INTRODUCTION

During recent years the theory of elasticity has found a wide application in the solution of engineering and technological problems. In three dimensional elasticity problems the solutions are available only for bodies of simple shapes. Most of the known solutions are those of axisymmetrical stress distribution problems. The classical examples are the Kelvin's problem (52) for the sphere and Boussinesq problem (16) for a semi-infinite region bounded by a plane. Certain problems of solids of revolution have been considered by Michell (70), Tedone (133), Morgan (78,79), Shapiro (110), Sneddon (115), Green (38) and Sternberg, Eubanks and Sadowsky (125,126). Papkovich-Neuber stress functions have recently been used for solving the problems of equilibrium of elastic shells by Lourje (65) and parallelopiped by Filonenko-Borodich (30,32). Few problems of stress concentration about an ellipsoidal cavity have been considered by Sternberg and Sadowsky (122,123). But the general three dimensional theory of elasticity is one of considerable analytical difficulty and still lacks methods of wide application. When one dimension of a body can be considered as small (large) in comparison with the other dimensions, plate theory or two dimensional theory of elasticity can be used to obtain a useful information about the stress distribution.

Plate problems occur very frequently in engineering structures and designing. For example a variety of problems concerning the bending of circular plates is encountered in the design of boilers, locomotive engines and steam turbines. Some engineering applications of circular ring plates are for diaphrams and high pressure expansion members and plate valves in compressors. Steel plates of ship hulls submitted to the action of water pressure, concrete and reinforced concrete slabs under the action of lateral loading are other notable examples. Thin walled structures are finding a wide application in the modern development of airplane structures and thin walled tanks and containers submitted to the action of internal or external pressure.

Two dimensional biharmonic problem occurs in plate and plane elasticity problems. Plane problems attracted considerable importance when it was recognised in 1907 that the first boundary value problem was completely equivalent to the fundamental biharmonic problem. The fundamental biharmonic problem is the problem of determining a biharmonic function for given values of its derivatives on a contour. This problem is intimately connected with the transverse deflection of a clamped plate in plate theory. The analogies between the plate problems and plane elasticity problems are noted by various authors. This enables the methods developed for one class of problems to be applied to the other class. It has been noted in Muskhelishvili's book (80) that clamped (free) plate problem corresponds to first (second) boundary value problem of plane elasticity.

A general correspondence of the plate problems with the plane problems has been discussed by Southwell (121) and in more detail by Prager (94) recently.

Various methods of attack have been employed to tackle both plate and plane problems by several investigators.

In plane problems the wellknown classical method of approach is that of Airy stress function, by which stress distribution is determined from a single function. Many problems have been solved by this method in cartesian coordinates by Levy (63) and Filon (31); elliptic coordinates by Timpe (138), Coker and Filon (20); polar coordinates by Michell (70) and Timoshenko and Goodier (137); bipolar coordinates by Jeffery (50), Mindlin (175), Karunes (51), Sengupta (104) and Saleme (97). Nash (86,87) has used a similar approach to discuss the bending of elliptic and elliptic ring plates loaded by edge moments, by using elliptic coordinates. Bipolar coordinates have been employed by Krieger (56) recently to discuss the transverse bending of certain thin plates.

The method of transforms has been used by number of investigators. The notable classical example is that of wellknown Levy's solution (136). The advantage here is the well developed theory of representation of an arbitrary function as a Fourier series or integral. Fourier transforms have been used by Jaramillo (49) to obtain the exact solution in terms of improper integrals for the deflection and moments due to a transverse concentrated load acting at an arbitrary point of an infinite cantilever plate and by Sneddon (115,116,117) to develop solutions of a wide class of problems for the half plane with a straight boundary and some problems for the infinite strip. Mellin transforms have been employed by Krieger (57) and Koiter (53) for infinite sectorial plate problems and by Tranter (129) for wedge problems. The stress distribution in a parabolic plate has been obtained by Paria (92) by assuming stresses in the form of Fourier integrals. Fourier sine transforms have been used by Fletcher and Thorne (33) to discuss the problem of bending of thin rectangular plates on elastic foundations and by Deverall and Thorne (27,28) to obtain the solutions for normally loaded rectangular and ring sector plates subjected to various physically important edge conditions.

The method of inversion has been introduced by Michell (71) to solve plane stress and plate problems. In addition to the basic work by Michell, mention should be made of the papers by Fillunger (29), Sonntag (120), Sternberg and Eubanks (124) and Olszak (88,89,90) who generalised this method to obtain solutions of a certain class of new problems. This method has been recently applied by Olszak and Mroz (91) to obtain the solutions for several cases of bending of circular plates with eccentric holes.

The solutions of boundary value problems for plane regions of arbitrary shape and also solutions of certain new types of plane problems, for example those of composite bodies, are developed by means of integral equations by Mikhlin (72,73) and Sherman (111,112,113,114).

