# CHAPTER 1

INTRODUCTION

#### CHAPTER 1

#### INTRODUCTION

#### 1.1 Introduction to Bond Graphs

The present work is aimed at studying dynamic systems through substructuring and bond graph techniques. In this work, attempt has been made to enhance the art of bond graph simulations in the analysis of complex dynamic systems, taking advantage of the nature of subsystems and their interactions. The instances of application of bond graphs in system modelling are continuously increasing. At the same time, the geographical territories, over which bond graphs are gaining popularity, are also expanding. However, there are still some places where bond graphs have just appeared on the horizon of system analysis. Thus, to improve readability of the thesis, bond graph mnemonics are briefly explained alongwith suitable references in Appendix A.

Bond graphs are used to represent pictorially the energy exchange, storage and dissipation among interacting physical systems in an efficient manner. They are also extended to nonenergetic systems. Usefulness of bond graphs in system modelling and simulation lies in the precise way the assumptions of system model and their modifications are incorporated. They are used to model, with the help of a small number of basic elements, systems over different energy domains, namely, electrical, mechanical, hydraulic, pneumatic, magnetic, fluid, thermal and others in a unified manner. The bond graph techniques give rise to selection of state variables, derivation of system equations and their solution systematically. The whole procedure may be automated if advantage is taken of the existing computer programs such as ENPORT E67,683. Computer programs like TUTSIM E51,883 and CAMP E21,353, which are based on continuous system simulation languages, may also be used for this purpose. The growing acceptance of bond graph techniques, for their power in modelling, analysis and simulation of physical systems, may be observed from the updated bibliography by Bos and Breedveld E43.

#### 1.2 Substructure Synthesis

In analysing a system, very often it is advantageous to regard it as an assemblage of subsystems or substructures (in the present work both terms will be used interchangeably). These substructures are analysed separately and later coupled to give the overall system model which may be studied for the required dynamics of the system. This technique of analysing a system considering it to be a collection of subsystems, each represented suitably, is referred to as substructure synthesis E54].

The usefulness of such an approach is increasing with the advent of small computers and that capable of parallel processing. This is because subsystems may be analysed independently and their reduced order models may be used in studying the overall system. This is also suitable for re-analysis incorporating changes in subsystems, which is a very significant facet of computer aided design of engineering systems.

There are different techniques of substructure synthesis,

depending on the mode of representation of the subsystems. discussed these principles. Meirovitch Meirovitch [54] and Hale [23,24,55] proposed the use of admissible functions n f the subsystems for analysing complex systems. Grief and Wu [22] summarised different such techniques and their applications to a wide range of systems.

# 1.3 Bond Graph Adapted Subsrtucturing

Kron [36] introduced the concepts of substructuring for piecewise analysis of large systems. Paynter [64] and Karnopp [29] discussed the relationship between bond graph methods and Kron's concepts of diakoptics.

Karnopp and Rosenberg [27] and Margolis [41] presented to study the interconnected distributed and lupmedmethods systems using normal modes and bond graphs. Margolis and Young [42] proposed a technique of reducing the system equation obtained from bond graphs to the conventional second order form suitable for eigenanalysis. In Ref. [44] Margolis discussed the method of analysing distributed systems with mixed causal inputs (in bond graph sense). Margolis E45J also presented an algorithm for synthesis of subsystem models to obtain the overall system representation exploiting the causal information of the subsystem bond graphs. This approach of bond graph adapted substructuring was illustrated in Ref. [46] by considering a large scale structural model. Pastrnak [61] presented a procedure for simulating dynamics of large distributed structure through bond graph modelling techniques using experimental modal analysis data οf the substructures. Margolis and Karnopp [43] presented a bond graph based techni-

que of analysing multibody disrtibuted systems with geometric nonlinearities. Margolis [48] summarised such bond graph based investigations on the analysis of systems consisting of both distributed and lumped subsystems.

1.4 Classification of Dynamic Systems Studied in this Work

In the present work linear dynamic systems have been studied through substructuring and bond graph techniques. In this, a system is regarded as an assemblage of subsystems (substructures) each of which can be modelled suitably. The subsystems are assumed to have definite interactions. The systems are classified according to the type of subsystems and their interactions.

The types of systems considered in this work mav Ьe broadly divided into two groups. The first group consists οf systems with identical subsystems, and for the second, the subsystems may not be identical. Systems with identical subsystems are referred to as "repeated systems". Repeated systems are further classified as "parallel systems" which consist of identical subsystems connected in a topologically identical manner with long-range and non-identical interactions. They different from the "periodic systems" in a sense that the are interconnections may not be identical and they need not Ьe restricted to the immediate neighbourhood.

