Chapter 1

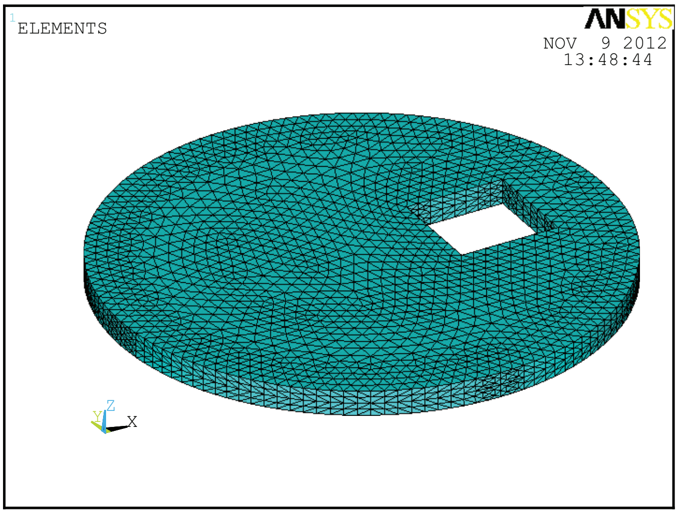
Introduction

This thesis is motivated by an engineering design problem. Consider two possible designs (shapes and dimensions) for some engineering component, to be made of some known, lightly dissipative material. Which design has better vibration damping?

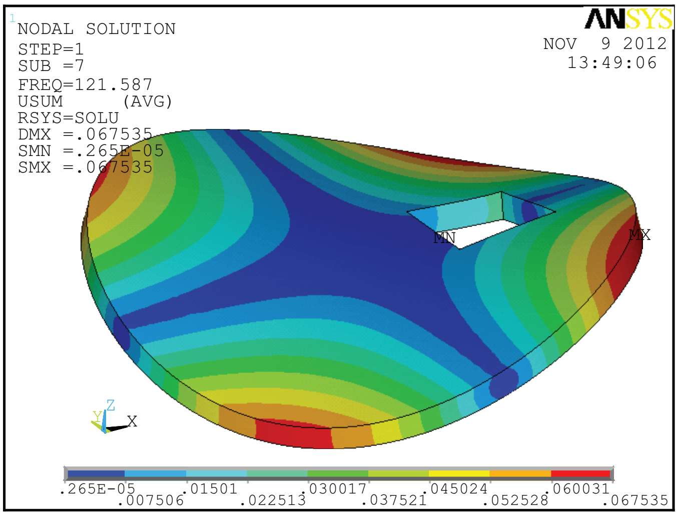
The above problem can be relevant to many engineering design situations. Design of automotive components for reduction of the vibration levels is an easy example. Another example would be to control the vibration levels in spacecraft components to improve the accuracy of optical and guidance systems. Improvement of damping behavior of aircraft and space structures equipments still remains an important topic. Therefore, the above question has its own importance.

One can think of using modern computational facilities to address this question. A computational approach would begin with straightforward finite element based modal analysis. However, the commercial finite element packages give only frequencies and mode shapes. Computation of damping values is not routine in such packages. Computation of damping requires a constitutive relation for material damping under time-periodic triaxial inhomogeneous stresses. As discussed below, such a relation is not presently available.

To fix ideas, let us consider the solid object of Fig. 1.1 (a), modeled using a finite element package, with its first mode as shown in Fig. 1.1 (b) (details of this model will be discussed in section 2.5.4). We want to compute the damping ratios for the first several modes of such an object. To do that, we require a multiaxial damping formula that can be used along with the modal analysis results. Such a multiaxial damping relation is not presently available in the literature.



(a)



(b)

Figure 1.1: (a) Finite element model of an arbitrarily chosen object. (b) Its first vibration mode.

In this context, in this thesis we have developed, for the first time in the literature, multiaxial macroscopic damping models based on assumed underlying micromechanical models of internal dissipation. In addition, we have demonstrated how these macroscopic models can be used to compute modal damping of any arbitrary solid object within a finite element environment. Therefore, this thesis adds to the existing knowledge of material damping within the framework of multiaxial dissipation in structural components.

