# Chapter 1

# Introduction

## 1.1 INTRODUCTION

The finite element method has evolved into one of the most powerful and widely employed techniques for finding the approximate solutions to engineering problems. However, the accuracy of the computed solution is estimated largely on qualitative basis by an experienced user of the finite element method. Traditionally, experience on similar previous computations, rules on permissible element shapes or sizes and the behaviour of the results have been frequently employed by the experts. This approach is too expensive to use in frequent application of the finite element method that is a common process while designing a product. Moreover, the size and complexity of the problems now being solved has increased to such an extent that even the most knowledgeable user finds it difficult to assess the validity of the computed solution.

Under the above circumstances measuring the *discretization error* or simply *error* of a computation is evidently desirable. The estimation of the error not only provides the information about the accuracy of the finite element solution, but also serves as an indicator for the mesh refinement, so that the current mesh can be further refined to the desired level of accuracy. In fact, techniques have been developed so that in many applications this can be taken care automatically without the intervention of the user. Such methods are well known today as *adaptive finite element methods*. The aim of much of the current research is to extend such automation to all areas of practical applications.

The adaptive finite element or *adaptive refinement* is recursive in nature. These methods automatically adjust themselves in order to improve the solution of the problem to a specified accuracy. This approach provides a level of confidence in the accuracy of the finite element results that would not otherwise exist. This confidence does not depend on blind faith in the output of the computer program. The progress to the desired level of accuracy can be traced in the steady refinement of the model and the accompanying reduction in the errors. Of the traditional approaches, the *h-refinement* or *h-adaptivity* is most common and natural to the engineers. It is particularly suitable when *singularities are present in the domain as it increases the convergence rate of the estimated error*. Figure 1.1-1 shows a typical self-explanatory *h*-adaptive finite element scheme. The figure also shows the various components involved in a typical scheme.

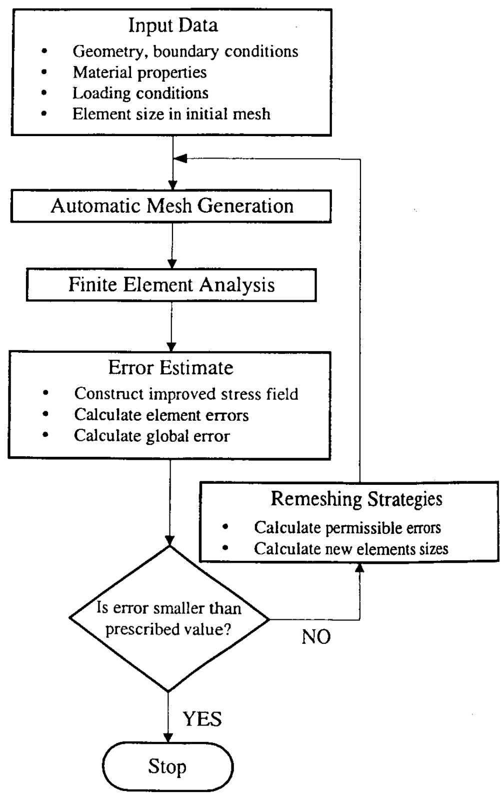


**Figure 1.1-1 An *h*-adaptive finite element sequence**

It can be observed from the above figure that pre-requisite for a reliable and robust *h*-adaptive scheme is a reliable error estimator. It should meet certain criteria such as accuracy and low cost of evaluation. The procedure of estimating the discretization error should also be practical and simple. Recently introduced *ZZ-error estimator* (Zienkiewicz and Zhu, 1987 and 1992b) has received much attention owing to its capability to accomplish the above requirements under certain conditions. In this approach, *recovery techniques* or *stress smoothing techniques* are frequently employed in order to estimate the discretization error as function of the *difference* between the *improved stress solution* and the original finite element solution. Here the improved stress of the mesh under consideration acts as an exact solution to find the error.

It is evident that, it is important to know the performance of the recovery techniques before the ZZ-error is intended to employ for problems containing the singularities. The *superconvergent patch recovery* (SPR) technique is the most widely employed recovery technique for ZZ-error estimation in the linear elasticity problems. Robust mesh generation techniques and efficient refinement strategies are also needed for successful application of the *h*-adaptive finite element methods to practical problems.