Parallel systems may be further classified on the basis of interactions among subsystems. When the ineractions are such that they can be derived from a set of potential functions, they are referred to as "potential interactions",

otherwise they are called "non-potential interactions". The potential interactions are divided into two groups. They are "commutative", when the interactions are such that the operators or matrices representing them commute. The interactions are "non-commutative", when the corresponding matrices do not commute. The "commutative potential" interactions have some special relations that they are amenable to simultaneous decoupling (diagonalisation) by a single transformation. This property is fully made use of in the present study. The "noncommutative potential" interactions have been studied using the technique of operator perturbations. In the category of non-potential interactions, only the commutative case has been discussed.

The systems with non-identical subsystems and a general kind of interactions have also been considered in this work. This category of systems are studied by a technique known as dual formulation. The dual formulation is based on the momentum coordinates unlike the usual Newtonian formulation based geometric coordinates. In this, subsystems are modelled on individually with proper boundary conditions at the interfaces. They are later coupled utilising the constraints of continuity and equilibrium at the interfaces. Here the subsystems may be from different energy domains. In the present study two important representative classes are considered. They are structural-structural and structural-acoustic systems respectively. The difference between the two classes lies in the method of dualisation of the subsystems. However, the procedure may be extended analogously to other systems.

Figure 1.1 shows the classification of systems studied





and their interactions .

in this work, according to the subsystems and their interactions. Periodic systems are not considered in the present investigation and are shown with a different boundary in Fig. 1.1. In the following sections a brief review of relevant literature on above systems is presented. This helps in identifying the scope of present work.

# 1.5 Survey of Relevant Literature

In this section a brief survey of literature on relevant topics is presented. This helps in identifying the scope of the existing methods and, in turn, the need for new techniques. However, to improve readability of each chapter, the references are cited again at proper places in the text.

## Periodic Systems

Periodic systems have been studied by many investigators. Brillouin [5] initiated the study of the propagation of harmonic waves through periodic structures in discrete crystal lattices. Mead [49] introduced a general theory of wave propagation in linear periodic systems with multiple coupling. This was subsequently enhanced in a two-part work [50] to study the relationship between the bounding frequencies of the propagation zones and natural frequencies of the individual substrucfor both "mono"- and "multi"-coupled systems. Denke et tures presented a matrix difference technique to obtain al. [14] harmonic response of the system. Meirovitch and Engles extended this technique, to study periodic systems, by using method Z-transform and modal analysis [18,53]. SenGupta [76,77] of summarised theoretical and experimental work on this subject.

However, there exists a class of repeated systems in which the interconnections may not be identical and they need not be restricted to the immediate neighbourhood. The interactions may be elastic, elastic with damping or visco-elastic. In such systems, termed here "parallel" systems, the periodic structure theory will not be applicable as such. Assumption of existence of harmonic solution is also a restriction in the present case. It is desirable to adopt a suitable method of analysis for these systems taking advantage of the identical subsystems and nature of their interactions properly.

# Parallel Systems with Potential Interactions

parallel systems with multiple interactions among the In subsystems, the method of analysis depends on the nature of interactions. Parallel systems with commutative, potential substructures have been studied interactions between by Samanta and Mukherjee [71]. Turbine blades interconnected with shrouds and lacing wires are among the examples of parallel systems with non-commutative, potential interactions. Natural frequencies of banded group of turbine blades were studied bν Prohl E66], and Weaver and Prohl E89] considering the discretised blades coupled through massless elastic shrouds and adapting the procedure to digital computation. Thomas and Belek [84] presented a technique for obtaining the vibration characteristics of a multiblade packet from the frequency inference diagram of a two-blade packet. This inference diagram can be constructed from the analysis of independent modes of uncoupled component blades and shrouds.

Tuncel et al. [87] analysed such systems in two steps.

First, the uncoupled blades were analysed and then the effects of coupling, due to the interconnections, were incorporated as perturbations. Though this method is attractive for its novel approach, it lacked in both quantitative and qualitative results, owing to the apriori assumption of weak coupling. Chen and Wada E6] used matrix perturbation techniques for studying the effect of small changes on the dynamic response of structural systems. Su-Huan et al. E783 developed a procedure based on this technique to obtain the free-free modes and frequencies of structural systems from the measured data of the constrained systems.

However, such systems can be studied in a more effective manner taking advantage of both bond graph techniques and a suitable pre-treatment followed by operator perturbation, without resorting to the assumption of weak coupling right from the beginning.

## Parallel Systems with Commutative, Non-potential Interactions

Problems of fluidelastic vibrations of tube arrays in heat exchangers and that of fuel rods in nuclear reactors belong to this class of parallel systems with non-potential interactions. Blevins [1] discussed different mechanisms like turbulence, vortex shedding, jet switching which can induce instabilities of tube arrays under the action of cross flows. In this, Blevins presented the quasi-static fluid force coefficients for calculating the critical flow velocities.