Having discussed briefly the goal and the contributions of this thesis, we now give a detail discussion of the topic and the relevant literature below.

1.1 Material damping and relevant prior work

Solid bodies undergoing cyclic deformation lose energy. Some energy is lost to the surrounding atmosphere; some goes into supports, joints, and such other dissipative elements; and the rest is dissipated inside the material. This thesis considers the latter means of dissipation, known as material damping or internal friction.

In our study of material damping, we have considered solids undergoing harmonic stress cycles and the energy dissipated therein. It has been empirically observed that, in many solids, the energy dissipated per unit volume and per cycle of deformation is proportional to the stress amplitude raised to some power $m \geq 2$, and largely independent of frequency in the low frequency range (say, on the order of 100 Hz or less). This can be written as

$$D = \xi \sigma_{eq}^{m} \tag{1.1}$$

where D stands for specific material damping, σ_{eq} represents a suitable stress amplitude, and ξ is a material constant. For comparison, we note that the commonly assumed linear viscous damping (such as a $c\dot{x}$ term representing damping in a harmonic oscillator) leads to per-cycle dissipation proportional to frequency; while a macroscopic rate-independent dry friction element gives dissipation with $m = 1$. Thus, Eq. (1.1) is not as intuitively obvious as it might initially seem.

There is a huge literature on damping. We have considered works only directly relevant to Eq. (1.1).

Frequency independence in material damping was first observed by Lord Kelvin (1865). Rowett (1914) reported careful static and dynamic experiments in torsion of thin walled tubes and observed frequency independent behavior. He also reported power law damping with $m = 3$ for the tubes as supplied, and found increased values of damping for annealed tubes. For the annealed tubes, his numerical data suggests $2 < m < 3$.

Broader experiments with several different materials involving horizontally supported spinning rods with vertical end loads, were reported by Kimball and Lovell (1927). Internal dissipation caused lateral deflection of the rod. Both frequency independence as well as a rough power law were observed ($m \approx 2$). The authors cited some other data that suggests $m > 2$.

Mead and Mallik (1976) suggested quite reasonably that replacing stress with strain in Eq. (1.1) leads to better units for ξ. Although they used an empirical dissipation model with a sum of two power-law terms, their actual experimental results for harmonic torsional oscillations would pass as straight lines on log-log plots, with $2 < m < 3$ (see their figures 4 through 6, reproduced below in Fig. 1.2 by manually extracting the data from their figures). Interestingly their results show that m can have non-integer values.

In a more recent experimental work, Maslov and Kinra (2005) have reported frequency independence (within their experimental scatter) in carbon foams over a large frequency range. Though nonlinear damping behavior was observed for large amplitudes, a power law was not fitted and appears inappropriate. Their data adds one more solid material with frequency independent damping to the remarkable list begun by Kimball and Lovell (1927).

When we come to modeling dissipation, we first mention Lazan's well known book in 1968 (Lazan, 1968). He took a phenomenological approach, noted power-law behavior for intermediate stress ranges for several materials (including fractional powers), and also discussed a variety of simple models. These models included elements with friction and plasticity, but not the statistical distribution of strengths that we assume here in order to theoretically obtain the power law behavior with $m \geq 2$.

From a more theoretical background, Granato and Lücke (1956) proposed an explanation of material damping based on dislocation pinning by impurity particles. Dawson (1978) considered an unknown function of nondimensionalized stress, for-

