SYNOPSIS

Catastrophic and unexpected failures, and the need to analyze them in order to prevent future occurance, were the principal stimuli to the development of Fracture Mechanics. The emphasis in these problems is an introduction to the field of fracture mechanics from the view point of application. In recent years, structural failures and the desire for increased safety and reliability of structures have led to the development of various fracture criteria for many types of structures including bridges, airplanes, and nuclear pressure vessels.

The development of fracture-control plans for new and unusual structures such as offshore drilling rigs, floating nuclear power plants, space shuttles etc., is becoming more widespread. Each of the topic of the fracture criteria and fracture control is developed from an engineering view point including economic and practical considerations.

Like other branches of Continuum Mechanics, the principles of Fracture Mechanics can also be applied for studying the fracture strength of hard biological tissues e.g. bones. Apart from forces like the gravitational forces and pressure exerted by neighbouring organs, bones are often subjected to external forces in our everyday life. Depending on the intensity and manner of the application of the applied loads, fracturing of bones may occur. As in the case of engineering structures, bone fracture does certainly depend on the mechanical properties of osseous-tissues.

A major portion of the material presented in the thesis is concerned with the calculation of the distribution of stresses in the neighbourhood of cracks and the stress intensity factor near the tip of the crack. The other useful physical quantities such as crack energy required to form the crack, critical pressure at which the crack starts spreading and the shape of the crack for various configurations of cracks in isotropic as well as anisotropic media have also been paid due consideration.

The thesis consists of seven chapters of which the first one is introductory. This chapter includes brief discussions on the scope and development of Fracture Mechanics, types of fracture, different theories on the initiation and propagation of cracks, basic equations and mathematical formulation of crack problems, anisotropic elasticity, composite materials, the constitution, structure mechanical properties and piezoelectric effect of bone, a survey of related previous studies and the scope of thesis.

The main object of the second chapter is the determination of the distribution of pressure necessary for producing a Griffith crack of prescribed shape in an orthotropic solid. The problem is studied analytically and is applied to a particular class of crack shapes for which numerical results are presented graphically.

The third chapter deals with a two-dimensional problem of an anisotropic elastic strip having an infinite row of Griffith cracks. By using integral equation approach, the problem is treated analytically. Expressions for the stress intensity factor, the critical pressure and the energy required to open the crack, are obtained for two cases - (a) when the edges of the strip are in contact with smooth and rigid plates, and (b) when the edge of the strip are free of tractions. Numerical results for the aforementioned quantities are obtained for both the cases for an anisotropic material ; a comparison is also made with the corresponding results for a given isotropic material.

In the fourth chapter, a problem of stress analysis for a long circular cylinder has been taken up. The cylinder is assumed to be made of an elastic material which is not isotropic but the elastic properties are considered to be similar in directions perpendicular to the axis of the cylinder. The body under consideration is supposed to contain an infinite row of penny-shaped cracks which are parallel to each other, and located periodically along the cylinder-axis. All the cracks are assumed to be opened by the same distribution of internal pressure, on their surfaces. By choosing appropriate potential functions, the problem is reduced to that of solving a Fredholm integral equation of the second kind, which is solved numerically to obtain various physical quantities of interest.

A crack-problem related to a composite structure is of concern in the fifth chapter of the thesis. The composite structure involves a solid cylindrical body made of a given material, surrounded entirely by a dissimilar material. Attention is paid to a situation in which the core cylindrical body contains an infinite number of transverse penny-shaped cracks. By choosing axisymmetric biharmonic functions, the problem is formulated in terms of dual integral equations which are reduced to Fredholm integral equations ; they are treated further numerically to achieve the final solution of the problem. Some computational results for the physical quantities which are of interest from the fracture mechanics point of view are also presented.

The last two chapters of the thesis are concerned with studies related to the mechanics of bone fracture. Of special consideration is the fracture of patella, which may occur due to direct or indirect violence. As pointed out by Salter[+] , in the direct type of violence (e.g. from a local blow), the patella is forcibly jammed against the lower end of the femur and sustained a crush fracture which is usually stellate and may be severely comminuted.

The sixth chapter is directed to a study on the theoretical estimation of the stress-field in a bone medium (patella, in particular) containing a star-shaped crack. The anisotropy of osseous-tissues (as per experimental observations) has been taken into account in the analysis. The star-crack is assumed to be formed by the interaction of a number of Griffith cracks of equal lengths when the face of the crack is subjected to prescribed pressure. By choosing suitable Airy-stress function, a fourth order differential equation is solved. The boundary conditions formulated mathematically, reduce the problem to that of solving a pair of dual integral equations which are treated further by resorting to numerical techniques. Numerical computation is made for studying the intensified stress-field in the bone medium in the vicinity of the crack. The effect of bone anisotropy on the generated stress-field is also studied.

[+] Salter, R.B., 'Text Book of Disorders and Injuries of the Musculoskeletal System', The Williams and Wikins Company, Baltimore (1970), p. 518 .

An analysis for an exterior star-shaped crack contained in a patella, has been put forward in the seventh and the concluding chapter of the thesis. As in the preceding chapter, the study has been made by using a simple mathematical model. It is believed that the analyses presented here will find applications in estimating the fracture strength of osseous tissues, in general and patellar bone, in particular.