CHAPTER - I

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INTRODUCTION

1.1 Introductory remarks

The study of Newtonian and non-Newtonian fluid flows with or without free convection and mass transfer is of great physical interest because of its varied applications in natural sciences and engineering. An attempt has been made in this thesis to analyze the influence of both free convective flow and mass transfer on the steady and unsteady flow characteristics of viscous and elastico-viscous liquids under different physical circumstances and also on the periodic flow due to an oscillating cylinder placed in a viscoelastic liquid. Chapter I is of review nature and deals with the introduction to the thesis. Chapter II is concerned with some problems of steady and unsteady free convection flow and mass transfer of viscous fluid in a rotating porous medium under various physical situations. These problems are important due to their applications in many fields of engineering and natural sciences dealing with subjects such as oil recovery, soil mechanics, catalytic chemical reactions, material adsorption on solids, filtration, polymer property measurements and agricultural sciences and are discussed here to study the effects of pertinent parameters, (rotation parameter, permeability parameter, frequency parameter, Grashof number, modified Grashof number, Prandtl number, Schmidt number) on the flow characteristics under variety of situations. The problems are investigated in two parts : (i) Mass transfer and free convective flow past a vertical porous plate in a rotating porous medium considering the viscous dissipation and Darcy's dissipation terms on the flow field and (ii) Unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through a porous medium, bounded by an infinite vertical porous surface, in a rotating system.

The study of the non-Newtonian fluid flows has become essential due to their importance in modern industries and various geophysical and engineering problems. In Chapter III we study the steady/unsteady free convection flow and mass transfer of a rotating incompressible elastico-viscous liquid under varied configurations. This chapter is divided into two parts. In the first part, we discuss the free convective flow and mass transfer of a rotating incompressible elastico-viscous liquid bounded by an infinite vertical porous plate when the plate is subjected to a constant suction velocity and heat flux at the plate is constant. The temperature and the species concentration at the free stream are constant. The second part of this chapter deals with unsteady free convective flow and mass transfer of a rotating elastico-viscous liquid past a vertical porous plate subjected to a constant suction velocity and the plate temperature varies harmonically with time.

The last chapter is devoted to the study of a boundary layer analysis of secondary flow induced by an oscillating cylinder in a viscoelastic liquid. This type of problem is of general and continuing interest and is discussed here to study the flow characteristics of viscoelastic liquid which leads to a considerable reduction in the drag co-efficient. A detailed investigation of the steady secondary flow and its final decay to zero are also analyzed.

Before we discuss various problems, we present below a brief survey of relevant literature on rotating fluid flows, flow through porous media, free convection flow and mass transfer and non-Newtonian fluids so that the work presented in this thesis could be seen in its proper perspective.

1.2 Rotating fluid flows

The stimulus for scientific research on fluid systems in rotating environments is originated from geophysical and fluid engineering applications. Many aspects of the motion of terrestrial and planetary atmospheres are influenced by the effects of rotation. The broad subjects of oceanography, meteorology and atmospheric science all contain some important and essential aspects of rotating flows. Rotating flow theory is utilised in determining the viscosity of fluid and in the construction of turbines and other centrifugal machines. The complete literature pertaining to rotating fluids is enormous and excellent review can be found in the monograph by Greenspan [1]. Rotation in a fluid system produces two effects ,viz., the coriolis and the centrifugal forces, on the fluid particles. The balance between the coriolis forces and the pressure gradient with correction for the viscous action at the boundaries emerges as the back bone of the entire theory of the rotating flows. In considering flows in rotating environments we come across situations where the entire fluid is in a solid body rotation or only the solid boundaries are rotating. In the later case , it is preferable to use an inertial co-ordinate system fixed in space. On the other hand the flow behaviour in the former case can be described in a co-ordinate system which rotates with the fluid, and in this frame of reference the fluid is at rest.

In a steadily rotating system, a balance is struck between coriolis and frictional forces in a thin layer over the horizontal boundaries. This layer, called the Ekman layer, was first noticed by Ekman [2] and plays a very fundamental role in the rotating fluid flows. In this thesis we consider certain problems in rotating systems due to their varied applications in the fields of cosmical, technological and agricultural sciences.