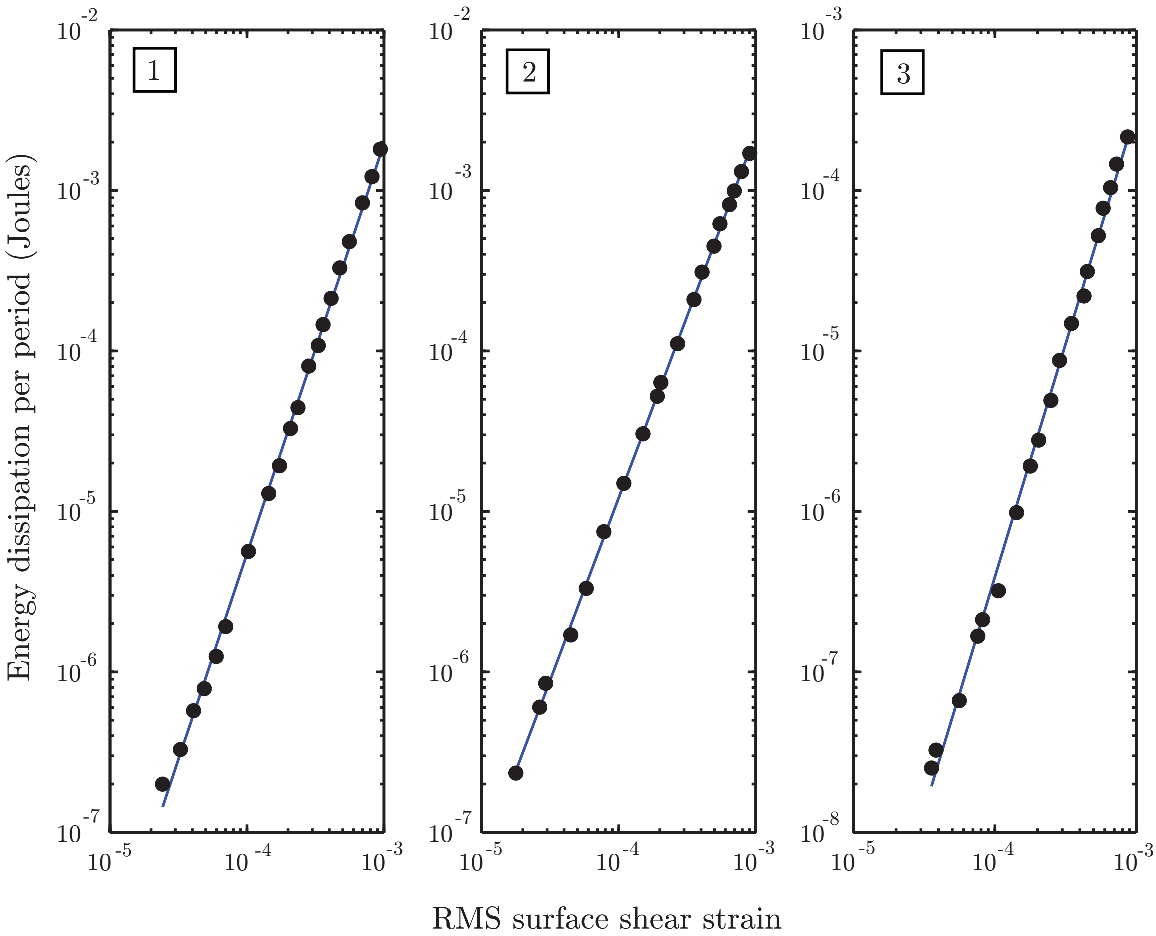


Figure 1.2: Figures 4 to 6 of Mead and Mallik (1976) are reproduced here by manually extracting the harmonic excitation data (black circles) from those figures. The numbers 1, 2, and 3 here correspond to figures 4, 5, and 6 respectively in their paper. The fitted straight lines on these log-log plots show that m is between 2 to 3 (2.56 for figure 4, 2.27 for figure 5, and 2.90 for figure 6).

mally expanded in a Taylor series using even powers only, leading by assumption to $n = 2$ for small stresses.

In addition to the above empirical and theoretical papers and approaches, there are some *procedural* papers that develop *ad hoc* one-dimensional nonlinear differential equation formulations that model frequency independent dissipation. These approaches could be useful in time domain simulations of damped systems. We mention Muravskii (2004) and Spitas (2009) as examples. In the context of such papers, we note that while $m = 2$ in the power law whenever the damping is linear, the converse is not true: nonlinear damping relations can give $m = 2$ as well.

