Abstract

An extension of Lagrangian-Hamiltonian mechanics is presented that incorporates dissipative and non-potential fields, and non-integrable constraints in a compact form, such that one may obtain invariants of motion or possible limit trajectories through an extension of the Noether's theorem. A new concept of an additional time, so named as the umbra-time has been introduced for such an extension. This leads to a modified form of the equations of motion called the umbra-Lagrange's equation. The underlying variational doctrine, which is based on recursive minimization of functionals, is proposed for the umbra-Lagrange's equation. The introduction of the concept of umbra-time extends the classical manifold over which the system evolves. An extension of Lagrangian-Hamiltonian mechanics for finite systems over vector fields in this extended space is presented. The idea of umbra-time is then carried forward to propose the basic concept of umbra-Hamiltonian, which is used along-with extended Noether's theorem to provide an insight into the dynamics of systems with symmetries. An interesting case of a bisymmetric electromechanical system is analyzed through this approach and limiting dynamics is obtained and validated through simulations. Gauge-functions for umbra-Lagrangian are also introduced to symmetrize unsymmetric systems. Extension of the Poincaré-Cartan analysis for umbra-Lagrangian is also proposed and its implications have been discussed with several examples. Finally, the umbra-Lagrangian formulation for a continuous set of generalized coordinates is developed in order to describe continuous systems. Variational formulation for one-dimensional continuous systems is presented considering gyroscopic forces and rotary inertia with internal and external damping. The invariance of umbra-Lagrangian density is obtained through an extension of Noether's theorem over manifolds. The dynamic behaviour of internally and externally damped rotor system with dissipative coupling is analyzed through this approach. In another case study, the effects of discrete external damping are investigated and some interesting results are obtained theoretically as well as numerically.