SYNOPSIS

Mathematical modelling can be regarded as the art of applying mathematical methods in solving various problems. The methodology of developing mathematical models includes a careful formulation of the definitions of the concepts to be discussed and an explicit statement of the assumptions based upon which a particular problem is analysed. The problem taken up for analysis is formulated symbolically through the use of these definitions and assumptions, while conclusions are drawn by employing a rigourous logic, basing upon the mathematical analysis.

Mathematical methods are now-a-days being extensively employed in exploring a variety of informations in various branches of physical sciences, engineering sciences and also life sciences.

Biomechanics is a branch of science which makes use of the various principles of continuum mechanics for investigating the functional behaviour of living organisms. The aims and objectives of biomechanics include attempts to establish the mechanical and physical basis of biological activities. The interpretations of the mathematical solution contribute significantly to a better understanding of physiology. Mathematical modelling of the mechanical behaviour of various biological tissues has the potential to contribute to an accurate determination of healthy and pathological states of the tissues.

In this thesis, it has been our object to put forward mathematical analyses of several interesting problems of continuum mechanics having physiological importance. The primary objective in each case has been to study the mechanical behaviour of the respective systems. Particular attention is devoted to examine the dynamical behaviour of the skull and the fracture strength, vibration characteristics as well as remodelling properties of bone tissues.

The thesis consists of five chapters. The first chapter includes a systematic description and presentation of various topics relevant to the different investigations incorporated in the thesis.

Of concern in the second chapter is the problem of the vibration of the skull-brain system, by considering the layered structure of skull. The three layers of the skull are the outer table, diploë and the inner table. Brain is treated as an inviscid irrotational fluid. The analysis also takes care of the experimentally established anisotropic behaviour of the skull. By assuming the sphericity of the skull, the elastic behaviour of the skull has been paid due attention. Both free and forced vibrations have been analysed. Particular attention has been paid to four different types pulse shapes which are reported to be encountered in vehicular impact situations. The various pulse shapes considered here are (i) square pulse, (ii) half-sine pulse, (iii) triangular pulse and (iv) skewed impact pulse. The quantities of physical interest are computed numerically. The results are displayed through graphs and an appropriate discussion is included.

The third chapter deals with a mathematical analysis of the distribution of stresses in the neighbourhood of an exterior crack in a specimen of bone. Experimental studies on different bone specimens assert that osseous tissues are non-isotropic . This observation has been duly accounted for in the analysis. The expressions for the crack profile and the normal stress in the plane of the crack have been derived. The problem is first reduced to that of solving a pair of dual integral equations which are subsequently reduced to a Fredholm integral equation. Quantities of physical interest from the view-point of fracture mechanics are computed numerically. The results are believed to be of interest for studying the fracture strength of bone structures.

The torsional problem of a tubular bone has been analysed mathematically in the fourth chapter. On the basis of experimental observations, bone tissues have been treated as a non-homogeneous medium. A power law variation of the elastic moduli of bone tissues has been examined and thereby the frequency spectra are studied. Also the induced electric and magnetic fields due to wave propagation in the tubular bone specimen are derived.

The fifth and concluding chapter deals with the task of analysing mathematically the interesting phenomenon of bone remodelling . Metallic pins are sometimes used as a part of the prosthetic device in the surgical procedure for long bones. In this process, the pin is fitted into the medulla of a long bone as a means of attachment. As a result, the local structure of the bone specimen around the nail is altered. Bone remodelling refers to the continual processes of growth, reinforcement and resorption. By virtue of this, bone adapts its histological structure to changes in long term loading. By taking care of the non-homogeneity, anisotropicity and piezoelectricity of bone tissues (all ascertained by experimental investigators), the remodelling of bones induced due to intra-medullary nailing has been studied theoretically in this chapter. By using the analytically derived expressions, numerical values of the remodelling characteristics are computed with an aim to quantify them. The numerical results presented depict the extent to which the remodelling stresses are affected due to bone piezoelectricity.