Application of the *h*-adaptive finite element techniques to fracture mechanics area forms an important domain in the adaptive finite element technology. Most of the unfortunate accidents involving ships, bridges, offshore oilrigs, gas pipelines, pressure vessels and other major constructions are only a few examples of the casualties of the *brittle fracture* (fast and unstable fractures). It has been discovered that, such catastrophic accidents of engineering structures are due to the pre-existing flaws or high stress concentration zones that are responsible for initiation of the cracks and fractures.

Fracture mechanics has become an interdisciplinary subject so as to prevent brittle fractures. The concept of the fracture mechanics have been increasingly accepted and utilized extensively by the various industries. It is widely applied to sophisticated fields like, aerospace, nuclear plants, offshore structures, submarines, locomotives, spaceships, rockets and ship structures. The principles of fracture mechanics have also been applied to other structures such as machineries, cranes, automobiles, household goods, bridges and piping etc.

Thus, engineering design that is based on the fracture mechanics approach is predominantly based on the concept of brittle failure. The basis of such design is the elastic analysis of the cracked bodies commonly known as *linear elastic fracture mechanics* (LEFM). A large number of engineering problems of practical interest fall in this category. The *stress intensity factor* (SIF) $K$ of a cracked structure is the primary quantity to be determined in LEFM. The deformation of the surface of the crack in a structure can be categorized into three basic modes each of which is associated with the corresponding stress intensity factor. Mode-I is the *opening mode* $(K_I)$, mode-II is the *sliding mode* $(K_{II})$ and mode-III is the *tearing mode* $(K_{III})$. The superposition of these three modes is sufficient to describe the most general case of crack surface displacement or crack problems commonly known as the *mixed-mode* problems.

Many problems of practical interest are of mixed-mode type. It has been noticed that (Meguid, 1989) the cracks, which start in single mode, may in fact become mixed mode later on during their useful life. Furthermore, actual cracks are generally irregular in shape and thus a combination of two modes may actually exist at a crack-tip. For the case of two-dimensional mixed-mode problems, only opening and sliding mode exist at a crack-tip.

The stress intensity factor constitutes the crucial parameters in engineering design against the brittle fracture. It is a function of the geometry of the cracked body, size of the crack and the associated loading. The elastic analysis of the cracked body has revealed that the stresses become singular as the crack-tip is approached in an inverse square root $(1/\sqrt{r})$ fashion (where $r$ is the radial distance from the crack-tip). The displacements, however, approach zero in a square root $(\sqrt{r})$ fashion. The stress intensity factor is a unique design parameter that represents the magnitude of the stress field severity near a crack-tip. From a mathematical point of view, the stress intensity factor gives a measure of strength of the singularity. Stress intensity factor provides a unique description of the stress, strain and the displacement fields in the vicinity of a crack-tip.

The stress intensity factor can be employed to determine the nature of the fracture process i.e., ductile or brittle (Meguid, 1989). More importantly, in the brittle fracture situations, the designer can come to know whether a crack in a component is likely to grow or not. In mixed-mode loading conditions, SIFs are also useful in finding crack extension direction and critical load calculations. It can also be used to determine the residual life in a component subjected to dynamic loading and it characterizes sub-critical growth due to fatigue and corrosion.

Due to the mathematical complexity of elastic analysis of the cracked bodies, analytical solutions for SIFs are available for simple configurations (Murakami, 1987 and 1992; Sih, 1973; Tada *et al.*, 1973; Rooke and Cartwright, 1976). Accurate values of the SIFs of the cracked engineering structures are desirable as they are of prime importance in preventing possible catastrophic brittle failure, which essentially involves loss of capital investments and more importantly loss of human lives. However, more techniques that are general are needed for realistic complex configurations with mixed-mode loading situations (that are usually encountered in practice). Numerical methods such as the finite element method have been employed extensively in computational LEFM.

Thus, a finite element model of the cracked engineering structure must be properly designed to determine extremely accurate values of the SIFs. However, accurate extraction of the SIFs depends also on the method of extraction. Even after extreme care is taken in selection of the proper method and design of the mesh, as has been stated earlier, it could be a very difficult task to assess the accuracy of the computed solution. Qualitative assessment of accuracy of the finite element solution and subsequent results of the SIFs might pose danger in view of the nature of the fracture problem. Evidently, application of the *h*-adaptive finite element techniques to the LEFM problems is more appropriate and it greatly reduces the risk factors that are usually involved in the traditional approaches. As the *h*-adaptive finite element techniques are scientific, they can provide more confidence on the accuracy of the extracted SIFs than otherwise.