1.3 Flow through porous media

Research on fluid flow through porous media finds great application in geothermy, geophysics and technology. But the basic property of the porous media is very complex because of the complicated pore structure of the porous medium and the pores in the porous medium are, in general, interconnected three dimensional network of capillary channels of non-uniform sizes and shapes [3]. To overcome certain inherent difficulties, some of the successful practitioners in the field of flow through porous media have tried, as much as possible, to stick with the continuum approach in which no attention is paid to pores or pore structure and a simplified model has been adopted in most of the studies of fluid flow through porous media [4,5,6].

In the analysis of flow through porous medium, Darcy's law usually been assumed to be the fundamental equation [7,8]. This law expresses that the (seepage) velocity is proportional to the pressure gradient and it does not have a convective acceleration of the fluid. This law is, therefore, considered to be valid for low speed flows, whereas the speed in the filter or the flow in the region where the velocity changes abruptly are not always small and the convective force may be important. For a high speed flow, we may also pay attention to the force acting on the fluid by the porous medium. The force may deviate from the usual Darcy drag which is proportional to the velocity [3]. However, in the case of very porous media such as filters, the deviation will be small enough to be neglected. Brinkman [9] has proposed this convective force assuming that the force on a particle situated in the cloud of particles could be calculated as if it were a solid particle embedded in a porous mass. He represented the porous mass by modifying Stokes' equation adding Darcy resistance term to it so that the effect of all other particles is treated in an average sense and the resulting equation is a modfication of Darcy's equation. Since the latter equation is emperical, Brinkman's result has not been generally regarded as a completely rigorous theoretical model to the problem even though by comparing with the experimental data the success of Brinkman's formula is indisputable. Tam [10] put Brinkman's method in better theoretical shape by treating the swarm of particles as point forces in Stokes flow and ensemble averaging over all particle position except that of primary particle.

The resulting generalized law is found to be useful in the study of flow in highly porous media such as pappus of dendelion and fibres. Several studies [11-15] have been made on the basis of generalized Darcy's law accompanied by convection term. Yamamoto and Yoshida [16] have made an analysis of suction and injection flow through a plane porous wall giving importance on the vortex layer attached to the surface in the wall and on the flow outside the vortex layer. Later Yamamoto and Iwamura [17] investigated the asymptotic behaviour of a general suction flow into a porous medium with arbitrary but smooth surface. Chawla and Singh [18] analyzed flow past and against a porous bed when the free stream oscillates with or without a non-zero mean.

In this thesis we study certain problems of fluid flows in a rotating porous medium with the basic assumption that the porous medium is regarded as an assemblage of small identical spherical particles fixed in space with σ and N representing the radius of a particle and the number density of the particles respectively. Since the porosity of the medium is taken to be unity, σ is taken so that $\sigma^3 N \to 0$ while σN remains finite as $N \to \infty$. Then for a solid particle, the Reynolds number $R = \sigma V_* / \nu$ is very small even when V_* is not small, where V_* is the reference speed and ν is the kinematic viscosity. The drag, on a sphere is given by Stokes' formula $\vec{F} = 6\pi\sigma\mu\vec{V}$, where $\vec{V}$ is the filtration velocity and μ is the viscosity of the fluid. The swarm of these solid particles exerts a force $6\pi\sigma\mu N\vec{V}$ per unit volume on the fluid. Clearly this body force remains finite in this limit. But the porosity of the medium $(1-\frac{4}{3}\pi\sigma^3 N)$ tends to unity as $N \to \infty$. If the representative length scale of the macroscopic flow be L, which is, in general, much larger than σ. Then, the Reynolds number $R_L = V_* L/\nu$ in the present consideration is not necessarily very small so that convective terms should be taken into account. Further for such a medium where fluid occupies almost all parts of the porous medium, the viscous stress τ_{ij} is given by $\tau_{ij} = \mu(V_{i,j}+V_{j,i})$, V_i being the component of velocity [17].

