CHAPTER 1

INTRODUCTION

Phased array systems have made their place in many applications due to their usefulness in carrying out functions which could not be performed with mechanically scanned antennas. Conventionally, phased array is a regular arrangement of identical antenna elements over a plane surface. The analysis of such an array is greatly facilitated by employing the rule of pattern multiplication and the Floquet's theorem. The array factor of the equally spaced array is elegantly expressed in terms of a polynomial of a complex variable leading to well developed design methods following Schelkunoff's theory [1]. The element pattern which is also needed for the array analysis is usually assumed identical for all elements. The analysis of the effect of mutual coupling amongst elements is essential as it causes significant modification of the element field pattern. The current distribution on the elements and the reflection coefficient at their input ports are also modified due to the mutual coupling thus leading to the requirement of modifications in the matching system at the element inputs to optimise the array efficiency.

A normal requirement on the radiation pattern of the phased array is to have a single beam radiation and to avoid the appearance of any secondary beams of equal strength called grating lobes in the visible range [2]. This limits the spacing between elements in the array not to be greater than a specified value determined by the range of beam scanning and the particular type of the element. Usually this spacing is limited within a wavelength. A minimum number of elements are, therefore, necessary to obtain the phased array of a given aperture area. This consideration also improves a higher limit on the frequency of operation of a given array.

Smaller spacings are also avoided in the phased arrays as the mutual coupling becomes stronger with such spacings. The strong coupling renders individual control of the amplitude and phase of the element excitation increasingly difficult and, thus, giving loss of effective control on the antenna aperture illumination which is a main purpose of introducing array concept [3] . The requirement of spacing not smaller than half wavelength (a normal limit) gives a lower limit of the frequency of operation of an array.

Above considerations indicate that the bandwidth of the conventional arrays is limited [4] and a minimum number of elements are a must for a given array size, that is, for a given beamwidth. The cost of large phased arrays is mainly dependent on the number of elements in it and the minimum limit of the latter frustates the economising efforts in such systems. This limitation is solely due to the grating lobe phenomenon which is the outcome of periodicity of the element position distribution.

The relative phases of excitation of elements follow a tapered distribution which varies with the direction of scan. The change in the scan direction, thus, reflects a corresponding change in the amplitude and phase of the coupled power in each element resulting in the variation of the reflection coefficient at its input port with scan angle. The variation in the field distribution on the element aperture (or current distribution in case of wire elements) due to change in the relative contribution of the higher order modes with scan angle also adds to the active reflection coefficient variation [5] . In case of the periodic arrays, a majority of elements look into identical variations of reflection coefficient with scan angle [6] . It implies that the increase of reflection coefficient, whenever it occurs, will take place in all the elements simultaneously, causing an overall reduction of the efficiency of the array.

It has been shown that even if the change in the field distribution on the element aperture is neglected, the reflection coefficient in a periodic array rises to unity when scaned such as to cause a grazing grating lobe to appear [7] , [8] . In practical arrays, high reflection coefficients (nearing unity) have been observed to occur at smaller scan angles away from the broadside than the grazing grating lobe appearance angle [5], [9] - [12] . This phenomenon, called blindness in the periodic phased arrays, has been viewed as an active array problem and also as the element pattern problem [13] . Accordingly, the explanations of the blind spots are in terms of resonance (or cancellation effect) and element pattern nulls (or leaky wave effect) [14] . The theories explaining the blind spots assume three essential factors for the appearance of the latter.

These are

1. Mutual coupling between elements.
2. Induction of higher order modes in the field distribution on the element aperture.
3. Periodicity of elements resulting in identical environment for all elements under any active condition.

The phase scan is obtained in practice by electronic control of the phases of excitation of element inputs. Digital phase shifters are commonly used for this purpose. The phases in such cases are to be quantised giving rise to a quantisation error. Due to the periodic spacing of the elements, the distribution of the error also becomes periodic with the period which may be larger than the element spacing. The error space factor resulting from the quantisation errors will thus give peak effects in the directions of the grating lobes corresponding to the period of the error distribution. Thus an increased sidelobe level is likely to appear due to the phase quantisation effect. Special techniques are, therefore suggested for dispersing the quantisation lobes [15] .

