Thus the reproduced speech is quiet, highly intelligible and of excellent quality for the bandpass case. A similar effect of linearization has been achieved with a closed-loop system where a negative feedback is given around the nonlinear circuit through a lowpass filter. The self-oscillation or the "limiting" cycle linearizes the system here. The theoretical and experimental results for the two types of signals discussed above have been shown in Table I.

Since the negative feedback is found to be quite effective in reducing the "burst" noise in two-level quantizers, it was felt that similar feedback techniques may also be used in time-amplitude quantized systems, such as ΔM/SQ-PCM and PCM. The two cases, however, have certain basic differences: as in ΔM/SQ-PCM, the sampling rate is much higher than the minimum Nyquist rate, but in PCM, the sampling rate is normally slightly more than the Nyquist rate. Thus, even though the samples are quantized into

TABLE - I.

SNR characteristics of amplitude limited systems.

Nature of the circuit.	Lowpass Signal (0.3 - 3.5 Kc/s. Complex Baseband.		Speech.			Bandpass Signal (SSB) 7.3-10.5 Kc/s. Complex Baseband.		Speech.		
	SNR in db at Full Load Input(0 db)	SNR in db at an Input -10 db.	Intelligibility.	Quality.	Noise.	SNR in db at Full Load Input (0 db)	SNR in db at an Input (0 db)	Intelligibility.	Quality.	Noise.
Nonlinear Unsymmetrical switch with a dead zone.	12* / 18**	0 / 9.4	Poor	Poor	Large	14 / 18	5 / 10	Good	Poor	Large
Partially Linearized With external high frequency input.	16 / 22	20 / 24	Good	Poor	Little	18 / 22	24 / 24	Very Good	Good	Very Little.
Partially Linearized With Self-oscillation.	20 / 30	20 / -	Good	Fair	Little	26 / 30	28 / -	Very Good	Good	Very Little

* Experimental results.

** Approximate theoretical results.

two levels ($\pm$ 1 or 1/0), the ΔM/SQ-PCM system may be approximated by a linear feedback model because of the higher sampling rates used. The SNR obtained in an SQ-PCM system is[9]

$$(SNR)_{SQ-PCM} = 0.5 \frac{K f_r^{3/2}}{f_{m*}^{1/2} \cdot f_m} \quad \ldots (7)$$

where f_m is the frequency of the message signal. The main disadvantage of the ΔM and SQ-PCM systems is the inherent frequency distortion associated with them. This distortion has been effectively removed by providing a secondary feedback loop around the basic coder, which provides a negative feedback through a 12 db/oct. lowpass filter. The error analysis of SQ-PCM, more elaborate than those made earlier, shows that the error spectrum at the coder output is given by:

$$G_N(\omega) = \left[\overline{V_1^2} + y^2 \frac{2\pi}{T_0} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T_0}\right)\right] \frac{|F_1(j\omega)|^2}{T_0} + \overline{V_2^2} \frac{|F_2(j\omega)|^2}{T_0} \quad \ldots (8)$$

where $F_1(jw)$ and $F_2(jw)$ are the Fourier transforms of the constituent pulses and $\overline{v_1^2}$, y, $\overline{v_2^2}$ are constants related to the pulse heights. The error spectrum rises from a minimum at zero frequency to a maximum value at about half the sampling frequency and becomes a minimum again at a frequency a little more than the sampling frequency and is repetitive thereafter. The secondary feedback loop now reshapes, as in a linear feedback system, this rising error spectrum, and makes it flat inside the message band and sharply rising outside. This also makes the output constant over a large frequency band and the corresponding receiver now becomes only a lowpass filter. Such an optimized ΔM system does not possess any frequency distortion and the SNR remains constant over the message band. This SNR is given by[20]:

$$(SNR)_I = 20 \log \left[0.3 \, (1 + GH) \left(\frac{f_r}{f_{ms}} \right)^{3/2} \right] \text{db} \qquad \ldots (9)$$

where GH is the loop gain of the system.

Because of the presence of a nonlinear amplitude quantizer in the loop, it would be worthwhile to attempt a more exact analysis of the nonlinear closed-loop system..Using the method of Booton[17], the quasi-linear equivalent gain K_{eq}. is determined for the comparator of the coder having a particular input spectrum. The distortion at the output of the nonlinear circuit is considered as an additive noise to the closed-loop system and a linearized analysis is made of the inner coder to determine the output noise spectrum. This noise spectrum is similar in shape to that obtained from the linear analysis discussed earlier. Now the inner coder is replaced by a source of noise whose spectral density is known and the secondary loop, which incorporates the sampler, is solved as a closed-loop sampled-data system. The modified noise spectrum now becomes flat inside the message band and is peaky outside, as also verified experimentally.