Many aspects of the traditional computational fracture mechanics are need to be considered before attempting to develop such adaptive refinement procedures. Of various aspects, finite element modeling at the crack-tip is found to be having profound effect on accurate extraction of the SIFs. It is a well-known fact that the use of conventional elements at the crack-tip decreases the convergence of the computed solution due to the lack of the $1/\sqrt{r}$ singularity representation (which is the characteristic of LEFM). A large number of conventional elements need to be deployed around the crack-tip for sufficient representation of the singularity and accurate extraction of the SIFs. This has led to the development of various singular elements that can incorporate the necessary $1/\sqrt{r}$ stress singularity either explicitly or implicitly in the formulation (Wahba, 1985). Amongst these elements, the *quarter point element* (QPE) (Barsoum, 1974 and 1976; Henshell and Shaw, 1975) is the most popular and widely employed at the crack-tips.

Owing to their good performance, many mixed-mode SIF extraction techniques such as *quarter point displacement technique* (QPDT), *displacement correlation technique* (DCT) and *modified crack closure integral techniques* (MCCI) have been devised in conjunction with development of the QPEs. Other methods like *path independent integrals* and *virtual crack extension methods* are in principle independent of the affairs at the crack-tip. However, it has been shown that accuracy of the extracted SIFs by these methods can be improved using the QPEs at the crack-tip.

Varieties of two-dimensional QPEs such as the quadrilateral, collapsed quadrilateral and natural isoparametric quadratic triangle have been developed for application in LEFM. Of all the different QPEs, the natural isoparametric triangular quarter point elements have shown better performance due to their ability to represent $1/\sqrt{r}$ singularity along all the radial lines that originate from the crack-tip (Wahba, 1985; Freese and Tracey, 1976; Lim *et al.*, 1993). It is interesting to note that, a large portion of the literature is dominated by the isoparametric quadratic quadrilateral element and its corresponding collapsed quadrilateral quarter point elements, in spite of the fact that, both the natural isoparametric quadratic triangular element (T6) and its corresponding quarter point elements are efficient in representing complex boundaries and crack-tip singularity respectively.

When the QPEs are employed at a crack-tip they demand an interesting necessity in the design of the mesh, i.e., the shape and arrangement of the QPEs around the crack-tip. It has been shown that these aspects have profound effect on the accuracy of the extracted SIFs. It has been found that certain crack-tip mesh patterns have returned inaccurate values of the SIFs than others for the same number of elements in the mesh. It is reasonable to expect that a small number of QPEs surrounding the crack-tip would result in inadequate and erroneous modeling of the field variables circumferentially. However, increasing the number of elements around a crack-tip decreases the span angle. Subsequently this intıoduces errors due to excessive element distortion.

Extensive investigations on shape and arrangement have been conducted by many researchers (Saouma and Schwemmer, 1984; Murti and Valliappan, 1986b; Menandro *et al.*, 1995). One of the most widely accepted suggestions for mixed-mode problems is that symmetric mesh should be employed around the crack-tip and eight number of QPEs each preferably having a span angle of $45^{\circ}$ should be used around the crack-tip. The arm ratio (ratio of maximum QPE side length to its minimum length) should also be close to unity for best results (Saouma and Schwemmer, 1984; Murti and Valliappan, 1986b). The commercial software ANSYS has also recommended a specific pattern of arrangement of QPEs. As a result of these studies two basic patterns (Figure1.1-2(a)) have been employed at the crack-tips in the literature. However, no literature is available on comparison of various such recommendations.