Above discussions reveal that the limitations of the minimum number of elements due to grating lobes, maximum available scan range due to blind spots and provisions for the dispersion of the quantisation error lobes are essentially the product of the equal spacing or periodic element lattice employed in the phased arrays. It is obvious that the elimination of the periodicity of element placement can be an effective means to get over these limitations. The unequally spaced arrays, thus, become of interest to be examined for phased array applications to combat the limitations cited above.

Unequally or nonuniformly spaced arrays have drawn considerable interest in the past with a view to obtain an extra design parameter for optimising the array designs. Array of arbitrarily spaced elements was first analysed by H.Unz [16] who expressed the space factor of such an array in terms of a set of algebraic equations. A significant amount of work has been reported since then. Excellent reviews of earlier developments in nonuniformly spaced arrays appear in standard books [17] - [19].

Arrays of nonuniformly spaced elements have been broadly classified as deterministic and statistical or random. The element spacings in the deterministic arrays are obtained by a definite computational procedure or a mathematical expression. In the random arrays, element positions are obtained following a statistical procedure giving, in general, different results at each attempt. The performance of such arrays can be described in terms of probabilistic averages only.

Computational deterministic procedures are directed to realise a desired space factor by employing the interelement spacing as a design parameter. The usual procedure of designing the equally spaced arrays is based on synthesising the continuous aperture illumination function [17] , [19] - [21] for the desired pattern and sampling the same at the element locations which are either given or determined by some other consideration [17] , [18] . A variation in this procedure is possible by utilising the equivalence of element density function with the illumination function [22] - [24] . The new procedure, thus, consists of designing the continuous illumination function and realising the same in the array by the equivalent element density distribution. The excitation of the individual elements is assumed prefixed, usually, as uniform. This procedure results in a nonuniformly spaced array with a single (or a few) level of excitation of elements. Such element density tapered arrays are more efficient than the corresponding illumination tapered arrays [25] and are preferable from the practical view point also when constrained feed is used because only one (or a few) input source level is required to be generated in element density tapered arrays [26] , [27] .

The nonuniform element spacings are also obtained by solution of a set of algebraic equation in which the spacings are unknown parameters. The equations are formed to satisfy the requirements of the desired radiation pattern [25] . Better approximations of the pattern are obtained by employing more number of elements. However, formation of suitable equations and their solution as simultaneous equations becomes more complex with increasing number of elements. Some iterative methods of solving these equations have been therefore suggested [28] - [33] .

An important computational procedure for nonuniformly spaced arrays is based on dynamic programming in which each element pair location is selected successively to optimize the resulting pattern [34] . The approach is, however, difficult to apply to the case of large arrays.

A perturbational technique was first introduced by Harrington [35] in which the element positions in a given predesigned equally spaced arrays are perturbed to modify the space factor of the array in the desired direction. Many procedures for successive perturbation to obtain an optimum design have been further suggested [36] - [39] . In these procedures, the resulting array heavily depends on the initial array taken as the starting point.

A different class of determinstic nonuniformly spaced arrays is obtained by arranging the elements in accordance with a mathematical function and study its properties. Such types of arrays are not proved to be as optimum as the arrays obtained by dynamic programming, but they are quite near to the optimum conditions and, as such, are useful when large number of elements are involved and the dynamic programming or other computational methods become increasingly cumbersome. The mathematical functions which have attracted attention are exponential function [40] and sinusoidal function [41] . The latter is incorporated in conjunction with the uniform spacing as its modifying term. Two distinct distributions are suggested for linear arrays. In one, the index number of element is viewed as a continuous number variable which is expressed as sum of a linear function and a sine function of distance. Element locations are obtained wherever the number function assumes an integer value. The other distribution employing sinusoidal function is obtained by expressing the position of the element as superposition of a linear and a sine function of the index number variable of the element. This is also conceivable as the modulation of the positions of the elements in an equally spaced array in accordance with a sine function [42], [43].

It is interesting to note that the mechanism of the elimination of the grating lobes is physically visualised in the nonuniformly spaced arrays of function specified element positions. This is so because the equally spaced array forms a special case in such distributions and is approached by controlling a specific parameter in the distribution function. It is seen that as the array deviates from its equally spaced condition, the grating lobes smear gradually to form plateaux around their initial positions. The sidelobes occuring in this region have their peaks within the envelop of the plateau. The exponential spacings give theoretically constant plateau level [40] while the sinusoidal functions give plateaux of slowly varying levels.