We note an important point here that the literature we discussed above only considers dissipation in uniaxial cases. However, we are interested in dissipation in multiaxial stress states. No convincing mechanically-based engineering model for the same is presently available. For example, the stress state in Kimball and Lovell's experiment (Kimball and Lovell, 1927) was predominantly uniaxial. The empirical laws in Lazan (1968) also do not identify the equivalent stress of Eq. (1.1) under arbitrary triaxial load. Dislocation-based models as in Granato and Lücke (1956) involve several parameters related to the crystal structure, yet to be translated into measurable external macroscopic model parameters. The approach of Dawson (1978) also does not identify the role of multiaxial stresses in the dissipation model. As a final example, Hooker (1969) proposed that the equivalent stress amplitude should be computed as

$$\begin{aligned} \sigma_{eq}^2 &= (1-\lambda)(I_1^2 - 3I_2) + \lambda I_1^2 \\ &= (1-\lambda)\sigma_d^2 + \lambda\sigma_v^2, \quad \text{with } 0 < \lambda < 1, \end{aligned} \tag{1.2}$$

where I_1 and I_2 are the first and second stress invariants respectively, σ_d^2 and σ_v^2 are proportional to the distortional and dilatational strain energies respectively, and λ is a fitted parameter. The above is motivated by the fact that it is a linear combination of distortional and dilatational strain energies[1]. Similar combinations and interpolations were considered, with varying degrees of experimental support, by several other authors, including Robertson and Yorgiadis (1946), Whittier (1962), Torvik *et al.* (1963), and Mentel and Chi (1964). These attempts to incorporate multiaxial stresses are given below.

Robertson and Yorgiadis (1946) studied several materials (metallic and non-metallic), and found that dissipation per cycle was frequency independent and proportional to the *cube* of a suitable stress amplitude (unlike Kimball and Lovell (1927) but matching Rowett (1914)). More interestingly, Robertson and Yorgiadis (1946)

[1]It is tempting to conclude that such a split is intuitively obvious but we will show reasonable micromechanics that predicts otherwise.

sought an equivalence between damping in extension and torsion of tubes, and found that in order for the dissipation under the two kinds of loads to be identical, the shear stress in the torsion experiment needed to be k times the longitudinal stress in the extension-compression experiment, with k ranging from 0.48 to 0.60. It is notable that if damping was a function of distortional strain energy alone, then k would be 0.577.

Whittier (1962) presented a study of vibration amplitude decay rates of circular steel plates and rectangular steel beams in vacuum. The experiments were well conceived and executed with the objects supported on nodal circles and nodes respectively, and displacements measured using noncontacting capacitive probes. In our opinion, the attention to possible sources of error and demonstration of their smallness, just in themselves, make this excellent paper well worth reading. It was found that for somewhat larger stresses dissipation varies as the cube of stress amplitude, and that both distortional and dilatational strain energy contribute to the observed macroscopic damping.

Torvik *et al.* (1963) discussed biaxial test results for several materials. The tests, done at the University of Minnesota, involved axial and torsional loading of tubes through crank arrangements. Load and deformation measurements were through strain gages. Dissipation was measured by the area of the hysteresis loop for a range of principal stress ratios, and was therefore limited to somewhat larger dissipation ranges. The empirical theory proposed was based on dissipation through small plastic deformation, and biaxial damping models proposed were motivated in their mathematical form by various failure theories for engineering materials, with variable degrees of success. Different qualitative behaviors were obtained for different materials, the theories used were not developed from underlying micromechanics, and the results cannot be easily compared with ours.

Subsequent related experiments were performed by Mentel and Chi (1964), also in Minnesota. Damping was measured in thin-walled cylindrical specimens (made of manganese-copper alloy) subjected to combined internal pressure and axial loading. Crank assemblies were used for the load application. In this experiment, the biaxial stress ratio ($-1 \leq \sigma_2/\sigma_1 \leq 1$) covered a larger range than in Torvik *et al.* (1963). Dissipation was measured by measuring areas of hysteresis loops. The results showed a small contribution of dilatational strain in the dissipation.

