

## Abstract

Generalized monotonicity plays an important role in the study of variational inequality problems as well as equilibrium problems. The objective of this thesis is to study and analyze the existence of solutions to equilibrium problems, variational inequality problems, and their extensions under various types of generalized monotonicity assumptions.

The thesis consists of seven chapters along with a chapter of concluding remark and scope of future work. Chapter 1 is the introductory chapter where the brief literature survey on various topics of equilibrium problems and variational inequality problems and the research interface have been presented. We establish the existence solutions of the variational-like inequality problems with relaxed  $(\rho-\theta)$ - $\eta$ -invariant monotone and relaxed  $(\rho-\theta)$ - $\eta$ -invariant pseudomonotone mappings in Chapter 2. In Chapter 3, we introduce the generalized relaxed  $\alpha$ -pseudomonotone multifunctions to obtain the solutions of the generalized variational-like inequality problems. In Chapter 4, we use the generalized relaxed  $\alpha$ -monotonicity to prove the existence and uniqueness solutions of a general variational-like inequality problem. Furthermore, we use a generalized projection iteration scheme to find a common element of the solutions of general variational-like inequality problem and the set of fixed points of a finite family of quasi- $\phi$ -nonexpansive mappings. The existence and uniqueness solutions of the mixed equilibrium problems under  $(\rho-\theta)$ -monotone and generalized relaxed  $\alpha$ -monotone mappings are established in Chapter 5. Chapter 6 describes the existence solutions of equilibrium problems with  $(\rho-\theta)$ -pseudomonotone and generalized relaxed  $\alpha$ -psuedomonotone mappings. In Chapter 7, we establish the existence and uniqueness solutions of generalized mixed equilibrium problem (GMEP) by using the generalized relaxed  $\eta$ - $\alpha$ -monotonicity assumptions. We also present a generalized  $f$ -projection method to find a common element of the solutions of (GMEP) and the set of fixed points of an infinite family of quasi- $\phi$ -nonexpansive mappings in a uniformly smooth and uniformly convex Banach space. Moreover, the strong convergence of our proposed method under generalized relaxed  $\eta$ - $\alpha$ -monotonicity have been proved.

It has been observed that, in some particular cases our results reduce to some of the existing works available in the literature, which shows that our results are more general than the existing results.

**Keywords** Variational inequality problem; variational-like inequality problem; equilibrium problem; vector equilibrium problem; generalized mixed equilibrium problem; Hausdorff metric; KKM mapping; quasi- $\phi$ -nonexpansive mapping; generalized projection; generalized  $f$ -projection.