1.4 Free convection and mass transfer

There are many transport processes which occur in nature and various experimental set-up in which flow is driven or modified by density differences caused by temperature, chemical composition differences and gradients, and material or phase constitution. These processes are very important and have received considerable attention in literature. Ostrach [19-22] has analyzed the effect of viscous dissipative heat on steady free convection and also on combined free and forced convection flows between parallel plates under variety of situations. Gebhart [23] and Gebhart and Mollendorf [24] have shown that the viscous dissipative heat in the natural convection flow is important when the flow field is of extreme size or at extremely low temperature or in high gravity field. Stuart [25] studied the flow past an infinite porous plate when the free stream oscillates in time about a constant mean and also discussed the unsteady temperature field by assuming that there is no heat transfer between the plate and the fluid. The unsteady free convective flows have also received attention of many research workers. Notable works in this field are by Nanda and Sharma [26] and Pop [27,28]. They have assumed that the plate temperature undergoes time-dependent variation in temperature, but the viscous dissipative heat is neglected in these works. Soundalgekar [29] analyzed unsteady free convective flow past an infinite vertical porous plate with constant suction on taking into consideration the viscous dissipative heat. The flows past porous vertical plate subject to uniform suction normal to the plate have been also discussed by Soundalgekar [30-32].

However, there are other flow fields which arise from differences in concentration or material constitution alone and in conjunction with temperature effects. Atmospheric flows, at all scales, are driven appreciably by both temperature and H_2O concentration differences. Flows in bodies of water are driven through the comparable effects upon density , temperature, concentration of dissolved materials, and suspended particulate matter. There are many interesting aspects of such flows, viz., resulting transport characteristics, results of the opposition of the two effects, influence of the combined effect on the stability of laminar flows, and the effects of values of the relative transport parameters like Prandtl and Schmidt numbers. The poineer works of Somers [33], Mathers et al. [34], Wilcox [35], Gill et al. [36], Lowell and Adams [37], Adams and Lowell

[38], Cardner and Hellums [39], Lightfoot [40], Adams and Mc Fadden [41], Den Bouter et al. [42], Manganaro and Hanna [43] and Saville and Churchill [44,45] may be considered as the origin of the modern research on the effects of mass transfer on free convection flow. Further, Gebhart and Pera [46] studied the laminar flows which arise in fluid due to the interaction of the gravity forces and density differences caused by the simultaneous diffusion of thermal energy and of chemical species neglecting the thermal diffusion and diffusion-thermo (Soret-Dufour) effects because the level of species concentration is very low. Sparrow et al. [47,48] and Sparrow [49] have analyzed the free convection flow with Soret-Dufour effects. Further, Soundalgekar [50] studied the mass transfer effects on the free convection flow of an incompressible, dissipative viscous fluid past an infinite vertical porous plate with constant suction. Soundalgekar and Wavre [51] extended this problem considering the plate temperature oscillates in time about a non-zero constant mean.

Forous media are very widely used to insulate a heated body to maintain its temperature. They are considered to be useful in diminishing the natural free convection which would otherwise occur intensely on the vertical heated surface. Further, the effect of free convection on the flow through porous medium plays an important role in agricultural engineering and in petroleum industry in extracting pure petrol from the crude. Nield [52] was the first who analyzed the problem of the onset of the convection, induced by buoyancy effects resulting from vertical thermal and solute concentration gradients, in a horizontal layer of saturated porous medium considering the general form of Darcy's law. Later, under various physical considerations, Cheng and Minkowycz [53], Rudraiah and Nagraj [54], Raptis et al. [55-57], Raptis [58-63], Raptis and Kafousias [64-66] Raptis and Perdikis [67,68], Raptis and Tzivanidis [69], Horia and Polisevski [70], Ramanaiah and Malarvizhi [71], Nield [72,73], Nield et al. [74] and Manole et al. [75] have studied the free convection flow through porous media and estimated the effects on the heat transfer in order to make the heat insulation of the surface more effective. The effect of rotation on the convective flow and mass transfer in the porous medium does not seem to have received much attention in the literature. Gupta and Chakraborti [76] analyzed the nonlinear thermohaline convection in a rotating porous medium. Raptis [77] discussed the steady free convection and mass transfer through a porous medium bounded by an infinite vertical porous plate for a fluid rotating with a constant angular velocity when the heat flux at the plate is constant. Johri [78] gave a theoretical analysis of two-dimensional steady free convection flow of an electrically conducting and incompressible viscous fluid past an infinite porous vertical plate in a rotating frame of reference for a porous medium.

