Abstract of the Thesis

Nonlinear differential equations arise in all fields of applied mathematics, physical science and Engineering, hence being of fundamental importance the existence of methods to find their solutions. As analytical solutions are only available in a few cases, the construction of efficient numerical methods is essential. In the 1980's, George Adomian introduced a semi-analytical technique known as, *Adomian decomposition method*, for solving linear and nonlinear differential and integral equations.

In this thesis, some new modifications of the Adomian decomposition method (ADM) are studied for solving second-order singular boundary value problems (SBVPs), second-order singular Sturm-Liouville eigenvalue problems (SSLEPs), boundary value problems (BVPs) for differential equations (DEs) and integro-differential equations (IDEs). Successful application of the ADM is also shown to get exact and approximate solution of some special type of initial value problem (IVPs) for integro-partial-differential equations, namely, breakage and aggregation population balance equations.

Two new efficient methods are presented for solving second-order SBVPs with various boundary conditions. The first method, *a new decomposition method* (NDM), is based on a new two-fold integral operator and the ADM where all the boundary conditions are utilized to derive an integral equation before establishing the recursive schemes. The second method, *a decomposition method with Green's function* (DMGF), is based on a combination of the ADM and Green's function technique where we convert the original SBVPs into an equivalent integral equation before designing the recursive schemes. Thus, the two new recursive schemes are established without any unknown parameters while computing the components of the solution of SBVPs. Unlike the existing ADM or modified ADM (MADM), the proposed methods avoid unnecessary evaluation for unknown parameters, thereby reduces the computational work. More precisely, the proposed methods provide the direct recursive schemes for obtaining approximate series solution of the SBVPs. Moreover, the convergence and error analysis of the series solution obtained by the proposed methods are discussed. Convergence analysis is reliable enough to estimate the error bound of the series solution. In addition, the DMGF is successfully extended for solving second and fourth order BVPs for DEs as well as IDEs. The convergence and error analysis of the series solution of such BVPs obtained by the DMGF are discussed.

Furthermore, *a new modified Adomian decomposition method* (NMADM), for computing eigenvalues as well as eigenfunctions of singular Sturm-Liouville eigenvalue problems is studied. Unlike other numerical methods, the proposed method does not require any linearization, discretization and initial guess. It is capable of finding nth eigenvalues and eigenfunctions of the SSLEPs simultaneously. Lastly, we employ *a direct application of the Adomian decomposition method* for obtaining the exact and approximate solutions of the IVPs for integro-partial-differential equations known as breakage and aggregation population balance equations. The convergence and error analysis of the series solution using the ADM is also established.

Several examples of SBVPs, SSLEPs, BVPs for IDEs, and IVPs for integro-partial-differential equations are considered for illustration and the numerical approximations of their solutions. The numerical results show that the proposed techniques are not only simple, but also powerful and effective. In this thesis, besides numerical results, graphical illustrations are also presented to show that only a few terms of the series solution are sufficient to obtain accurate approximations to the solutions.

Keywords: Adomian Decomposition Method; Green's Function; Singular Boundary Value Problems; Approximations; Integro-Differential Equations; Boundary Value Problems; Singular Sturm-Liouville Eigenvalue Problems; Breakage and Aggregation Population Balance Equations.