## REVIEW OF THE PAST WORKS AND SCOPE OF WORK DONE:

When a time delay is included within the loop of a feed back system, a sharp increase in difficulty is experienced in analysing the resultant system as the literature attests. (Weiss 1959, Choksy 1960, Wheather 1963, Chyung 1965, Oguztoreli 1966, Nielson 1967).<sup>1-6</sup>The classical methods lead to transcendental characteristic equation in the S-domain. The necessity to invert Laplace Transform involving exponential terms in the complex variable makes it much more difficult to obtain the response. Many methods have been presented in the past for the analysis of linear and nonlinear time delay systems. Analysis of the system is further supplemented by computer simulation. It is a well known technique for the analysis of control systems to investigate the dynamic behaviour.

Analog model provides a simple and effective method for identifications and analysis of a wide range of physical systems, via transfer function concept. The validity of the mathematical model of the system can be checked by comparing the experimental values of system constants with those obtained by considering the physical parameters of the system. INTRODUCTION

For the stability analysis, both absolute and relative Chu(1952)<sup>7</sup> presented root locus methods with one free parameter, usually the gain, for the time delay systems. The method is usually laborious and time consuming for higher

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order systems as it involves constructions of various phase plane loci of the rational algebraic function of the open loop transfer function. Applications of Nyquist stability criterion or Mikhailov stability criterion may as well be extended to time lag systems to determine the absolute stability in the plane of parameter K and T (Majumder and Das Gupta 1969)<sup>8</sup>. They have shown the applicability of Pade approximation to the stability of time delay systems. Usual Routh criterion could be applied after approximating the exponential term with rational algebraic functions. The method though lengthy, is reasonably accurate for higher order approximation of time delay. Mukherjee and Das Gupta ( 1967)<sup>9</sup> obtained the relative stability curves for the retarded systems from the root locus drawn by Chu for different values/damping ratio. Kind Smith (1971)<sup>10</sup> utilized the root locus method supported by a certain amount of numerical calculations for the stability analysis of the linear time delay systems. Eisenberg (1966)<sup>11</sup> and Karmarkar (1970)<sup>12</sup> analysed the retarded systems using the parameter plane representation of the characteristic equation as introduced by Mitrovic(1959)<sup>13</sup> and generalized by Siljak (1964). 14 Simultaneous variations of the controller gain and time lag for higher order systems are dealt with as easily as low order systems.

Loo (1969)<sup>15</sup> and McKay (1970)<sup>16</sup> applied D-Partition method for dead time or distributed lag for mapping the region of stability or instability in K-T plane. The latter

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indicates how information on the influence of the system parameter on stability parameter conveniently be plotted using Bode diagram technique. The method is simple and gives more direct result. Roberts (1969)<sup>17</sup>described the applications of Beckers (1964)<sup>18</sup> real and imaginary curve method for determining the stability bounds of linear continuous data systems. The method is graphical in nature. The stability criterion is obtained from the real and imaginary parts of the characteristic equation of the system. The stability region for system containing two free parameters may be determined. The frequency of possible continuous oscillations are indicated in addition to the stability boundary. Ho and Chan (1969)<sup>19</sup> applied Pontryagin's (1955)<sup>20</sup> method for the determination of the zeros of the 3rd order retarded systems characterized by the normalized transcendental values of the parameters of the retarded Bhatt and Hsu (1966)<sup>21</sup> presented stability crisystems. terion of second order retarded systems. However; this method becomes difficult when the distributed lag is present in the system. Cook (1966)<sup>22</sup> outlined method for determining the phase margin and gain margin of the linear system based on Nyquist criterion, the method is easy to apply when a time lag is present in the system. Chen and Haas (1965)<sup>23</sup> indicated the stability of linear systems with time lag as well as stability of linear systems from the knowledge of generalized phase margin. The method resorts to plotting two loci and that is why it is called dual locus technique or two hodograph method.

More recently N-Nagy and Al-Tikriti(1970)<sup>24</sup> came out with a simple and elegant graphical method for determining the stability of linear control systems with dead time using Nyquist criterion. The stability is studied with respect to two variable parameters. The criterion employs chart constructed before hand. It required the magnitude in db and phase in degree of the open loop transfer function to be plotted separately in the same charts from which the stability criterion can be determined. The method considers the lag element in forward as well as feed back path.

Mukherjee  $(1974)^{25}$  applies the Nyquist criterion which is basically an one variable parameter method to time lag systems, by introducing dual locus diagram to include two variable parameter K and T. The method is quite simple but need charts constructed before hand. The Liapounov method, though powerful, lacked the procedure for choosing a suitable V-Function. Until recently Nagaraja and Chalam  $(1974)^{26}$  have proposed a systematic development of the Liapunov function. Extension of the above approach has been made/Mehrotra  $(1977)^{27}$  to transport lag systems. However, the mathematics and computations involved even for low order systems are not that simple.

Stability analysis of nonlinear control systems with time delay reported so far are based on one of the following methods. The phase plane method (Geldwani 1970a, 1970b) $^{28-29}$ does not resort to any approximation and as such is pretty accurate. However, the derivation of expressions for limit

cycle amplitude and period requires long time.