The stastical arrays usually refer to a conventional array of large number of elements in which many of the elements are removed following a statistical procedure. The amplitude distribution in the initial, or filled, array is considered as a weighting functing in the probability of removing an element. The resulting thinned array contains a considerably reduced number of equally excited elements [24]. The element spacings in this case, however, remain multiples of the spacing in the filled array and, therefore, the grating lobe corresponding to the later is not eliminated.

The random array, in general corresponds to the case where element locations are chosen randomly following a prescribed procedure involving the random number table. The resulting arrays at the end of each trial of the random location selection are very likely to be different from each other. The average properties of many trials following the same random experiment are described by the probability theory [18], [45] - [47].

Some more methods of obtaining nonuniform planar arrays have been suggested recently [48], [49]. They are aimed at the suppression of the grating lobe by rotating the specific parts of element groups and thus obtaining a nonuniformly spaced array. One way suggested is to locate the elements on a rotating rhombus structure [48] while another suggestion deals with rotating subarray groups of elements by different angles over the apertures [49].

The random arrays are attractive from the point of view of its mathematical representation. No unified theory is available for the deterministic type of unequally spaced arrays. However, it has been pointed out that the results of the random array theory are applicable, on average basis, to the general deterministic arrays also [50]. Since the random theory predicts the performance only as an average of many trials, a particular case of random design will not necessarily satisfy the specifications required. Thus, it is necessary to carry out many random design trials and choosing the one evaluated as best amongst them. Each trial, therefore, will be required to be tested fully. As a special example, consider the peak sidelobe level specification. It is obtained as the average of many trials by the probability theory. The location of the peak in the space factor, however, remains arbitrary. Therefore, it is required that each part of the space factor of trial design is carefully screened to look for the occurance of the peak sidelobe. Such screening becomes increasingly cumbersome in case of large arrays. Even in case of small arrays, though the screening will be fast, the number of trials required to be tested will be comparatively larger, thus, requiring a sizeable computational effort.

An important outcome of the studies of nonuniformly spaced arrays, deterministic or random, has been the demonstration of their potential in realising a space factor with comparatively smaller nnmber of elements at the cost of slight deterioration of the sidelobe level and directivity [51] - [53]. This is only due to the elimination of grating lobes by the nonuniform spacing which, in turn, makes it possible to employ average interelement spacing in the array. A criterion for comparison of two arrays can be, therefore, the higher average interelement spacing for specified sidelobe level in the given range.

Many reports on unequally spaced arrays take the phasing into account and beam steering has been mentioned in some. The latter, however, do not include details for the variation of its performance in regard to sidelobe level and directivity with phase steering of beam. An exception to the above is the random array which has been studied for its average properties as phased array [13]. It is again in randomly located element phased array, that the effects of mutual coupling have been studied [54]. In general, the excitation problems of the unequally spaced phased arrays are not addressed.

The purpose of the work reported in this thesis is to examine the utility of the unequally spaced arrays for the application to the large phased array systems. Since almost all the nonuniform element distribution schemes mentioned earlier give performance of similar nature, the simplest form is preferable to avoid unnecessary complications without significant advantage. A simple form is accordingly conceived as unequal changes incorporated in the element positions of an equally spaced linear array. These changes or modulation are made proportional to a sine function which is a conventional variation used in electrical communication network analysis. The modulation term is applied such that the element spacings around the centre of the array are close and it spreads towards the edges. It results in density tapered distribution of elements and some qualities of equivalent amplitude tapering are, thus, expected to be incorporated adhoc in the so formed position modulated linear array. A linear array of (2N + 1) elements will, thus, have the modulated element positions x_n for all elements such that

$$x_n = an - b \sin(n\pi/N) \qquad (1.1)$$

where n is the index number of an element such that $-N < n < N$ The centre element is indexed as $n=0$ and x_n in the distance of n^{th} element from the centre element along the line of the array (taken above as x-direction). The average spacing of the elements is a, and b gives the peak of the modulation term. The equally spaced array is approached as a particular case of the modulated array by choosing $b=0$. The modulated array as described here is essentially an aperiodic array except when $b=0$ and when N is infinite. The positions of three elements, two edge elements and one centre element, are not changed. Thus, the overall length of the array remains same as the corresponding equally spaced or unmodulated array.

The element position modulated array described above is recognised to be essentially similar to one of the cases considered, and only briefly discussed by Ishimaru and Chen [55] .

Attracted by its simplicity of description, the modulated array has been chosen as a representative case of aperiodic arrays for the purpose of studying the phased array properties of such arrays.