Chapter II of the present thesis is concerned with the steady and unsteady free convective flow and mass transfer of viscous fluid in a rotating porous medium under various physical circumstances considering the fluid motion in the porous medium governed by the generalized Darcy's law in which the convective acceleration of the fluid is taken into account. This chapter consists of two parts. The first part deals with mass transfer and free convective flow past a vertical porous plate in a rotating porous medium. We have considered the problem, formulated by Raptis [77], retaining the viscous dissipation and Darcy's dissipation terms in the energy equation. The analytical solution for the velocity, temperature, concentration and stress components are obtained. The effects of the pertinent parameters on the flow field are discussed. We find that the primary velocity gets retarding effect due to increase in R but the secondary velocity increases in magnitude with R. While both primary and secondary velocities increase in magnitude with the values of Gr, Gm and K. Further, it is observed that for constant Gr, Gm, K and R, an increase in E leads to increase both primary and seondary velocity profiles in magnitude. The results may be useful for very fluffy foam metal material as well as fibrous material. In the second part, the effects of the unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through a porous medium in rotating system are discussed. The porous medium is bounded by a vertical plane surface which adsorbs the fluid with variable suction velocity. The temperature on the vertical plane surface fluctuates in time about a non-zero constant mean and the temperature at the free stream is constant. Also, the species concentration on the plane surface and the free stream are constant. The analytical expressions for the velocity, temperature and concentration are obtained and the effects of mass transfer, frequency of temperature oscillation, rotation of the fluid, suction parameter and the permeability of the porous medium on the flow field are discussed. The present investigation is likely to have bearing on the geothermal problem of ground water flowing through a porous medium subjected to the earth's rotation.

1.5 Non-Newtonian fluids

A 'Newtonian' is one for which a linear relation exists between stress and the spatial variation of velocity. If changes in fluid density are not important, the constant of proportionality is viscosity, a characteristic constant of the material at a given temperature and pressure. So a 'Newtonian' fluid is characterized by a linear relation between stress and rate of strain of the form

$$p_{ij} = (\lambda\Delta - p)\ \delta_{ij} + 2\mu\ e_{ij}\ , \qquad (1)$$

$$e_{ij} = \frac{1}{2}\ (v_{i,j} \quad v_{j,i}\ )\ , \qquad (2)$$

where P_{ij}, e_{ij}, p, Δ denote the stress tensor, strain-rate tensor, pressure and dilatation respectively and λ, μ are co-efficients of the medium. Its theory has been extensively investigated during the last century. The relationship (1) explains reasonably well, most of the phenomena like drag, lift, skin-friction, seperation etc. occurring in the flows of fluids. However, it fails to explain the occurrence of Poynting effect [79], Merrington effect [80] and Weissenberg effect [81]. In order to explain these effects, the stress-rate of strain relation should be generalized so that the constitutive equation becomes non-linear. The fluids under this category are termed as non-Newtonian fluids. Non-Newtonian fluids form an extremely wide class of different materials, whose only common features are fluidity and a failure to obey Newtonian's viscous law [82]. The exceptions to Newton's viscous law are not of rare occurance, in fact the so-called non-Newtonian fluids are to be found close at hand everywhere. The fluids like blood, honey, condensed milk, liquid lubricants, printing inks, starch, resin, pastes, plastics, high polymers, salad dressings, butter, whipped cream and doughs, egg white, paints, certian varieties of oil and many other materials of industrial importance fall under the category of non-Newtonian fluids [83]. But the behaviour of such fluid is not so readily amenable to theoretical analysis due to the non-linearity of the stress-strain rate relations governing the fluid model and the dependence of the co-efficients occuring in them on physical properties of the fluids. Moreover, experiments are not many to throw sufficient light on the flow phenomena of this type of fluids.