A Liapunov functional and the global asymptotic stability calculations for the systems with multiple nonlinearities and lag are discussed by Nishimura et.al (1969).<sup>30</sup> The stability region dependent on time lags can be determined for simpler systems by using different types of Liapunov functional. Kim and Yeh (1964)<sup>31</sup> have tested the stability of time lag systems using z-transform to the time delay function. However, the method was suitable for linear systems only. The well known Popov method or extended circle criterion (Shepiro 1970)<sup>32</sup> is generally applicable to linear systems with a sector type nonlinearity for absolute stability determination. The presence of delay may, however, be circumvented by a suitable transformation. (Tagata 1968).<sup>33</sup>

In the parameter plane method for nonlinear systems with multiple nonlinearities but without time delay (Chen et.al 1971)<sup>34</sup> the equations for systems are formulated and used to map the imaginary axis of the S-plane on to the parameter plane for the stability conditions. The existence of the limit cycle in the systems are also studied. The above method is quite simple and accurate for stability analysis of linear systems.

Sensitivity, as a general concept refers to the change in an output variable which can be attributed to a change in one of the systems parameters. As a quantitative measure, sensitivity has a value in allowing the engineer to predict possible changes in the systems output based on proposed or actual changes in system parameters.Sensitivity becomes specially important in a feed back(recycle process, that is the process in which the possibility exists for the system output to influence itself.

The classical sensitivity in the Laplace transform space is defined by S  $\frac{I}{a} = ( \bigtriangleup I / I) (a/\triangle a)$ . When I is the performance index, 'a' is the system parameter and  $\triangle$ refers to the gross change.

Houpis (1965)<sup>35</sup> has extended the root locus method to determine the parameter sensitivity of simple control systems to yield information on the effect of variation in such parameters as first damping co-efficient. Sensitivity of performance index with respect of parameter variations in the state equations of the optimally designed time delay systems has been discussed by introducing the  $\rho$ -sensitivity (Malek-zavarei and Jamshidi 1975)<sup>36</sup> A system is said to have P-sensitivity with respect to a certain class of variations if the value of the performance index does not increase by more than factor of  $\rho$  for variation in that class. (McClamroach st.al 1969).<sup>37</sup> In an autonomous case, the sensitivity analysis of the system with time delay is similar to that analysis without delay. However, a small delay inside the feed back loop may dominate the stability domain (Gumowski 1974).<sup>38</sup> Conner ( 1971)<sup>39</sup> has studied the problem of minimizing the performance sensitivity to the



parameter variations in a nonlinear time delay system using a scalar case. However, he feels the study is equally suitable for vector equations. A design procedure for obtaining approximately optimal feed back controller with bounds on the sensitivity has been discussed by Sarma and Deekshatulu(1968).<sup>40</sup>

Inoue et.al (1971)<sup>41</sup> have shown the applications of sensitivity approaches to the optimization of time delay systems. While preserving a good performance, Zinobar and Fuller (1973)<sup>42</sup> considered the effect of small parameter variations on the system and have further shown that small parameter deviation of the plant parameter from its nominal value yield instability.

Process control engineers are often concerned with design and performance of a continuously operated chemical plants under steady state conditions. There are however many occasions when knowledge of dynamic behaviour of a process is of great importance. The study of such cases have increased in the recent years with the greater use of computing facilities. Simulation is a well known technique for the analysis and synthesis of control systems.Extensive work though has been done on the simulation of control systems, not much literature is however available for simulation of time delay systems. (Lineberger and Brenner 1965, Abraham and Collins 1966).<sup>43-44</sup> Chatterjee and Sikdar (1968)<sup>45</sup> have described a method for digital simulations of control

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systems. Though the result predicted are accurate for linear system, the technique is not suitable for time delay systems. However, the subroutine for the nonlinearities are quite useful. Paul and Steinberg (1972)<sup>46</sup>have described two methods to simulate the time delay digitally. However, both the methods use a hybrid computer and a lot of other hard wares and the methods are suitable only for a small delay. Davies (1972)<sup>47</sup> has discussed the applications of operational amplifiers to form time delay approximations for those applications, where the time delay is more or less constant. A number of such approaches are discussed and resulting approximations are brought to a common form and scale for comparison. An interactive digital simulation and optimism package developed by Roach and Chow (1975)<sup>48</sup> is guite suitable for optimization and minimization of the system. The package contains many subroutines including that for a time delay and a variabletime delay. The above programme, however, is quite complex and requires many changes to run on IBM 1130. A Bucket-Brigade technique was suggested by Rothstein (1973)<sup>49</sup> and Frank (1972)<sup>50</sup> to simulate a variable time delay on digital computer. In this method the sampled output is held stored in the memory for the delay period and the technque 19 suitable for simulating a variable time delay by subroutine. Chu (1969)<sup>51</sup> has widely discussed the digital simulation of continuous data control systems and he has also suggested a method similar to Frank for simulation of time delay by subroutine.