The properties of interest for phased array applications can be split in two major groups; one relating to the radiation properties and the other concerning with the engineering aspects of excitation of the array. Since the main motivation of the present work is to explore the possibility of economical advantage in large systems, the study confines itself to the arrays of large number of elements wherever possible. The radiation properties of large arrays are represented mainly by their space factor at least to the first approximation. It is with this view that the investigation of space factor properties of large element position modulated phased arrays is included in the scope of the present thesis.

The space factor studies by themselves are not sufficient to establish the practical feasibility of the system for a certain application. As indicated by the equally spaced phased arrays, the mutual coupling between the elements principally determines the efficiency of the system. There is no reason, why the same should not be true for the aperiodic arrays. Thus, the analysis of the modulated array including the mutual coupling effects is essential to draw any definite conclusion about its engineering utility. Such analysis is, therefore, also included within the scope of this work.

Apart from examining the excitation properties of the modulated array, the study is also aimed at bringing out the effectiveness of the position modulation in getting over the limitations due to the blind spots and the grating lobes as discussed earlier in this chapter.

The technique of mutual coupling estimation between radiating elements depends on the particular type of the elements. The excitation properties of the array, requiring mutual coupling considerations, are required to refer to a particular type of element for convincing illustration. Rectangular apertures in a ground plane formed by open ends of waveguides are considered for the purpose of such illustration. The choice is based on the fact that the open-ended waveguides are one of the common elements used for phased arrays and the analysis in the rectangular geometry is seemingly simple. Presence of any discontinuity in the waveguide other than the open end in the ground plane, and any dielectric discontinuity in the open space has been excluded from the scope of this study.

Experimental verification of the theoretical formulations employed for the analysis and developing the computer models, has been carried out wherever it could be possible.

A general formulation for the waveguide apertures of arbitrary cross-section, radiating from an infinite ground plane is presented in the chapter 2. The formulation is based on plane wave spectrum analysis of the field in the free space and modal analysis in the waveguides. A network representation in terms of admittance parameters between various modes in each waveguide is obtained conveniently employing the orthogonality of waveguide modes and boundary conditions at the aperture plane. The radiation field as well as the input reflection coefficients are obtained as the result of this formulation. The formulation is applicable to uniform as well as nonuniform spacings of elements in the array.

Chapter 3 formaly introduces the position modulated linear array and a specific case of position modulated planar array formed by linear array of linear arrays.

Theoretical and computational studies of the space factor of a large modulated linear phased array are presented in the chapter 4. The properties of the space factor are presented with special emphasis on application of the array in phase scanning. The theory is based on Anger function series representation of the space factor following the work reported by Ishimaru and Chen [55]. Computational studies are made for the cases where theory does not apply. The results of the combination of theoretical and computational investigations have been summarised in the form of various graphs giving universal curves wherever possible. Though, full space factors were computed for numerous cases, from which the aforementioned graphs are prepared, they themselves, barring a few, have not been given to save the space in this already voluminous report. A simple design procedure emerges out from the results given. It is shown in the example of 147 element array that the position modulated array is equivalent to equally spaced array of much larger number (55% to 85%) having the same permissible sidelobe level in the given range of scan and beamwidth/physical length; the illumination in both the cases being uniform [43].

Nonuniform illuminations of (i) cosine-on-pedestal, (ii) Taylor and (iii) Gaussian distribution on the modulated array have been considered.

The effect of nonuniform illumination in controlling, the main beam shape and its neighbouring sidelobes has been illustrated. It is shown with the help of a 20 dB sidelobe level design that a modulated array can be definitely more economical in a case where the equally spaced array has to employ a large number of elements to obtain a very narrow beam.

Chapter 5 deals with the position modulated planar phased array introduced in the chapter 3. A general three dimensional representation of the space factor is illustrated from which the phase scanning properties can be derived for a particular case of spacings.

Chapter 6, 7 and 8 are devoted to the admittance analysis of the aperture array problem introduced in chapter 2 for the case of identical rectangular waveguide apertures. A general expression for the mutual admittance between a mode in one waveguide aperture and another mode in some other waveguide aperture in a common ground plane has been derived generally following the method of Borgiotti [56] in chapter 6. The expression is simplified and specialised for the rectangular waveguide apertures. An asymptotic expression for the mutual admittance is also derived for large spacing between the two aperture.

