Abstract

Semi-inner product is a useful tool to establish Hilbert space like theory in Banach spaces. The main objective of the thesis is to study theory of frames, numerical range of operators, variational inequality and variational inclusion problems in semi-inner product spaces.

The thesis is comprised of eight chapters along with a chapter containing concluding remark and scope of future research. Chapter 1 is the introductory chapter, where brief introduction and adequate literature survey on semi-inner product spaces, frames, numerical range of operators, variational inequality and variational inclusion problems have been presented. In Chapter 2, we have introduced the theory of frames in classical semi-inner product spaces. We have studied Parseval frame, tight frame and established their reconstruction formulae without using the inverse frame operator. In Chapter 3, we have introduced the theory of frames in generalized semi-inner product spaces. We have studied X_d -Bessel sequence, X_d^* -Bessel sequence, X_d -frame, X_d^* -frame, X_d -Riesz basis, X_d^* -Riesz basis in this new prospective. Chapter 4 deals with numerical range of two operators in semi-inner product spaces. We have investigated some classical spectral theory results which are related with numerical range of operators. We have constructed an approximation method for solving an operator equation involving two nonlinear operators. In Chapter 5, we have solved extended F-implicit variational inequality and complementarity problems in semi-inner product spaces. Chapters 6, 7 and 8 deal with variational inclusion problems. In Chapter 6, we have discussed A-monotone operator and solved a class of variational inclusion problems involving A-monotone operator. In Chapter 7, we have solved implicit variational inclusion problems involving A-monotone, H-monotone and maximal monotone operators. In Chapter 8, a new type of variational inclusion problem involving H(.,.)-Co-monotone operator is solved. In all the Chapters 6, 7, and 8, we have used generalized resolvent operator technique to solve variational inclusion problems in 2-uniformly smooth Banach spaces.

Previously, researchers have taken the help of bounded linear functionals to study Hilbert space like theory in Banach spaces. The main benefits of using semi-inner product are of three folds. It is computationally easy, one can avoid the inconvenient of using arbitrary bounded linear functionals, and it helps in constructing examples. The frame theory that is developed in this thesis will draw the attention of researchers to work in this area.