Chapter-1

# A Review of Literature

## 1.1 Introduction

Assessment of structural performance and safety under realistic dynamic loads is one of the most serious challenges for a structural engineer. Many dynamical systems of significance in civil, mechanical, naval and aircraft industries are often subjected to excitations that are basically random in nature. It includes systems subjected to aerodynamic and fluid dynamic forces as well as those subjected to earthquake ground motion or machine induced noise environments. Additional sources of randomness may be attributed to the system uncertainties associated with the geometrical as well as material properties of the structure itself. As the external load and/or system randomness contributes significantly to the structural response, these effects must be accounted for. If the system is deterministic and the input (external forces) is random and time dependent, the output (response) is a stochastic process. The analyses of such processes seek to determine the statistics or the probability measure of response variables (or functionals) and involve the probability theory and in particular, the theory of stochastic differential equations (SDE-s). In estimating the failure probability of a dynamical system or the reliability indices, these statistics play an important role. For such studies, it is generally required to generate an ensemble of sample path solutions. Unfortunately, in most cases analytical or closed-form solutions of SDE-s modeling an engineering system are not available and one is forced to use numerical methods to approximate them. Recent developments on efficient numerical algorithms in the broad area of stochastic structural dynamics are based on Monte Carlo simulation, stochastic finite and boundary elements, finite differences, stochastic Green's functions, and other methods. The Monte Carlo simulation method is the most general approach and requires solving a large ensemble of deterministic sample problems corresponding to generated samples of the input processes. The resulting output sample processes may then be used to compute response statistics and thus estimate reliability. Though the method has the drawback of potential inefficiency, it has its versatile application to linear as well as non-linear stochastic analysis provided that sample pathwise or strong solution of the associated deterministic problem corresponding to sample input processes are known. Moreover, with several variance reduction and importance sampling strategies being presently available to increase the computational efficiency of MCS, there is an ever increasing research focus to explore new and more complex stochastic dynamic systems using direct simulation.

Sample path solutions are achieved either in strong or weak sense. A strong stochastic solution is a pathwise description of the response trajectory as it evolves from a specified (deterministic) initial condition under a specific realization of the stochastic excitation. On the other hand, when requirements are relaxed in approximating the response in such a manner that the first few statistical moments like mean, variance, skewness etc., of its ensemble (generated under an ensemble of realizations of the stochastic excitation) match with those of the 'exact' or strong response, then such an approximation is referred to as a weak solution. In the numerical treatment of SDEs, the weak approximations play an important role in efficiently computing functionals of the response, such as moments, functional integrals, invariant measures, and Lyapunov exponents. A weak solution is preferable if it can be shown that it is numerically simpler and faster. This is particularly true as most engineering solutions to problems in probabilistic mechanics need only to consider the first few statistical moments and not the pathwise realization of the response.

Sample path solutions to dynamical systems may be attempted either in time-domain or in frequency-domain. The ease of implementation along with the versatility of application to non-linear and higher dimensional problems has driven the popularity of time integration approaches. Given the ready availability of increasingly faster computers, there is a great thrust now on computational methods to expand and generalize the integration procedures and hence make them applicable to systems large enough for being of interest to a practicing engineer.

The essence of the present research work is to develop a few single and multi-step time integration strategies for a given mathematical model of a system with excitation uncertainty only, in their strong and weak forms, and consequently to apply them to non-linear stochastic dynamical systems driven under both external (additive) and parametric (multiplicative) white noise excitations.

## 1.2 Stochastic Processes and SDE-s

### 1.2.1 Some Concepts in Probability Theory

A large variety of characteristics are encountered in the broad area of stochastic structural engineering that may be categorized under static/dynamic stochastic analysis; low/higher dimensional systems in the state space; deterministic/uncertain systems; external/parametric with normal/non-Gaussian excitation; conservative/dissipative/hysteretic systems; stationary/non-stationary response; collapse/serviceability failure conditions, etc. Characterization of the response of such stochastic systems and/or stochastically excited systems requires solutions of SDE-s along with the knowledge of the input probability distributions. Random processes and random fields are key concepts in such studies. These are used, among others, as models for excitation and response time histories. The construction of probability density functions (PDF) and joint probability density functions are necessary to characterize random variables and random vectors respectively.

A stochastic process may be viewed as a mathematical abstraction defining a rule for the random selection of sample paths from a set of specified functions of time [Grigoriu 2000]. The stochastic field resembles a stochastic process but depends on a space rather than a time argument. In principle, random processes and random fields can be described completely in terms of infinite sequences of joint probability distributions of successively higher order, or in terms of infinite sequences of moment functions or cumulant functions. In practice, it may often be necessary to deal with very incomplete descriptions; e.g., with marginal probability distributions only, or with a limited number of statistical averages (moments).

In many engineering problems a partial characterization of random quantities, referred to as the second moment properties, can provide useful information on the trend and the magnitude of random fluctuations about this trend. The second moment properties consists of the mean and variance for random variables, the mean vector and covariance matrix for random vectors, and the mean function and co-variance functions for stochastic processes and fields. The square root of the variance is called *standard deviation*. The ratio of standard deviation to mean is the *coefficient of variation* and is defined for random variables with non-zero mean. The second moment properties of a random vector $X = \{x_i\}$ consists of the mean $\mu_i$, variance $\sigma_i^2$ of the random variables $x_i$, and a measure of the relationship between these random variables are given by the *covariance*, $\gamma_{ij} = \iint (x_i - \mu_i)(x_j - \mu_j) p_{ij} dx_i dx_j$ for all $i \neq j$, where $p_{ij}$ denotes the joint probability density function of random variables $X_i$ and $X_j$. The covariance $\gamma_{ii}$ coincides with the variance $\sigma_i^2$ of $x_i$. If the covariance $\gamma_{ij}$ are zero for all $i \neq j$, the coordinates of random vector $X$ are *uncorrelated*. The mean vector $\mu = \{\mu_i\}$ and the covariance matrix $\gamma = \{\gamma_{ij}\}$ define the second moment properties of $X$. The scaled covariance $\gamma_{ij} = \gamma_{ij} / \sqrt{\gamma_{ii}\gamma_{jj}}$ is called the correlation coefficient of $X_i$ and $X_j$. The covariance function of a stochastic process $X(t)$, denoted by $c(t,s)$, is equal to the covariance of the random variables $X(t)$ and $X(s)$ for all values of times *t* and *s*. If the mean function $\mu(t) = \mu$ is constant and the covariance function $c(t,s) = c(t-s)$ depends only on the time lag $t-s$, the process is said