The system is also seen to be a sampled-data system and hence, the coder output contains the message signal in the baseband and as sidebands around f_r and its harmonics. The noise in the sidebands may be reduced considerably by using a negative feedback of the message signals obtained through a bandpass filter, and an envelope detector. Since the low frequency contents of the coded signal is now reduced, it is possible to use cables and lines with poor low frequency characteristics to transmit such signals.

The coding efficiency of the optimized - ΔM system has been improved further by using a multi-stage coding of the message input. The multi-stage coder, called here as an optimized $\Delta - \Delta$ Modulation system[30], incorporates individual stages of optimized-ΔM coders where the message is coded in the first stage and the error in the approximation is coded in the succeeding stages. The information about the quantizing error is sent along with the main signal and is added suitably in the receiver to increase the overall SNR. Such a two-stage $\Delta - \Delta$M coder has an overall SNR given by

$$(SNR)_{II} = 40 \log \left[0.5 \, (1 + GH) \left(\frac{f_r}{f_m} \right)^{3/2} \right] - 9 \text{ db} \qquad \ldots (10)$$

where f_r is now half the transmitted bit rate. But this theoretical result cannot be obtained in practice, as there is always a finite phase lag between the error of the primary coder and the approximated error, in the receiver. The reduction in the error, for f_r = 30 Kc/s, is approximately 94%, 93%, 86% and 78%, for phase differences of 0, 1^0, 5^0 and 10^0 respectively. Experimental verification of above showed that it is possible to obtain an SNR of 47 db with only a crude phase matching in the receiver, giving a residual phase difference of 10^0 at a transmission rate of 60 Kbits/sec. Considering a basic sampling rate of 10 Kc/s, the SNR's for an n-digit ΔM coder rises exponentially with transmission rate (on a log-frequency scale) as is the case with the conventional PCM but the optimum SNR's in n-ΔM are much better for all information rates and the improvement is 12 db for the increase of a digit in n-ΔM as compared to 6 db only in PCM.

Though instantaneous companding has been used advantageously in ΔM[9,25], the accurate point-to-point matching of the compressor-expandor pair is a quite difficult problem. This difficulty can be avoided to a considerable extent if the instantaneous compressor-expandor pair is included in the secondary feedback loop of the optimized ΔM coder. When the pair is included inside the feedback loop, the harmonic distortion due to mismatch is reduced because of the negative feedback, and the SNR and the dynamic range are improved considerably. Such an optimized coder with companding has been used in multi-stage coding, where only the first stage need have a compressor-expandor pair in the feedback loop, since the inputs to the succeeding stages are constant. An experimental two-stage coder with companding and having a phase difference of 10° gives an optimum SNR of 38 db and a dynamic range of more than 40 db at a transmission rate of 60 Kbits/sec., which is better than the companded ΔM and conventional PCM systems.

As in the above Δ-ΔM systems, the quantizing noise in multi-digit PCM also can be reduced by a considerable amount with the help of negative feedback. But such a Feedback-PCM system will lose most of its appeal if the basic conventional PCM coder itself is too complex. In an attempt to simplify the overall circuits, PCM coder-decoders have been developed employing simple tapped LC-delay lines which form the heart of the whole system[31,32]. These delay lines, tapped at equal intervals and with proper weightage, act as the "coding" and "decoding networks", the number of tappings being equal to the number of digits used in coding, and the tapped outputs are summed to produce a tailored binary or ternary-weighted staircase waveform whose final amplitude is proportional to that of the input pulse. The ternary coder is a direct extension of the binary one, using similar principles and slightly modified circuits. It has also been possible to combine the coding-decoding networks into a single delay line, thus simplifying the coder considerably. In the basic binary PCM coder, the coding network gives a staircase waveform having amplitudes of 2^{n-1}, whereas the decoding network produces a staircase having amplitudes 2^n, at $n\tau$ secs. where n is the number of digits and τ is the delay between two consecutive tappings. The analogue input is sampled at the Nyquist rate f_r and the resulting amplitude modulated pulses are fed to the coding network to give rise to a staircase $r_1(t)$. The input to the decoding network is the 1/0 pulses from a gated comparator and the output is a synthesized waveform $r_2(t)$, which is continuously compared with $r_1(t)$ to minimize the error in approximation. Therefore the output of the comparator becomes an approximate binary representation of the samples of the analogue input.