Several *ad hoc* damping models proposed by these *biaxial* experimental measurements were later combined by Hooker (1969) to prescribe a formula (given in Eq. (1.2)) based on a weighted sum of the two strain energies (distortional and dilatational). No physical basis was presented for this model. Experimental investigations in this area were again reported, more recently, by Hooker and co-authors (see Hooker (1981), Hooker and Mead (1981) and Hooker and Foster (1995)). These studies were mostly targeted towards devising experimental setups for careful damping measurements in biaxial settings, and not geared towards formulating constitutive models for general triaxial stress states. Their experimental improvements mostly lie in lowering extraneous losses, achieving more uniform stress distributions for a full range of stress ratios, application of mean loading, simplicity in operation, etc.

In summary, we can say that there have been several prior studies of dissipation under biaxial stress states, especially for materials whose internal dissipation is relatively high or at stress levels where the dissipation is high. Theoretical development of models such as we are going to present in this thesis has been missing, as has any experimental verification of any model under triaxial stress states. Data for materials with low dissipation, in addition, suffers from large scatter as expected.

1.2 Contributions of the thesis

We conclude from the above discussions that it is worthwhile, both academically as well as towards possibly designing special damping materials, to develop physically based multiaxial damping models that lead to Eq. (1.1) with $m \geq 2$. In this thesis, we have developed multiaxial damping model that rationalizes, physically and mathematically, the material damping behavior of Eq. (1.1). Then those dissipation models are used in finite element prediction of modal damping ratios in arbitrary solid objects.

Our damping models may be summarized as follows. We assume that the dissipation within the vibrating object is due to a multitude of random distribution of microscopic, rate-independent, dissipation sites or flaws. We consider two mathematically simple micromechanical dissipative phenomena that can be modeled as rate-independent: (i) Coulomb friction, and (ii) ambient-temperature plasticity. In the first, we consider a flat crack in an elastic material, with Coulomb friction between the crack faces. In the second, we consider dissipation due to microscopic elasto-plastic flaws.

For the first model with frictional microcracks, we begin with a single-crack embedded within a linearly elastic solid under far-field time-periodic tractions. The material is assumed to contain many such non-interacting microcracks. Single-crack simulations, in two and three dimensions, are conducted using ABAQUS. The net cyclic single-crack dissipation under arbitrary triaxial stresses is found to match, up to one fitted constant, a formula based on a pseudostatic spring-block model. We use that formula to average the energy dissipation from many randomly oriented microcracks using Monte Carlo averaging for arbitrary triaxial stress. We develop a multivariate fitted formula using the Monte Carlo results. The fitted formula is used in finite element simulation of solid objects for the computation of the net cyclic energy dissipation via elementwise integration. The net dissipation is then used to compute equivalent modal damping ratios. We note that this model in the absence of pre-stress always gives $m = 2$ in Eq. (1.1).

In the second approach, we consider dissipation due to distributed microscopic elasto-plastic flaws. For analytical tractability, we choose ellipsoidal elastic perfectly-plastic flaws or inclusions which is embedded inside an elastic matrix. We use finite element simulation in ABAQUS to obtain the state of stress within the ellipsoidal inclusion under far-field cyclic loads. We also develop a semi-analytical method with the help of Eshelby's (Eshelby, 1957) formula for inclusion problems. Computed results using this semi-analytical approach are found to match, with excellent accuracy, the finite element simulation results. We identify two limiting special cases from these simulations and consider them for our analytical development of macroscopic dissipation formulas. In the first limiting case, plastic flaws or inclusions are assumed to be of spherical shape where the single flaw dissipation is governed by J_2, the second deviatoric stress invariant, which is proportional to the distortional strain energy. In the second limiting case, we consider flaws that are near-flat and thin wherein the dissipation is governed by the far-field resolved shear stress applied on the plane parallel to the near-flat surface of the flaw. In both these cases, we assume a random distribution of flaw strengths and orientations, and obtain the net macroscopic dissipation analytically. We show that in these dissipation models arbitrary $m > 2$ can be incorporated. We also find from the first case that the net dissipation is exactly described by a power of the distortional strain energy. However from the second limiting case, we show that, when m is between 2 and 6, the net dissipation is accurately described by a power of the distortional strain energy (and exactly so, for $m = 2$ and 4). For large m, separate asymptotic formulas are found for this second case, showing that the dissipation deviates from a function of distortional strain energy alone.