The study of non-Newtonian fluids has become essential due to their importance in modern industries. This has led to the formulation of various theories of non-Newtonian fluids.We present below a brief discussion of the constitutive equations for the models of non-Newtonian fluids, viz., Elastico-viscous liquid (Model B due to Oldroyd) and Walters' model fluid B' which are directly related to our present thesis.

a) Elastico-viscous liquids (Model B due to Oldroyd)

The class of liquids which possesses a certain degree of elasticity in addition to viscosity is known as the elastico-viscous liquid. Thus, when an elastico-viscous liquid is in flow, a certain amount of energy is stored up in the material as strain energy in addition to viscous dissipation. In an inelastic viscous liquid we are concerned only with the rate of strain, but in elastic liquids we cannot neglect the strain, however small it may be, as it is responsible for the recovery to the original state and for the reverse flow that follows the removal of stress. Only in elastico-viscous liquid there is a degree of recovery from the strain when the stress is removed whereas in other liquids the whole strain remains.

Evidences of liquid elasticity in an 1.5 per cent starch solution can be observed by Hess's experiment [84] and by the recoil of air bubbles in a mixture of Polymethyl Methacrylate and Cyclohexanone (made by dissolving 3 gms perspex in 100 ml of solvent) contained in a bottle which has been suddenly turned and then brought to rest.

Oldroyd [85] formulated a non-linear theory of a class of isotropic incompressible elastico-viscous liquids with the following rheological equation of state between the stress tensor p_{ik} with the rate-of-strain tensor e_{ik} :

$$p_{ik} = - p\delta_{ik} + p'_{ik} \tag{3}$$

where

$$p'_{ik} + \lambda_1 \left(\frac{D}{Dt}\right) p'_{ik} + \mu_0 \, p'_{jj} \, e_{ik} - \mu_1 \left(p'_{ij} \, e_{jk} + p'_{jk} \, e_{ij}\right) + \nu_1 \, p'_{jl} \, e_{jl} \, \delta_{ik} = 2\eta_0 \left[e_{ik} + \lambda_2 \left(\frac{D}{Dt}\right) e_{ik} - 2\mu_2 \, e_{ij} \, e_{jk} + \nu_2 \, e_{jl} \, e_{jl} \, \delta_{ik}\right] \tag{4}$$

and e_{ij} given by the equation (2).

In equation (4) η_0, λ_1, λ_2 denote the coefficient of viscosity, stress relaxation time and strain retardation time respectively. The other five material constants μ_0, μ_1, μ_2, ν_1, ν_2 are all of dimensions of time. The quantity (D/Dt) denotes the total derivative following a fluid element taking into account its translational and rotational motion. For any tensor b_{ik}, this derivative is defined by

$$\left(\frac{D}{Dt}\right) b_{ik} = \frac{\partial\, b_{ik}}{\partial t} + v_j\, b_{ik,j} + w_{ij}.b_{jk} + w_{kj}.b_{ij}, \qquad (5)$$

where the vorticity tensor w_{ij} stands for

$$w_{ij} = \frac{1}{2}(v_{j,i} - v_{i,j}) \ . \qquad (6)$$

It may be noted that the memory of the liquid is accounted for through the stress relaxation time λ_1 and rate-of-strain retardation time λ_2 and the linearity of the Newtonian constitutive equation is broken through introduction of quadratic terms in the strain rate components and the product of the stress rate and strain rate components.