The admittance formulation developed in chapter 2 and 6 is verified by applying it to determine the input admittance of a single aperture and mutual admittance of the two apertures in a ground plane. Results are compared with those available from the integral equation solution and experimental data, reported elsewhere [57] , and obtained by independent measurements. These results are presented in chapter 7 in which a tentative criterion for selecting higher order modes and the computation of the aperture field distribution is also given.

The admittance formulation is applied to develop a digital model of a linear array of the waveguide apertures along the line of the E-plane which is studied in the chapter 8. Arrays of 15 apertures, uniformly spaced and position modulated, have been studied. Only one higher order mode in 15 aperture array could be accommodated with the available computer facility.

The computer model gives the radiation field as well as the input reflection coefficient in each element determined from the aperture field distribution which in turn is dependent on the incident dominant mode voltages in each waveguide. Thus, the element radiation patterns, variation of reflection coefficient in each element with scan angle under active phased array operation and the radiation patterns for all phased conditions of the array are conveniently studied by the digital model. The results show that the grating lobe discontinuity and the blind spots are eliminated in case of modulated arrays and the overall efficiency of the array does not significantly fall over a much wider range of scan than in case of equally spaced array of same length. The radiation patterns demonstrate the successful phase scanning operation of the modulated arrays over such wide ranges [58] , [59] .

Chapter 9 is devoted to the presentation of the experimental results as verification of some of the results obtained in earlier chapters. Amongst other things, in includes measurement of mutual coupling between the elements in the array environment, element radiation patterns and space factor measurement under two phased conditions. The results support the theory based on the admittance analysis and space factor analysis of earlier chapters.

The results have been finally summarised in chapter 10 and discussed to arrive at the conclusion that the modulated arrays have the potential of administering economy in large phased array systems. This conclusive chapter also contains some suggestions about directions for further work.

References cited in each chapter are listed at the end of the particular chapter along with the appendices, if any, of that chapter. A consolidated list of symbols appearing in the beginning of the thesis includes all the major symbols used here. The duplication in the meaning of some symbols could not be avoided and they are to be understood in their proper context. Attempt has been made to follow only one type of English usage through out the thesis. However, there might be some mixture of other usage also appearing inadvertently at some places.

Some of the parts of this thesis are published in Journals of repute and presented in recognised international symposium/ conference [42] , [43] , [58] . One more paper [59] has been proposed to be published. Copies of the published papers are enclosed at the end of the thesis for ready reference.

REFERENCES

1. E.C.Jordan, Electromagnetic waves and Radiating Systems, Prentice-Hall, New Jersey, 1950, p. 424.

2. N.Amitaya, V. Galindo and C.P. Wu, Theory and Analysis of Phased Array Antennas, John Wiley, New York, 1972, p. 11.

3. ibid. p.1.

4. M.G. Andreasen, "Linear arrays with variable interelement spacings," IRE Trans. Antennas and Propagation, Vol. AP-10, No.2, pp. 137-143, March 1962.

5. B.L.Diamond, "A generalized approach to the analysis of infinite planar array antennas," Proc. IEEE, Vol.56, No.11, pp. 1837-51, Nov. 1968.

6. N.Amitaya,V. Galindo and C.P.Wu, Theory and Analysis of Phased Array Antennas, John Wiley, New York, 1972, p. 16.

7. H.A.Wheeler, "The grating lobe series for the impedance variation in a planar phased array antenna," IEEE Trans. on Antennas and Propagation, Vol. AP-14, pp. 707-714, Nov. 1966.

8. L. Stark, "Microwave theory of phased array antennas - a review," Proc. IEEE, Vol. 62, No. 12, pp 1661-1701, Dec. 1974.

9. I.W.Leentrek, "Effect of coupling accumulation in antenna arrays," IEEE Trans. on Antennas and Propagation, Vol. AP-16, pp 31-37, Jan. 1968.

10. R.H.T. Bates, "Mode theory approach to arrays," IEEE Trans. on Antennas and Propagation, Vol. AP-13, pp 321-322, March 1966.

11. E.V.Byron and J.Frank," Lost beams from a dielectric covered phased-array aperture, " IEEE Trans. on Antennas and Propagation, Vol. AP-16, pp 496-99, July 1968.

12. G.F. Farrel and D.H. Kuhn," Mutual coupling effects of triangular grid arrays by modal analysis," IEEE Trans. on Antennas and Propagation, Vol. AP-14, pp. 652-654, Sept. 1966.