We finally suggest that for engineering modeling purposes of metallic structures, for moderate m, a simple power law based on the distortional strain energy might be reasonable. We then demonstrate use of this dissipation model to compute the modal damping ratios of arbitrary solid objects using commercial finite element package ANSYS.

We first use $m = 2$ in the distortional strain energy formula for the computation of effective damping ratio (ζ_{eff}). The effective damping ratio is a measure of equivalent viscous damping in any lightly damped structure. We first use solid element (SOLID187) in ANSYS for our ζ_{eff} computation. Our finite element calculation of effective damping uses modal analysis results from ANSYS complemented by our own volume integrals for the dissipation. The results are then verified with known analytical results for the cases of several analytically tractable geometries. An interesting aspect of these damping results is that the ζ_{eff} shows a variation of over one order of magnitude over the cases we have considered even when the power (m) is 2. The torsion dominated mode has the greatest damping and for the radial mode of the solid sphere the damping ratio is the least of all.

Subsequently, noting that many engineering components can have thin-walled geometries, we extend our ζ_{eff} computation using shell elements (SHELL181) in ANSYS. The net cyclic dissipation in this case is also calculated from modal analysis followed by our own appropriate volume integrals. The results are verified with known analytical results.

Finally, we consider arbitrary $m \geq 2$. The ζ_{eff} results in these cases are normalized with respect to the average volumetric strain energy density for clean (size-independent) comparison. Effective damping results for various m values are reported for a pair of objects of less analytically tractable shape.

In the course of this work, we have also developed an automated Matlab based graphical user interface (GUI) for fast and reliable computations of the effective damping ratios using Matlab's graphical user interface development environment (GUIDE). Development of this GUI makes use of the advantage of ANSYS's APDL (ANSYS Parametric Design Language) and its easy interface with Matlab. Some details of this practical contribution are provided.

Therefore, the main contributions of this thesis may be viewed as following.

1. We develop, for the first time, multiaxial damping formulas based on micromechanically motivated rate-independent dissipative phenomena as opposed to *ad hoc* proposals like Eq. (1.2).

2. We show that Eq. (1.1) can result from dispersed microscopic dissipation sites in an appropriate statistical framework; and therefore, Eq. (1.1) is less arbitrary than it might initially seem.

3. We develop two multiaxial damping models considering two mathematically simple rate-independent dissipation phenomena. The first model considers dissipation due to frictional microcracks whereas the second model involves dissipation in elasto-plastic flaws.

4. Based on a theoretical study of materials with small elasto-plastic flaws we sug-

gest that for engineering modeling purposes of metallic components, a simple power law formula based on the distortional strain energy might be reasonable when the index (m) of the power law is between 2 to 6.

5. We finally demonstrate how these multiaxial damping formulas can be used for finite element computation of modal damping ratios of arbitrary solid objects using both solid and shell element in ANSYS.

1.3 Outline of the thesis

We give an outline of the rest of the thesis here. As discussed earlier, in this thesis, we initially develop multiaxial damping models based on physically based rate-independent damping mechanisms. And then, we have demonstrated how these models can be used in a finite element environment to compute modal damping of any arbitrarily shaped solid body.

In **Chapter 2**, the entire development of the dissipation model based on the frictional microcracks is discussed. Use of such formula in finite element computation of modal damping of an arbitrarily shaped solid object using solid elements in ANSYS is also presented for completeness. In **Chapter 3**, dissipation within a single elasto-plastic flaws has been discussed using both finite element simulations and a semi-analytical approach. **Chapter 4** discusses the development of the other multiaxial dissipation formula considering the dissipation due to a multitude of elasto-plastic inclusions. In **Chapter 5**, we adopt a distortional energy based damping formula and discuss in details the finite element computation of the effective damping ratios. We discuss both solid and shell element formulation for this dissipation model and verify our computation with known analytical results. In **Chapter 6**, the summary and the major conclusions of the present thesis have been presented.