In the PCM coders mentioned above, the output of the decoding delay line at a delay of $(n + 1).\tau$ secs. is proportional to the corresponding input sample and this output can be used for monitoring purposes and to provide a secondary feedback loop to reduce the quantizing noise by feeding back the approximated signal to the input for continuous comparison with the incoming signal[32,33]. The output of the decoding network at $(n + 1).\tau$ secs. is sampled and stretched in a box-car circuit to convert it into a staircase analogue signal, thus giving rise to a modified PCM system called here as the Feedback-PCM system. The inclusion of the box-car circuit in the closed-loop sampled-data system reduces it to a linear system with a total loop delay of $\left\{\frac{T_0}{2} + (n+1)\tau + \tau_1\right\}$ secs. where T_0 is the sampling period and τ_1 is any extra delay involved in the system. For an approximate analysis, the inner PCM coder is replaced by a linear amplifier with unity gain and constant delay of $(n + 1).\tau$ sec. plus a source of additive quantizing noise. The SNR with feedback is now given by:

$$(SNR)_{F.B.} = (SNR)_o \left[1 + A'\beta\right] \qquad \ldots (11)$$

where $(SNR)_o = (6n + 3)$ db, and $A'\beta$ is the loop gain of the system. The increase in SNR (for $A'\beta >> 1$) is proportional to the increased forward gain A' which is limited either by instability of the closed-loop system or by saturation of the inner coder. Both the limits are functions of the sampling frequency and the number of digits used in coding.

In another method, an integrator is used in place of the box-car circuit in the secondary feedback loop thus giving rise to a multi-digit ΔM system, called here a Δ-PCM system. The Δ-PCM is similar to the Feedback-PCM, except that it is a multi-digit ΔM employing a quantization of slopes (like that in SQ-PCM) instead of the quantization of amplitudes as is done in PCM. In Δ-PCM, the output and SNR are functions of message frequency, whereas in Feedback-PCM system they are independent of frequency. But the problem of stability arises in Feedback-PCM which is not so important in the case of a Δ-PCM system. In both the cases an optimum SNR of 42 db was obtained at an equivalent bit rate of 60 Kbits/sec. using a basic 2-digit PCM coder. For higher number of digits the coder delay increases and the maximum SNR reduces somewhat from the optimum value.

The above studies have, thus, shown that, in similarity with linear feedback systems, the analogue negative feedback may be used to improve considerably the performances of different types of time and amplitude quantized systems. In sampling circuits, the use of feedback simplifies the problem of designing the almost ideal filters and an economy of the bandwidth requirements is obtained for a certain class of wideband signals. A simple amplitude limiter may be used to transmit a continuous signal in the form of a two-level signal only, thus making the message more immune to the effect of channel noise and other distortions present in the chain of amplifying circuits. For the purpose of the bandwidth compression of complex signals, the use of the two-level quantizer simplifies considerably the elaborate division and multiplication circuits, at the same time keeping the intelligibility and quality, in the case of speech, satisfactory, for a frequency division of at least four.

In digital modulation systems, also, the effect of the simple negative feedback is to improve the SNR properties to a large extent. A summary of the important results of the ΔM/SQ-PCM and PCM systems is shown in Table II, where a relative comparison in terms of the coding efficiency of different systems may be made. It is seen that the Δ - ΔM system, where each coder is a two-loop optimized ΔM, has the best SNR and dynamic range, T A B L E - II.

SNR characteristics of different digital systems.

Modulation system.	Transmission Rate=60 Kbits/sec. Optimum SNR(db)	Maximum SNR with companding (db)	Dynamic Range with companding (db)	Transmission Rate = 40 Kbits/sec Optimum SNR(db)	Maximum SNR with companding (db)	Dynamic Range with companding (db)	Rate of increase of SNR.	Remarks.
Conventional PCM	45*	32	40	33**	22	-	6db/ digit.	* 7-digit ** [illegible]-digit.
Binary SQ-PCM	42	32	40	38	27	40	9db/ octave.	
HU-PCM	45	32	40	41	30	40	9db/ octave.	closed-loop compandor.
Δ - ΔM (2-digit, θ =10°).	47	38	40	41	32	40	11db/ octave.	closed-loop compandor in the first stage.
Δ - ΔM (2-digit, θ = 0°).	58#	-	-	49#	-	-	15db/ octave	# Theoretical results.
Feedback-PCM (2-digit)	42	32	40	38	28	40	9db/ obtave	
Δ - PCM (2-digit).	42	-	-	38	-	-	9db/ obtave.	

and at least theoretically, the optimum $\Delta - \Delta M$ will be superior to any other existing digital system. The simpler SQ-PCM and the conventional PCM have also improved through the use of analogue negative feedback. Such optimized systems may be used for digital processing of high quality TV and wideband FDM signals.