Oldroyd, in his paper [86], considered two particular types of liquid for the general model governed by the equation (4) with

$$\left.\begin{array}{c}\eta_0 > 0,\ \lambda_1 = -\mu_1 > \lambda_2 = -\mu_2 \geq 0, \\ \mu_0 = \nu_1 = \nu_2 = 0 \quad ,\end{array}\right| \qquad (7)$$

which gives liquid A, and the equation (4) with

$$\left.\begin{array}{c}\eta_0 > 0,\ \lambda_1 = \mu_1 > \lambda_2 = \mu_2 \geq 0, \\ \mu_0 = \nu_1 = \nu_2 = 0 \quad ,\end{array}\right| \qquad (8)$$

which gives B liquid.

It was observed by Oldroyd that the model represented by the constitutive equation (4) along with (8) exhibits Weissenberg climbing effect when sheared at a finite uniform rate between two co-axial cylinders and has a distribution of normal stress equivalent to an extra tension along streamlines with a isotropic state of stress in the plane normal to the streamlines. Thus the constitutive equation (4) subject to (8) retains essentially the rheological properties of a liquid. This model can predict normal stress and time-dependent visco-elastic behaviour which are in accord with physical observation. However, this model fails to account for the variation of apparent viscosity with rate of shear. Further, the constitutive equation (4) along with (8) holds for low rates of shear.

It was experimentally found by Oldroyd et al. [87] that a solution of a mixture of Polymethyl Methacrylate in pyridine obeys the constitutive equation (4) subject to (8) and for this solution $\lambda_1=0.065$ (sec) and $\lambda_2=0.015$ (sec) with $\eta_0=7.9$ poises and density 0.98 gm/ml. Thus for a real elastico-viscous liquid, the restriction $\lambda_1>\lambda_2$ is a valid one.

b) Walters' liquid B'

The constitutive equation for the Walters' liquid B' [88] (at small rates of shear) is given by

$$p_{ik} = - p\, g_{ik} + p'_{ik} \;, \tag{9}$$

$$p'^{ik}(x,t) = 2 \int_{-\infty}^{t} \psi(t-t') \frac{dx^i}{dx'^m} \cdot \frac{dx^k}{dx'^r}\, e^{(1)mr}(x',t')\,dt', \tag{10}$$

where p'^{ik} is the deviatoric stress tensor, p an arbitrary isotropic pressure, g_{ik} the metric tensor of a fixed co-ordinate system x^i, x'^i the position at time t' of the element which is instantaneously at the point x^i at time t, $e_{ik}^{(1)}$ the rate of strain tensor and

$$\psi(t-t') = \int_0^{\infty} \frac{N(\tau)}{\tau}\, e^{-(t-t')/\tau}\, d\tau \;, \tag{11}$$

$N(\tau)$ being the distributive function of the relaxation times τ. It may be noted that the elastico-viscous liquid of Model B due to Oldroyd is the special case of Walters' Liquid B', obtained by substituting

$$N(\tau) = \eta_0 \frac{\lambda_2}{\lambda_1} \delta(\tau) + \eta_0 \frac{(\lambda_1 - \lambda_2)}{\lambda_1} \delta(\tau - \lambda_1) \ , \qquad (12)$$

in equations (10) and (11). $\delta(\tau)$ denotes a Dirac delta function.

It has been shown by Walters [89] that in the case of liquids with short memories (i.e. short relaxation times), the equation of state can be simplified to

$$p'^{ik} = 2\ \eta\ e^{(1)ik} - 2\ K_0 \frac{\partial}{\partial t} e^{(1)ik} \ , \qquad (13)$$

where

$$\eta \ (= \int_0^\infty N(\tau)\ d\tau\ )$$

is the limiting viscosity at small rates of shear,

$$k_0 \ (= \int_0^\infty \tau N(\tau)\ d\tau\ )$$

is the elastic coefficient and $\frac{\delta}{\delta t}$ denotes the convected differentiation of a tensor.

It is interesting to note that the mixture of Polymethyl Methacrylate in pyridine at 25°C containing 30.5 gm of polymer per litre and having density 0.98 gm/ml fits well in the above model. For this mixture, the relaxation spectrum as given by Walters is

$$N(\tau) = \sigma\eta_0 \delta(\tau) + \frac{1-\sigma}{\beta}, \quad (0 \le \tau \le \beta) \qquad (14)$$

$$= 0 \qquad \text{for } \tau > \beta$$

where σ=0.13, η_0=7.9 poises (gm/cm.sec) and β=0.18 sec.

