Abstract

This thesis, as addressed in its title, analyzes the inflation mechanics and dynamics of a hyperelastic toroidal membrane of initially circular cross–section. The membrane material is considered to be homogeneous and isotropic Mooney–Rivlin solid.

First, we study the finite inflation analysis of a toroidal membrane of initially circular cross–section under internal pressure. The inflation problem is formulated as a variational problem for the total potential energy comprising of the strain energy with additional terms due to the transverse isotropy and internal energy of the gas. The problem is then discretized and solved up to high degree of accuracy through a sequence of approximations based on the Ritz expansion of the field variables combined with a potential energy density perturbation and Newton–Raphson method. Next, we reformulate the inflation problem with different set of field variables and the equilibrium equations obtained are solved by direct integration of the differential equations using shooting method by employing Nelder–Meads search technique. The results obtained are compared with those obtained using the discretization method. The effect geometric and material parameters on the finite inflation of a toroidal membrane have been studied.

Moving onto the contact inflation, we study the finite inflation of the toroidal membrane on a cylindrical rim. Both, compliant and rigid frictionless rims have been considered. The two point boundary value problem is solved using shooting method with two (for compliant cylinder) and three (for rigid cylinder) dimensional search space for the optimization function. The effect of the inflation pressure, material parameters and elastic properties of the contact surface on the state of stretch and geometry of the inflated torus have been studied.

Next, we first study the general small amplitude perturbation (out–of–plane and in–plane) dynamics of a toroidal membrane around the finite inflation configurations. The effect of the inflation pressure on the eigenspectrum and modes of vibration of the membrane have been studied. Finally, we study the in–plane dynamics of an inflated toroidal membrane in a simplified setting. The covariant form of the governing equation for the general in–plane small amplitude displacement field has been derived from the variational formulation which clearly shows the effect of curvature on the dynamics. The curvature in the equation of motion may be interpreted as quadratic potential in the Lagrangian with a coupling proportional to the Ricci curvature scalar of the membrane. The variational problem is discretized, and is analyzed to obtain the eigenfrequencies and mode of vibration. The effect of anisotropy on the modal dynamics of the torus has also been studied.

Keywords: Toroidal membrane, large deformation, impending wrinkling, shooting method, frictionless contact, cylindrical rim, out–of–plane dynamics, in–plane dynamics, curvature elasticity, anisotropy, general perturbation