13. A.A. Oliner, "Surface wave effects and blindness in phased array antennas," Phased Array Antennas, ed. A.A. Oliner and G.H. Knittel, Artech House, 1972.

14. G.H.Knittel, A. Hessel, A.O.Oliner, "Element pattern nulls in phased arrays and their relation to guided waves," Proc. IEEE, Vol. 56, No.11, pp 1822-1836, Nov. 1968.

15. G.J.Buck, "Quantization and reflection lobe dispersion," Phased Array Antennas, ed. A.A. Oliner and G.H. Knittel, Artech House, 1972, pp 222-229.

16. H.Unz, "Linear arrays with arbitrarily distributed elements," IRE Trans. on Antennas and Propagation, Vol. AP-8, pp. 222-223, March 1960.

17. M.T. Ma, Theory and Applications of Antenna Theory, John Wiley, New York, 1974, pp. 37-42.

18. B.D.Steinberg, Principles of Aperture and Array System Design, John Wiley, N. York, 1976.

19. R.E. Collin and F.J. Zucker, (ed.), Antenna Theory, Vol. I, McGraw Hill, 1969.

20. R.J. Marhefka, E.L. Pelton and W.D. Burnside, "An iterative approach for computing an antenna aperture distribution from given radiation pattern data," 1979 International Symposium Digest, Antennas and Propagation, pp 50-53.

21. K. Hidaka, " A pattern synthesis method with low sidelobes over a wide frequency range," ibid, pp 26-29.

22. S.S. Sandler, "Some equivalences between equally and unequally spaced arrays," IRE Trans. on Antennas and Propagation, Vol. AP-8, No.5, pp 496-500, Sept. 1960.

23. A.C. Schell and A. Ishimaru, "Antenna pattern synthesis," Antenna Theory, Part 1, ed. R.E. Collin and F.J. Zucker, McGraw Hill, New York, 1969, pp 235-305.

24. M.I.Skolnik, "Nonuniform arrays," ibid., pp 207-234.

25. D.D. King, R.F. Packard and R.K. Thomas, "Unequally-spaced, broadband antenna arrays,", IRE Trans. on Antennas and Propagation, Vol. AP-8, No.4, pp 380-384, July 1960.

26. R.E. Willey, "Space tapering of linear and planar arrays," IRE Trans. on Antenna and Propagation, Vol. AP-10, No.2, pp 369-377, March 1962.

27. M.T. Ma, Theory and Applications of Antenna Arrays, John Wiley, 1974, p 147.

28. Y.V. Baklanov, V.L. Pokrovskiy and G.I. Surdutovich, "A theory of linear antennas with unequal spacing," Radio Engineering and Electronic Physics (Translation from Russian Journal), No. 6, p. 905, June 1962.

29. J. Sahalos, "A solution of the general nonuniformly spaced antenna arrays, " Proc. IEEE, Vol. 62, No.8, pp 1292-1294, Aug. 1974.

30. W.L.Stutzman, "Shaped beam synthesis of nonuniformly spaced linear arrays," IEEE Trans. on Antennas and Propagation, Vol. AP-20, pp 499-501, July 1972.

31. H.K. Schuman and B.J. Strait, "On the design of unequally spaced arrays with nearly equal sidelobes," IEEE Trans. on Antennas and Propagation, Vol. AP-16, No. 4, p. 493, July 1968.

32. J.P.Basart, "A short design method for nonuniformly spaced antenna arrays," IEEE Trans. on Antenna and Propagation, Vol. AP-22, pp 822-823, Nov. 1974.

33. F. Hodjat, S.A. Hovanessian, "Nonuniformly spaced linear and planar array antennas for sidelobe reduction," IEEE Trans. on Antennas and Propagation, Vol. AP-26, No.2, pp 198-204, March 1978.

34. M.I. Skolnik, G. Nemhauser and J.W. Sherman III, "Dynamic programming applied to unequally spaced arrays," IEEE Trans. on Antenna and Propagation, Vol. AP-12, pp 35-43, Jan. 64.

35. R.F. Harrington, "Sidelobe reduction by nonuniform element spacing," IRE Trans. on Antennas and Propagation, Vol. AP-9, pp 187-192, March 1961.

36. [illegible] Lo, "Note on nonuniformly spaced arrays," IEEE Trans. on Antennas and Propagation, Vol. AP-11, No.4, p [illegible], July 1963.