Very little is known, as far as we are aware, about the dynamics of free convective flow and mass transfer of rotating non-Newtonian fluids which is the subject of investigation presented in Chapter III of this thesis. We consider two problems of steady/unsteady free convective flow and mass transfer of a rotating incompressible elastico-viscous liquid under various configurations in order to make an entry into this field of study.

In the first part of Chapter III, we discuss the free convective flow and mass transfer of a rotating incompressible elastico-viscous liquid bounded by an infinite vertical porous plate when the plate is subjected to a constant suction velocity and heat flux at the plate is constant. The temperature and the species concentration at the free stream are constant. We find that the presence of the elasticity character of the liquid increases the order of the equations of motion to three from two which is for classical viscous case.

Also we observe that many researchers have analyzed the solution by the method of successive approximation taking the Newtonian solution as the initial solution following the method of Beard and Walters [90] which is valid only for small values of the elastic parameters. However, in the present investigation we have considered the solution assuming the viscous and elastic effects to be of equal importance. The analytical expressions for the velocity and stress components are obtained and the results are presented graphically. The second part of this chapter deals with the unsteady free convective flow and mass transfer of a rotating elastico-viscous liquid past a vertical porous plate subjected to a constant suction velocity and the plate temperature varies harmonically with time. The analytical expressions for the velocity, temperature and concentration are obtained and the effects of the elastic parameters and other pertinent parameters on the flow field are analyzed through the graphical representation. It may remarked for the problems (Part one and Part two) that both the primary and secondary velocity components represent higher profiles than the viscous fluid for small values of the elastic parameters ($\alpha_1=0.6$, $\alpha_2=0.2$) while as the elastic parameters (α_1, α_2) increase, the velocity components decrease.

1.6 Oscillating viscoelastic liquid flows

There are many natural phenomana such that when a fluid is set into oscillation, as in the presence of an acoustic wave or an oscillating boundary, steady streaming motions are created. These steady streaming belong to a class of secondary flows. For a purely viscous liquid this kind of phenomenon has been reported over a century ago by Faraday [91] and Rayleigh [92]. More recently, this steady streaminghas been considered both theoretically and experimentally by authors like Schlichting [93], Andres and Ingard [94,95], Stuart [96] and Glauert [97]. An extensive list of the more significant contribution has been given by Riley [98]. Stuart [99] and Riley [100] investigated the phenomenon of steady streaming flow when a circular cylinder oscillates normal to its generator in an unbounded Newtonian fluid. Problem of this nature they have described by finding inner and outer solutions; and then using some kind of matching process. Each author employed a different method of solution: Stuart extended a method due to Fettis [101] while Riley developed a series solution analogous to the Blasius series in classical boundary layer theory. Each method leads, essentially, to the same results following a similar amount of manipulative labour.

In spite of the numerous investigations of the steady streaming phenomenon in purely viscous flows, there are comparatively few investigations of this kind for elastico-viscous flows [102-107]. Frater [104] has considered the effect of elasticity of the liquid on the steady streaming produced by an oscillating cylinder. Chang et al. [108-110] have experimentally confirmed that the addition of small amount of polymer solution to a Newtonian fluid can significantly reverse the secondary flow. Chang [111] also made a theoretical study of the problem using Walters' liquid B' as model for viscoelastic liquid. However, he has enormously omitted the term $-\frac{\partial^3 u}{\partial t\, \partial y^2}$ for his boundary layer equation.

In chapter IV of this thesis we have reconsidered the problem of periodic flow due to an oscillating cylinder placed in a viscoelastic liquid using the correct boundary layer equations. As a result we find that the resulting flow field is completely different from that obtained by Chang [111]. We have also made a detailed investigation of the steady secondary flow and its final decay to zero. The analysis is patterned on the lines of Stuart [99] and Riley [100]. The problem of this nature may find useful for relative characterization of additives which reduce drag in turbulent flows.