Abstract

This thesis develops a few fuzzy geometrical ideas to analyze fuzzy multi-objective optimization problems. The main contribution of this thesis is twofold. Firstly, it introduces fuzzy geometrical ideas with two newly defined concepts—same points and inverse points. Secondly, the developed fuzzy geometrical ideas are applied to give a clear geometrical view on fuzzy multi-objective optimization problems.

In analogy to the well-known classical definitions, fuzzy geometrical entities are formulated. In basic fuzzy geometry, detailed studies on fuzzy reference frame, fuzzy point, linear combination of fuzzy points, fuzzy distance between fuzzy points, coincidence of two fuzzy points, fuzzy line segment, fuzzy point on a fuzzy line segment and fuzzy angle between two fuzzy line segments are proposed. Four different forms of fuzzy lines: a two-point form, a point-slope form, a slope-intercept form and an intercept form are formulated. Fuzzy circles are constructed in two different ways.

Using the developed fuzzy geometry, feasible regions for fuzzy multi-objective optimization problems are constructed on two different spaces—decision space and objective space. Under a monotone assumption on the constraint functions and on the membership functions of the coefficients, it is shown that the same points and inverse points of fuzzy geometry can highly reduce the computational cost to obtain the fuzzy decision feasible region. In the fuzzy objective feasible region, fuzzy non-dominated set is constructed. In order to capture the complete fuzzy non-dominated set, a method, named as fuzzy ideal cone method, is proposed. Formulation of the proposed method essentially depends on a classical method, ideal cone method, which is introduced to obtain the complete non-dominated set of a classical multi-objective optimization problem. The proposed ideal cone method is a non-gradient direction based sequential procedure which bears a necessary and sufficient condition for global Pareto optimality. Moreover, for any pointed convex-cone D, a simple modification of the subproblem constraint set can ensure D-Pareto optimality and more importantly ϵ-Pareto optimality of the outcome solutions. Using the proposed classical method, the entire fuzzy non-dominated set of a fuzzy multi-objective optimization problem is generated in a sequential way. After capturing the fuzzy non-dominated set, a fuzzy proper non-dominated set is identified for final decision making. The thesis is supported with several numerical and pictorial illustrations for all the introduced ideas and methodologies.

Keywords: Extension principle ∘ Fuzzy number ∘ Fuzzy geometry ∘ Fuzzy point ∘ Same and inverse points ∘ Fuzzy distance ∘ Fuzzy line segment ∘ Fuzzy angle ∘ Fuzzy line ∘ Fuzzy circle ∘ Multi-objective optimization ∘ Fuzzy multi-objective optimization ∘ Fuzzy ideal cone method ∘ Fuzzy non-dominated set ∘ Fuzzy proper non-dominated set.