Chapter 1

Introduction

The determination of natural frequencies of blading is of considerable importance in the design of turbo-machines. It is known from investigations that the failure of turbo-machinery blading is quite often due to fatigue, and it occurs when vibrations take place at or near resonant conditions. Recent experiences of blade failures include those in the HP turbines of RMS 'Queen Elizabeth 2' as reported by Fleeting and Costs (1970). This ship left the manufacturers on November 19, 1968 and the failures took place as early as on December 24, 1968 during the first major voyage of the ship from Tail O' the Bank. The 9th stage starboard HP turbine rotor was completely damaged. Partial damage occurred to the 10th stage starboard HP turbine rotor and the 9th stage port HP turbine rotor. Another blade failure has been reported by Bharat Heavy Electricals Ltd., Hardwar, India. The 27th and 31st stage blades of a 200 MW steam turbine failed when the turbine speed varied beyond the stipulated range of 49 to 51 cycles per second.

The magnitude of the problem encountered in the design of turbo-machinery blading is evinced by the fact that in some turbines there can be as many as a few thousand fixed and rotating blades of different characteristics and the failure of even one of them will force a shutdown. Several analytical methods which have been developed in the field of blade vibrations involve lengthy computational procedures and can be applied efficiently only by the use of digital computers. These methods thus are not well suited for use by a designer for a fairly rapid and accurate prior estimation of the natural frequencies of blading at a preliminary stage of design. In general, the blades as used in turbo-machines are pretwisted and tapered having an asymmetrical aerofoil cross-section and are mounted on a rotating disc at a stagger angle. In some cases a certain number of blades are grouped together by a shroud to form a packet of blades (also known as bucket of blades or cascade of blades). To increase the damping capacity it is again customary to use a lacing wire around the blades and mount them somewhat loosely on the disc.

For the convenience of determining the natural frequencies of turbo-machinery blading, the designer can first consider the blade as a straight, uniform and stationary cantilever beam of symmetric cross-section, rigidly fixed at the root, and then find out the effects of the following factors on the natural frequencies:

1. Taper of the blade cross-section
2. Shear deflection and rotary inertia
3. Asymmetry of the blade cross-section
4. Rotation of the disc
5. Disc radius
6. Stagger angle of the blade
7. Pretwist of the blade
8. Shrouding

9. Lacing wire

10. Root fixing

Considerable theoretical and experimental works have been carried out by various investigators to enable a designer to calculate the natural frequencies of blading. Still it is too early to say the last word on the problem. Since one cannot entirely depend upon the theoretical calculations, it is advisable to make full-scale model tests on the prototype and study the vibration characteristics, i.e., the natural frequencies and their corresponding modal patterns. The experimental procedure, however, is time-consuming; also, it is highly expensive to conduct tests on a number of designs. Thus it becomes essential for a designer to have theoretical knowledge as well as practical means for carrying out tests on models for a prior estimation of the natural frequencies of blades at an earlier stage of design.

Turbo-machinery blades execute either uncoupled bending or torsional vibrations or coupled bending-torsional vibrations depending on the blade characteristics. Coupled bending-torsion vibrations take place when the centre of flexure of the blade cross-section is not coincident with its centroid. When the blade is pretwisted, vibrations are further coupled between the two bending modes. The differential equations of motion for the case of pretwisted blades with asymmetric cross-section are too complex to be amenable to an easy solution. Also, the natural frequencies are influenced by shear deflection, rotary inertia, fibre bending in torsion, warping of the cross-section, root fixing and Coriolis acceleration. In general, the equations of motion will be six coupled partial differential equations coupled between the two bending deflections, the two shearing deflections, the torsional deflection and the longitudinal deflection as derived by Rao and Rao (1973), allowing for the effects of warping, rotary inertia, etc. Since the longitudinal natural frequency of turbo-machinery blades is usually very high and there is a less possibility of exciting a longitudinal mode of vibration of the blade, this deflection can be ignored. Carnegie (1967a) considered the Coriolis accelerations due to the inward displacement of the blade element and derived the nonlinear partial differential equations of motion. These equations are very complex and it may be almost a futile exercise at present to seek for a solution of them taking all the effects into account. When the blades are shrouded, the coupling between the blade and the shroud is to be further taken into consideration. Thus it becomes a highly difficult task to determine theoretically the natural frequencies of actual turbo-machinery blading allowing for all the effects simultaneously.

Various investigators in the field of blade vibrations have solved the differential equations of motion by taking into account the individual aspects just mentioned, and determined their effects on the vibrational characteristics. Rao (1971b) considered some of the aspects mentioned, viz., taper of the blade cross-section, shear deflection and rotary inertia, rotation of the disc, disc radius, stagger angle, pretwist, asymmetry of the blade cross-section and Coriolis accelerations, either individually or with a few aspects combined, in determining the natural frequencies or the response of the blades.

This thesis is aimed to present the effects of disc rotation, disc radius and asymmetry of blade cross-section on the coupled bending-torsional natural frequencies of straight uniform blades of aerofoil cross-section (symmetric about one principal axis but asymmetric about the other) mounted on the periphery of a rotating disc. The differential equations of motion for such blades are derived and expressed in the non-dimensional form. Galerkin method is used for the theoretical evaluation of dimensionless frequency parameters for different non-dimensionally expressed values of variables commonly observed in turbo-machine blades. Secondary effects due to rotary inertia and shear deformation are also taken into account by Rayleigh-Ritz method of analysis, and the effect on the coupled natural frequencies due to variations in the radius of gyration about the axis of symmetry of blade cross-section expressed non-dimensionally with respect to blade geometry is studied. Further, the methods of Myklestad (1944b) and Rao (1969, 1970b) are extended to take into account the asymmetry, disc radius and rotation, by appropriate use of polynomial expressions and dividing the blade into discrete masses and inertias. To verify the theoretical calculations, an experimental rig is designed and fabricated to test rotating blades. Experiments are conducted at different speeds of rotation. The results obtained are compared with the theoretical results and a favourable agreement is shown between them. The theoretical results are also compared for some special cases with those available from earlier works and a good agreement is observed between them. The thesis consists of the following chapters:

the important points of which are given below.

In chapter one the problem of turbo-machinery blade vibration is introduced. Chapter two contains a survey of literature available to the author. The third chapter is devoted to the analysis of coupled bending-torsion vibrations of asymmetric cantilever blades as treated by Galerkin method. The coupled bending-torsion vibration problem of rotating asymmetric cantilever blades incorporating the effects of rotary inertia and shear deformation is dealt with in chapter four by Rayleigh-Ritz method. The fifth chapter analyzes the problem of coupled bending-torsion vibrations of rotating asymmetric cantilever blades by the polynomial frequency equation method. Chapter six deals with the experimental verification of the theoretical results. Concluding remarks are given in chapter seven. Chapter eight contains an outline of the scope of further work. References are listed in chapter nine.