Abstract

To study certain topological and geometric properties of some modular sequence spaces is our central theme in the present thesis. Geometric properties such as Kadec-Klee, rotundity, coordinatewise uniformly Kadec-Klee, uniform Opial, k-nearly uniform convex are of special interest for our study. Beside this, the results related to first Baire category set, dense and F_σ-set for some modular sequence space have been discussed. Further, completeness, Schauder basis, α-, β-, γ- duals, characterization of certain matrix mappings of some generalized paranormed difference sequence spaces are obtained. The study of scalar valued sequence spaces is carried out by several mathematicians such as Simons (1965), Maddox (1967), Iyer (1948), Woo (1973), Srivastava (1980), Musielak and Orlicz (1962), Orlicz (1963), Malkowsky and Savaş (2000), Et and Bektaş (2004), Başarir and Altundağ (2008), Savaş and Savaş (2004), Başar and Altay (2002), Altay and Başar (2003, 2006), Ahmad and Mursaleen (1987), Çolak and Et (1997), Başarir and Öztürk (2008), Mursaleen and Noman (2010), Karakaya and Polat (2010), Mursaleen and Noman (2011a), Polat et al. (2011), Demiriz and Çakan (2012) and many others either in an analogous way or in different way using various concepts. Infact, most of them have studied structural, topological & geometric properties of the spaces, inclusion relations, Schauder basis, α-, β-, γ- duals and characterization of matrix transformations among them.

Kamińska (1981, 1986) did the investigation for rotundity and uniform rotundity in Musielak-Orlicz sequence spaces. Later on, several authors such as Hudzik and Ye (1990); Hudzik and Pallaschke (1997), Cui and Hudzik (1998, 1999a), Cui and Hudzik (1999b, 2001) Cui et al. (1999), Cui and Meng (2000); Cui et al. (2005), Wang et al. (2006) and many others studied various other geometric properties, e.g., smoothness, extreme points, Maluta's coefficient, packing constant, uniform Opial property, Kadec-Klee property, locally uniform rotundity etc. for Musielak-Orlicz spaces.

The study of geometric properties are not only confined in Musielak-Orlicz spaces but also investigated for the spaces such as Cesàro, generalized Cesàro, Cesàro-Orlicz, Cesàro-Musielak-Orlicz etc. by Suantai (2003), Petrot and Suantai (2004), Petrot and Suantai (2005b), Petrot and Suantai (2005a), Sanhan and Suantai (2003), Wangkeeree (2003), Cui and Hudzik (1999b, 2001), Cui et al. (2005), Foralewski et al. (2008, 2010) and by many others. Recently, Simşek et al. (2010), Simşek (2011) have studied some geometric properties for the spaces defined by using de la Vallée-Poussin means. Šalát (1980) have obtained some results related to first Baire category set and F_σ-set for the classical sequence space l_p, $p \geq 1$ while Ewert and Šalát (1986) obtained similar results

for modular sequence spaces.

The work of the present thesis is aimed to study some topological and geometric properties of newly introduced sequence spaces $V_\Phi(\lambda)$, $V_\lambda^\varphi(\Psi, \Delta^m)$, $X(r, s, t, p; B^{(m)})$, $\Delta^{(m)} l_\alpha\{\varphi_n\}$ and $\Delta^{(m)} l^\alpha\{\psi_n\}$ defined by using de la Vallée-Poussin means, difference operator and generalized means with suitable topologies.

A scalar valued sequence space $V_\Phi(\lambda)$ is introduced by using de la Vallée-poussin means and Musielak-Orlicz function Φ as a generalization of known scalar valued sequence spaces such as Cesáro sequence space ces_p, generalized Cesáro sequence space $ces(p)$, Cesáro-Orlicz sequence space ces_φ, Cesáro-Musielak-Orlicz sequence space ces_Φ, Orlicz sequence space l_φ, Musielak-Orlicz sequence space l_Φ etc. Some equivalent conditions for non-triviality of $V_\Phi(\lambda)$ are obtained. It is proved that $V_\Phi(\lambda)$ endowed with the Amemiya norm has the Fatou property and consequently it is a Banach space. Separability, order continuity as well as sufficient criteria for geometric properties such as the coordinatewise uniformly Kadec-Klee, Uniform Opial, k-nearly uniform convex for the spaces $V_\Phi(\lambda)$ are obtained.

To generalize known sequence spaces such as T_φ^{a*}, T_φ^a, $[V, \lambda]_0$ and $[V, \lambda]_0(\Delta^m)$ studied by earlier authors, modular spaces of difference sequences $V_\lambda^\varphi(\Psi, \Delta^m)$, $\widehat{V_\lambda^\varphi}(\Psi, \Delta^m)$ are introduced. These are defined by using de la Vallée-poussin means, m-th order difference operator Δ^m and φ-functions. It is shown that the sequence space $V_\lambda^\varphi(\Psi, \Delta^m)$ has a linear structure. By defining a suitable topology on $V_\lambda^\varphi(\Psi, \Delta^m)$, it is proved that $V_\lambda^\varphi(\Psi, \Delta^m)$ a Frèchet space. The countably and uniformly countably modulared spaces have been introduced by using de la Vallée-poussin means and sequence of φ-functions. Equality and completeness for these spaces are also investigated.

An unified class $X(r, s, t, p; B^{(m)})$ for $X \in \{l_\infty(p), c(p), c_0(p), l(p)\}$ as a generalization of known classes is defined by using generalized means and generalized B-difference operator $B^{(m)}$. A suitable topology on $X(r, s, t, p; B^{(m)})$ is defined under which it is a complete paranormed space. The Schauder bases for the spaces $X(r, s, t, p; B^{(m)})$ for $X \in \{c(p), c_0(p), l(p)\}$ are obtained. The characterization of matrix transformation from $l(r, s, t, p; B^{(m)})$ to l_∞ is established. Finally, it is shown that the sequence space $l(r, s, t, p; \Delta^{(1)})$ is rotund for $p_n > 1$ and has the Kadec-Klee property for $p = (p_n), p_n \geq 1$ for each $n \in \mathbb{N}_0$.

For any sequence of non-degenerate Orlicz functions $(\varphi_n)_{n=0}^\infty$ and $(\psi_n)_{n=0}^\infty$, some modular difference sequence spaces $\Delta^{(m)} l_\alpha\{\varphi_n\}$ and $\Delta^{(m)} l^\alpha\{\psi_n\}$ are defined by using the m-th order difference operator $\Delta^{(m)}$ and a sequence $(\alpha_n)_{n=0}^\infty$ of strictly positive real numbers. It is shown that these spaces are separable Banach spaces and has Schauder basis. The results related to dense, F_σ-, first Baire category set in s are investigated.

Keywords: Modular space, Musielak-Orlicz function, de la Vallée-poussin means, Luxemberg norm, Amemiya norm, Kadec-Klee property, Uniform Opial property, Nearly uniform convexity, Rotundity, Riesz weighted mean, Difference sequence space, φ-function, Sequential modulus, Generalized means, α-, β-, γ-duals, Matrix transformations, Baire category.