The accuracy of extraction of the SIF is considered as a basis for selection and design of the quarter point element mesh patterns. It is likely that the choice of a particular SIF extraction method has significant impact on the selection of a particular crack-tip mesh pattern. To the best of the author's knowledge no report is available on whether any particular crack-tip mesh design is needed in order to improve the SIF extraction by the path independent integrals and virtual crack extension methods. In spite of this fact, many researchers have employed in practice the basic crack-tip mesh patterns shown in Figure 1.1-2(a) along with certain number of concentric rings. Reason may be most obvious as the patterns in the above figure provide more secured mesh design around the crack-tip and it seems that there is no reason why they should not be employed while using the path independent integrals and virtual crack extension methods. Thus, employing either of the basic forms shown in Figure 1.1-2(a) is a conservative approach irrespective of method of extraction of the SIFs. Indeed, the suggested recommendations of commercial software like ANSYS are needed for any method of extraction of the SIF.

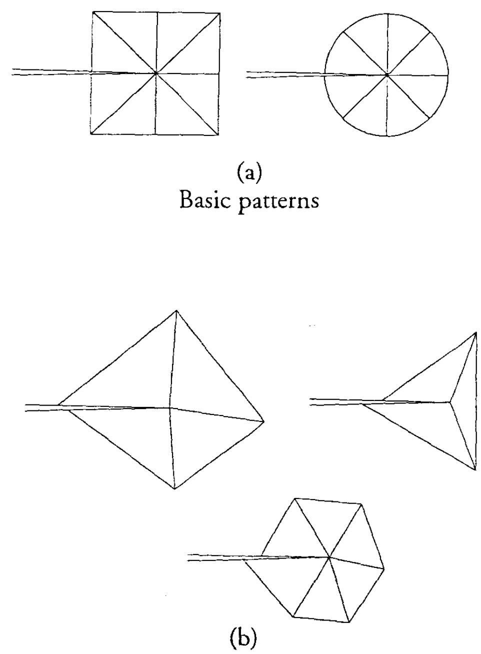


**Figure 1.1-2 (a) Different widely employed basic crack-tip mesh patterns; (b) typical crack-tip mesh patterns from general two dimensional automatic adaptive mesh generator**

Apart from these issues, individual requirements and performance characteristics of the mixed-mode SIF extraction methods on the adaptively refined meshes play a crucial role in extraction of the reliable mixed-mode SIFs of practical problems. For example, the methods such as DCT, QPDT and MCCI techniques pose certain restrictions on the arrangement of the QPEs along with usual refinement requirements. On the other hand, various parameters of the path independent integrals and virtual crack extension methods also affect the accuracy of the SIFs. Thus, it is desirable to conduct a thorough investigation on performance characteristics of a large number of SIF extraction methods on the adaptively refined meshes. Such an investigation reveals about the methods that can be safely employed along with the *h*-adaptive finite element techniques in order to extract accurate mixed-mode SIF values of practical problems.

Very limited amount of investigation (Lo and Lee, 1992; Min *et al.*, 1994; Sandhu and Liebowitz, 1995) has been conducted on application of the *h*-adaptive displacement finite element techniques to LEFM in order to extract accurate values of SIFs. These earlier investigations lack the ability to extract very accurate SIF values. Furthermore, the proposed *h*-adaptive schemes cannot be extended to the domains with complicated boundaries and intricate orientations of the cracks due to their inefficient mesh generators, refinement strategies and the error estimation procedures. The results of Lo and Lee (1992) clearly indicate that the inaccuracy in the extracted SIF values are primarily due to the inability of their automatic adaptive mesh generator in providing any of the basic crack-tip mesh patterns as shown in Figure 1.1-2(a).

Figure 1.1-2(b) shows schematic illustration of typical crack-tip element patterns that are likely to appear at a given crack-tip by a general two-dimensional automatic adaptive mesh generator. Clearly, these are not desirable for many reasons. However, it has been demonstrated that (Lo and Lee, 1992), the relative percentage error in the mesh can be drastically reduced by using the quarter point elements (QPEs) instead of standard isoparametric elements around the crack-tips. As a result, the refinement needed for achieving the same level of accuracy is also reduced.

Another important aspect of the earlier work in this direction is the absence of *h*-adaptive finite element analysis of mixed-mode crack problems. Despite the importance of these loading situations in practical problems, all earlier researchers have attempted this approach for only pure mode-I problems. In addition, to the best of the author's knowledge, no report is available to date on application of an *h*-adaptive displacement finite element technique to two-dimensional mixed-mode crack problems. Moreover, no attempt has been made by the earlier researchers to suggest (with some confidence) the SIFs of a problem that does not have any closed form solutions.