37. [illegible] Murthy and [illegible] Kumar, "Synthesis of linear antenna arrays," IEEE Trans. on Antennas and Propagation Vol. AP-24, No.6, pp [illegible]-870, Nov. 1976.

38. [illegible] Kumar and P.[illegible] Murthy, "Synthesis of equally excited linear arrays," IEEE Trans. on Antennas and Propagation, Vol. AP-25, No.3, pp. 425-[illegible], May 1977.

39. M.T. Ma, Theory and Application of Antenna Arrays, John Wiley, New York, 1974, pp. 1[illegible]6-156.

40. Y.L. Chow, "On grating plateaux of nonuniformly spaced arrays," IEEE Trans. on Antennas and Propagation, Vol. AP-13, pp 208-215, March 1965.

41. A. Ishimaru, "Theory of unequally spaced arrays," IRE Trans. on Antennas and Propagation, Vol. AP-10, pp 691-702, Nov. 1962.

42. G.S. Sanyal and M.G. Sharma, "An element position modulated antenna array," 6th European Microwave Conference, 1976, Rome, Italy, pp 349-353.

43. M.G. Sharma and G.S. Sanyal, "Position modulated phased array - space factor considerations," IEEE Trans. on Antennas and Propagation, Vol. AP-27, No. 3, pp 373-375, May 1979.

44. T.M. Maher and D.K. Cheng, "On random removal of radiators from large linear arrays," IRE International Convention Record, Part I, 1962, pp 183-189.

45. Y.T. Lo, "A mathematical theory of antenna arrays with randomly spaced elements," IRE Trans. on Antennas and Propagation, Vol. AP-12, No.3, pp 257-268, May 1964.

46. M.I.Skolnik,J.W. Sherman III, F.C. Ogg, "Statistically designed density tapered arrays," IRE Trans. on Antennas and Propagation, Vol. AP-12, pp. 408-417, 1964.

47. B.D. Steinberg, "The peak sidelobe of the phased array having randomly located elements," IEEE Trans. on Antennas and Propagation, Vol. AP-20, pp 129-136, March 1972.

48. F. Hodjat, "Electronic scanning antenna using a nonuniformly spaced planar array," Proc. IEE, Vol.125, No.3, pp 185-189, March 1978.

49. V.D. Agrawal, "Grating lobe suppression in phased arrays by subarray rotation," Proc. IEEE, Vol. 66, No.3, pp 347-349, March 1978.

50. B.D. Steinberg, Principles of Aperture and Array System Design, John Wiley, New York, 1976, pp 135-136.

51. G.W. Swenson, Jr. and Y.T. Lo", The university of Illinois radio telescope," IRE Trans. on Antennas and Propagation, Vol. Ap-9, pp 9-16, Jan. 1961.

52. C.H. Tang, "On the optimum performance of nonuniformly spaced arrays, "IEEE Trans. on Antennas and Propagation, Vol. AP-14, No.5, pp 651, Sept. 66.

53. A.L.Maffet, "Array factors with nonuniform spacing parameters," IRE Trans. on Antennas and Propagation, Vol. AP-10, pp. 131-136, March 1962.

54. V.D. Agrawal and Y.T. Lo, "Mutual coupling in phased arrays of randomly spaced antennas, "IEEE Trans. on Antennas and Propagation, Vol. AP-20, pp 288-294, May 1972.

55. A. Ishmaru and V.S. Chen, "Thinning and broadbanding of antenna arrays by unequal spacings," IEEE Trans. on Antennas and Propagation, Vol. AP-13, pp 34-42, Jan. 1965.

56. G.V. Borgiotti, "A novel expression for the mutual admittance of planar radiating element," IEEE Trans. on Antenna and Propagation, Vol. AP-16, No.3, pp 329-333, May 1968.

57. R.J. Mailloux, "Radiation and near-field coupling between two collinear open-ended waveguides," IEE Trans. on Antennas and Propagation, Vol. AP-17, No.1, pp 49-55, Jan. 1969.

58. M.G. Sharma and G.S. Sanyal, "Admittance analysis of nonuniformly spaced phased arrays of waveguide apertures in a ground plane," 1979 International Symposium digest, Antennas and Propagation, pp 629-632, (summary of the paper).

59. M.G. Sharma and G.S. Sanyal, "Admittance analysis of non-uniformly spaced phased arrays of waveguide apertures in a ground plane," to be published.