Complex variable methods developed mainly by Kolosoff (54,55), Muskhelishvili (81,82,83,84), Lekhnitzky (60,61,62) and Stevenson (127,128) are most widely used for plane and plate problems. Complex variable methods have been employed in one form or another by Sen (98,99), Milne-Thomson (74), Lurie (67), Green (39,40), Holgate (46,47), Seth (109), Livens and Morris (64), Conrad (21), Rothman (96), Tiffen (134,135) and a host of other workers. Sen (98,99) used complex function theory and potential theory to develope the method of direct determination of stresses from the stress equations. This method has been further developed by him (100) and Sengupta (105) and it has been used by Sengupta (105,106) and Arkilic (3) for solving certain particular problems. Muskhelishvili and Stevenson have expressed the unknown biharmonic equation, in terms of two functions of a complex variable. Corresponding to given boundary conditions these functions fulfil certain functional relations on the boundary of a given region. The complex potentials are then determined from these functional relations. Different investigators have solved these functional relations for the complex potentials, proceeding on independent lines. Stevenson's tentative method is to transform the functional equations to curvilinear coordinates, suitable for the boundary of the cross-section and to assume the possible forms of the complex potentials which would fit the functional relations for certain definite values of the constant coefficients involved in the complex potentials. This method has been extensively used by him (127,128) and many others. Green proceeds on different lines to define two functions of the complex variable in such a way that the real part of one and the imaginary part of the other give the edge stresses from which he determines the solutions with the aid of Fourier integrals. He (41,42) has used this method to solve certain first boundary value problems.

The general and direct methods of obtaining the complex potentials have been developed by Muskhelishvili and Sherman who have used these methods for solving plane problems. Later these methods have been applied to plate problems by Lekhnitzky. These methods have been discussed in detail in the two books by Muskhelishvili (80,85) and also in a book by Green and Zerna (43). Applications of these methods are abondant in the recent literature on the subject.

These methods supplemented by additional developments have been employed by Winslow (144) to find the stress solutions for rectangular plates. Radok (95) has used these methods to deduce a general method for the solution of problems of reinforced cutouts in infinite thin sheets. Hoskin and Radok (48) have solved the problem of the root section of swept wing by using Muskhelishvili's method. Paria (93) and Madan Mohan (68) have used complex function theory to solve certain mixed boundary value problems of plane elasticity. Complex variable techniques have been employed in a series of papers by Yu (145,146,147,148,149) to solve certain plane and plate problems. Bending of flat slabs suproted by square-shaped columns, and clamped, has been discussed by Krieger (58) by making use of complex variable methods. Aggarwala (1,2) has used complex function theory and vortex analogy to calculate the shears due to a concentrated load and deflections due to concentrated moments applied at particular points of rectilinear plates. Solutions for a number of important boundary value problems of thin plates have been obtained by Buchwald and Tiffen (18), Buchwald (17) and Gladwell (35,36) by using complex variable techniques. In a recent paper adoption of Muskhelishvili's method for clamped rectilinear plates has been discussed by Deverall (26). Complex variable methods have been successfully employed in a series of papers by Bassali and Dawoud (14) and Bassali (4,5,6,7,8) for discussing the bending of partially and normally loaded circular plates subjected to general boundary conditions which include the usual clamped and simply supported cases. Bassali (9,10,11) has also discussed the problems concerning the bending of partially and normally loaded infinite and semi-infinite plates by employing the same methods. Muskhelishvili's method has been used by Tamate (131,132) to discuss the transverse flexure of infinite and semi-infinite thin plates containing an infinite row of equal circular holes.

Complex variable methods have been used in the present work which includes the solution of the following problems:

(1) Stresses in certain thin plates rotating steadily about an axis lying in their middle plane. The plates of following shapes have been considered:

(i) Curvilinear polygonal plates, the following particular cases of which have been discussed in detail:

a) Circular plates, b) Dumb-bell shaped plates,

c) Cycloidal shaped plates,

d) Cog-wheel shaped plates,

(ii) Plates in the shape of a cardioid.

(2) Gravitational stress distribution in a heavy cyclinder with a central hole, resting on a horizontal plane. The general solution obtained contains the solution for the following practical shapes of the cross-sections of the cylinder:

a) Circular ring, b) An elliptic ring,

c) Approximately an ellipse or an ovaloid with an ovaloidal hole,

d) Approximately a circle with a square hole with rounded corners.

(3) Bending of certain thin plates by concentrated edge couples and forces. The problem includes the solution of several shapes of plates, namely:

(i) Plates which can be mapped onto the unit circle by polynomial type mapping functions with particular reference to regular polygonal plates subjected to arbitrary number of edge couples and forces. The following problems are considered in detail:

(a) approximately square plates subjected, respectively, to two bending couples, to two twisting couples both applied at the ends of a diagonal and to four concentrated forces applied at the four corners.

(b) approximately equilateral triangular plates subjected, respectively, to three bending couples, to three twisting couples both applied at the three corners and to six concentrated forces applied at the corners and middle points of the sides.

(ii) Rectilinear regular polygonal plates with a central circular hole subjected, respectively, to two bending couples, to two twisting couples both applied at two opposite points of the outer boundary.

(iii) Circular ring plates subjected, respectively, to two bending couples, to two twisting couples both applied at the ends of a diameter and to four concentrated forced applied at the ends of two perpendicular diameters.

(4) Bending of normally loaded circular ring plates. The plates are elastically restrained at both the boundaries and are subjected to normal loading of the type

$$p = p_0 r^{n-2} \cos s\theta \quad (n \geq 2,\ s = 0, 1, 2, \ldots)$$

over the whole plate.

(5) Bending of certain clamped thin curvilinear plates normally loaded over or along a concentric circle. The circular plate, plate bounded by an inverse of an ellipse with respect to its centre, an inverse of an ellipse with respect to its focus, a cardioid and a plate clamped along two semi-infinite lines which are the only boundaries of the plate are included as special cases.