Tanaka et al. [60,81-83] measured fluid forces and used them to get critical velocities for different arrangements of

tube arrays and fluid flow. Chen [9,10] discussed two main mechanisms of instabilities based on displacement and velocity dependent fluid forces. Chen [8,11] also summarised the available literature emphasising on the analytical techniques, experimental results and future trends of research in this field.

Blevins' theory [1] is quite helpful in understanding the phenomena of flow induced vibrations with moderate computation. In the analysis of Blevins, effects of different constraints among adjacent tubes in the array and the dynamic details of individual tubes are not considered. The semianalytical technique of Tanaka et al. and Chen requires a significant amount of measurement and computation to determine the fluid force coefficients which are necessary for further analysis.

However, Blevins' theory can be utilised in a better way if the non-potential nature of the interaction among the tubes, due to fluid flow, is identified and exploited properly. This helps also in reducing the measurement and computation costs considerably.

#### Bond Graph Based Dual Formulation

Most of the analyses of dynamic systems are based on usual geometric co-ordinates. However, the possibilty of an alternative (dual) formulation based on momentum co-ordinates was pointed out by Trent E86], Toupin E853 and Chrandall et al. E123. Karnopp E263 extended the idea to study coupled vibratory systems. In Ref. E323, Karnopp studied system of acoustic filters using dual formulations and normal modes.

Lebrun utilised this approach in analysing hydraulic lines and fluid flow networks E37,38].

In Ref. [34], Karnopp presented four alternative formulations based on mass displacements, momenta, spring impulse and spring displacement coordinates respectively. The order of the equation varies with the type of formulation, as discussed in [33]. Karnopp also proposed an alternative procedure of Ref. modal analysis of the first order state equations derived from the system bond graphs, leading to the representation of the system in form of another bond graph. It retains the feature of symmetry of bond graph state equations in both types of energy variables (displacements and momenta) and both types of excitations (effort and motion). The bond graph so obtained is somewhat unusual in a sense that each modal oscillator (a combination of I-C connected to an S or 1- junction) aets I and a C element connected by a TF depicted by an with modulus equal to the corresponding natural frequency. It gives rise to an increase in the number of bonds in the final bond graph.

However, there exists a possibility of another procedure which is similar to the usual formulation. It is also necessary, at the subsystem level, to eliminate the high frequency modes due to coupling compliances incorporated among the subsystems for ease of analysis E44,453, in case of distributed systems. This will be particularly important in analysing response of such systems. A necessity is felt to study the dual formulation taking full advantage of the bond graph techniques and develop it as a useful tool for system analysis

through substructuring.

Margolis and Tabrizi E473 proposed a procedure of repeated modal decomposition of the system equation to study the dynamics of interacting lumped and distributed systems. It is found necessary to extend the same concept to the bond graph adapted dual space formulation.

## Acoustoelastic Systems

Systems consisting of acoustic and structural componentes, termed "acoustoelastic" systems [16] were studied by Lyon [40], Dowell and Voss [15], and Morse and Ingard [56] for simple harmonic motions of structures. Dowell et al. [16] equations for arbitrary wall motions developed the usina Green's theorem for the acoustic region. The coupled system equation could be obtained from the in vacuo structural modes and rigidwall acoustic modes. Dowell [17] surveyed the available literature on vehicle interior noise prediction. Nefske et al. [58,59] presented reviews of current practices for vehicle interior noise analysis based on finite element method. [80] presented a dual formulation for acoustoelastic Tabarrok . It is felt advantageous to extend concepts of systems. bond graph adapted dual formulation to the analysis of such systems.

# 1.6 Scope of the Present Work

From the above discussion, it is felt that there exists a scope for an organised study of a wide range of dynamic systems, with help of bond graph techniques, taking full advantage of their flexibilty in modelling multi-energy dynamic

systems and uniqueness in formulation of system equations. The uniqueness lies in the symmetric manner of treating momentum and displacement variables and both flow and effort type of excitations. This feature helps in extracting useful information, from the bond graph of the system, for further processing in simulating its dynamics.

The present work is discussed in the next eight chapters. Chapters 2 to 4 deal with the analysis of repeated systems. In Chapters 5 to 8 non-repeated systems have been studied using dual space formulation. Contents of each chapter are summarised in the following paragraphs.

#### Parallel Systems with Commutative, Potential Interactions

In Chapter 2 a procedure is discussed to study the dynamics of parallel systems with commutative, potential interactions. Subsystems are decoupled by a similarity transformation influencing only the interactions. The process of decoupling consists of two steps. In the first step, a pilot bond graph is constructed replacing each subsystem by a unit mass at the point of interconnection and retaining only one interaction at a time. The second step consists of construction of a decoupsubsystem bond graph with the constraining elements led obtained from the eigenanalysis of the pilot bond graph. In case systems with more than one commutative interactions, of the pilot bond graph with one interaction need be considered, and effects of other interactions may be suitably incorporated in the decoupled subsystem model. The overall system dynamics can studied with the same computational effort necessary be in analysing a single subsystem.

