Abstract

This thesis describes a number of higher order iterative methods and their convergence analysis for finding solutions/fixed points of nonlinear equations in Banach spaces. A number of theorems are established for the convergence under different sufficient conditions along with the derivation of a priori error bounds and the existence and uniqueness domains for their solutions. The main contributions of the thesis can be summarized as follows.

The Chapter 3 gives a class of third order derivative free exponential iterative methods used to solve nonlinear equations in $\mathbb{R}$. The proposed method is improved further by combining it with the regula falsi method to establish that both the sequence of diameters and the sequence of errors converge to 0 simultaneously.

The Chapter 4 describes a sixth order method developed by extending a third order method for solving nonlinear equations in $\mathbb{R}$. In terms of computational cost, it requires evaluation of only two functions and two first derivatives, thus avoiding the second derivative evaluation per iteration and gives an improved efficiency index of $1.565\ldots$.

Next two chapters are concerned with the semilocal convergence analysis of quadratically convergent Stirling's method used to find fixed points of nonlinear operator equations in Banach spaces. Using recurrence relations, the convergence analysis is done under the Hölder, weak Hölder, ω- and weak ω- continuity conditions on the first Fréchet derivative of the involved operator. It can be shown that the Lipchitz and Hölder continuity conditions are special cases of ω-continuity condition. A priori error bounds along with the existence and uniqueness domains of fixed points are derived. The R-order of convergence is also shown to be equal to $(1+p)$ for $p \in (0,1]$.

The last three chapters are concerned with the semilocal convergence analysis based on the recurrence relations of a third order Stirling-like method used to find fixed points of the nonlinear operator equations in Banach spaces. The convergence analysis is established under the Lipschitz, Hölder and ω- continuity conditions on the first Fréchet derivative of the involved operator. A priori error bounds, the existence and uniqueness regions and the R-order of convergence equals to three are given for each of them.

A number of numerical examples are worked out in each chapters and results obtained are compared with those obtained by other similar methods. In all cases, our approach either gives better results or behaves similarly.

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