Parallel Systems with Non-commutative, Potential Interactions Chapter 3 deals with a technique to study the dynamics a class of repeated systems with non-commutative inteοf ractions. The system equations are formulated using tensor product algebra. A pre-analysis of the system equations is carried out to reduce the interactions to a small perturbing operator. The procedure of pre-treatment is adapted to bond graph techniques for the ease of representation of dynamic systems. It is shown that the method of operator perturbation may also be handled very conveniently by bond graph approach. The overall system dynamics can be studied from the analysis of the subsystems coupled through the perturbing operator thus reducing the size of the problem to be considered at a time. procedure helps in assessing the effects of different The interactions in a systematic manner.

#### Parallel Systems with Commutative, Non-potential Interactions

Chapter 4 is devoted to the discussion of a procedure for studying the dynamics of parallel repeated systems with nonpotential interactions within the framework of bond graph techniques. This type of interaction, when decoupled, gives rise to complex elements which cannot be used for further processing in the conventional bond graph analysing packages like ENPORT. This shortcoming is removed by reformulating the decoupled subsystem equations to an equivalent form with all real parameters leading to reduction in size of the problem to studied at a time. The procedure is illustrated by the be of fluidelastic vibration of tube arrays in example heat exchangers. The technique is suitable for interactive design

purposes because of its ability in assessing the effects of different system parameters on design variables with moderate computation and reasonable accuracy.

#### Bond Graph Adapted Dual Space Formulation

technique is presented to study the In Chapter 5 a overall characteristics of complex dynamic systems from those the subsystems which are easy to model individually. The σf complex system is divided into a number of simple subsystems which may be represented by using either positioneach αf velocity or impulse-force descriptions. The appropriate boundary conditions are assigned at the interfaces. The subsystems are then interpreted in form of bond graphs using a finite mode representation. These bond graphs are coupled through proper elements such that the conditions of equilibrium and continuity at the intrefaces are preserved. The overall system dynamics are studied from the resulting bond graph. The procedure is illustrated through simple examples. The entire analysis is based on bond graph techniques and thus allows the effective use of ENPORT packages. The main advantage of the proposed technique lies in unambiguous substructuring of multi-connected distributed systems and removal of redundant state variables. It is also suitable for "stiff" systems because of modal reduction at subsystem level.

# Repeated Modal Decomposition in Bond Graph Adapted Dual Space.

The overall system dynamics can be studied using dual formulation as discussed in the previous paragraph. From the

coupled subsystem bond graphs in dual space the overall system model can be reconstructed in an equivalent uniform c o ordinate space. This again can be represented in form of bond graph taking advantage of a second stage modal another decomposition. The effects of damping and external excitations can be incorporated in this model suitably. The system can be obtained over the required frequency range response retaining the appropriate modes. The procedure is particularly suitable for "stiff" systems because of its scope of modal truncation at both the stages of analysis. Chapter 6 deals with this aspect of bond graph adapted dual formulation.

# Role of Zero Frequency Modes in Bond Graph Adapted Dual Space Formulation

The technique of analysing dynamic systems making use of formulation and bond dual graphs has been presented in 5 and 6. The representation of the subsystems and Chapters. their subsequent modal decomposition have been based on their dynamic modes, removing altogether the non-dynamic modes characterized by zero frequencies. However, sometimes in a momenbased formulation there are zero frequency modes which tum play a significant role in the overall dynamics of the systems when eliminated, lead to anomalous results. In chapter 7, and role of such nondynamic modes in bond graph adapted dual the is discussed. An algorithm is presented for formulation retaining these modes in the overall system model. The procedure is extended to obtain system response taking advantage of second stage modal decomposition. The procedure is illusa trated by suitable examples.

#### Acoustoelastic Systems

discusses a procedure for analysing Chapter 8 the dynamics of acoustoelastic systems within the framework of bond graph techniques. The substructures, acoustic and structural, are modelled individually in form of bond graphs which are coupled through suitable elements satisfying the conditions at the interfaces. From this bond graph a second stage decomposition is performed to represent the overall modal in yet another bond graph which can be svstem analysed to the overall system dynamics. The procedure is illusobtain trated by the examples of coupled cavities. The svstem different external responses due to excitations are also obtained. The technique is suitable for acoustoelastic systems which are "stiff" in nature due to the wide range of their natural frequencies.

# Conclusions

This chapter is devoted to summarise the present work as to its aplicability in analysing dynamic systems of different classes alongwith its salient advantages. The future scope of the work is also discussed.

The work concludes with a list of references cited in the text.