CHAPTER I.

I N T R O D U C T I O N.

1.0. GENERAL.

Beginning with an abstract representation of a message by symbols, the transmission of telegraph signals in 1838 ushered in the era of modern communications. With the increase in traffic, the economy of the multiplexed systems was appreciated and a successful Time Division Multiplex (TDM), in the form of Baudot telegraph system, was commercialised as early as 1874. However, the lack of high speed switching devices and the practical difficulties in instrumentation held up any further progress in the TDM system for transmission of speech and other complex signals. The tremendous interest in the transmission of speech over long distances led to the development of telephony and the associated techniques of modulation, transmission and reception developed with the progress of electronics. By 1918, many messages could be transmitted over the same link by using Frequency Division Multiplex (FDM) methods. For conserving frequency space, the Single Sideband transmmission has proved to be the most efficient among the available FDM methods. Wideband Frequency Modulation (FM), on the other hand, has been found to possess considerable immunity from noise and interferences in the communication link.

With the availability of faster and newer devices, further investigations in the TDM systems were carried out, and by 1942, new Pulse Modulation systems were developed. These systems provide some protection from noise and interference in the link, though at the cost of larger bandwidths. The Time Division methods employ the principle of quantization in time, and the message is carried as a change in amplitude, length, position, slope or frequency of some pulses, thereby giving rise to the systems[1,2], PAM, PLM, PPM, PSM and PFM, respectively. For RF transmission, compound modulation schemes, such as PAM-FM, PPM-AM, etc., have been developed, and PAM-FM has been found to be the most efficient among these wideband systems. These systems are essentially analogue schemes, and the message is conveyed in terms of any value in a continuous range of values. The analogue schemes are susceptible to external noise, they have an average error, and the errors are cumulative in a long distance link in which repeaters are used. Even then, these schemes have been used to advantage in Radio Relays and Satellite Communication systems where a compromise between bandwidth and system noise has been possible.

So far, the transmission of speech and complex signals have been in the analogue form only, but with the immense development in the fields of telemetry, digital computers, and remote and automatic controls, the message signals have been diversified in form, and discrete signals, two valued and multivalued, are being widely used. Digital transmission of message signals has many advantages over analogue transmission, such as, lesser susceptibility to channel noise, ease of amplifying the signal pulses with regenerative repeaters, and the possibility of a TDM of many channels without using costly filters. The advantages of digital transmission resulted in the development of systems in which the analogue signals are first converted into a number of discrete levels. The effect of this quantization is to introduce an error in the beginning of the processing itself but leading to a subsequent error-free transmission, so that, the error encountered in the received signal is mainly due to the quantization. This quantization error can, however, be reduced to any desirable degree by choosing smaller spacings between the discrete levels.

The general problems of communication and signal processing may be classified broadly into two groups, such as, (1) signal transformation, and (2) their subsequent transmission. An analogue signal converted into a multi-level quantized form has many difficulties, both in the transformation and transmission, due to the lack of multistate storage and switching devices, and their vulnerability to noise interference. Necessity, therefore, arises for some form of coding after quantization in order that the transmission is made more efficient. The code, currently used in practice, is the Binary code, and it has found wide applications in digital computers, hybrid computers, memory and storage devices, telemetry, digital feedback systems, electronic exchange and speech communication. Notable among the systems, which use quantization in time and amplitude and are binary coded, are the Pulse Code Modulation[3] (PCM), Delta Modulation[4] (Δ-M), and the Delta-Sigma Modulation[5] (Δ-ΣM). Here the presence, or absence, of a pulse in a sequence of on-off pulses carries the information, and, therefore, these systems are insensitive to noise and disturbances, to the extent that the received signal is almost unaffected by the circuit noise.

The need for the efficient utilisation of various media for transmission of digital signals has stimulated the investigation of sophisticated coding and modulation techniques. The channel capacity of 2 bits/second/cycle of bandwidth of the binary code could be increased by using a q-ary code, but again, the lack of suitable devices restricts the feasibility of many such schemes. However, Alexander[6] has shown that, a Ternary code will be more efficient, even with the use of binary devices to generate a ternary code. The amount of equipment required, to achieve the same function, will be less with the ternary than that with the binary code, and the channel capacity will be larger, being 3.16 bits/sec/cycle of bandwidth. Ternary Digital computers and other Ternary coded systems are being developed, and it is expected that, they will have some advantages over the Binary coded ones.

A good coding scheme, however, does not necessarily lead to a more efficient system for transmission. Apart from the noise produced in the process of quantization, the input at the distant-end receiver consists of extra noise contributed by the channel, and a wideband modulation scheme for transmission is preferred, because of its valuable channel-noise-reducing properties. In fact, many compound coding and modulation schemes have been developed to improve the transmission efficiency and the Orthogonal-matched-filter communication systems require less power to transmit at a given information rate than any other system and approach Shannon's ideal.[8] The coded pulse modulation signals could either be transmitted by cables, or modulated into a wideband system like FM, or

Phase Modulation (PM), or some other compound modulation scheme. With the same channel noise, the power in these wideband systems can be reduced considerably at the cost of larger bandwidths - a condition particularly useful in satellite communication and other similar situations.

1.1. CODED PULSE MODULATION SYSTEMS.

The digital transmission systems, as is well known, offer many advantages, and it is necessary to evaluate their performances continuously and seek to improve upon them by newer and better techniques. In the familiar Binary PCM, called here Amplitude-Quantized Pulse Code Modulation (AQ-PCM), the samples of the message signal (taken at the Nyquist rate of $2W_m$ per second) are amplitude quantized into ℓ levels, and each level is coded into a set of 'n' binary 1/0 pulses giving $2^n = \ell$ levels. Several practical systems of AQ-PCM[9,10] (refer to Appendix A) with n = 6 or 7 have been widely used. In the Δ-M system (refer to Appendix B), a waveform matching by simultaneous quantization of the signal in time and amplitude (at a considerably higher pulse repetition frequency) is achieved by using a negative feedback loop. The coding is done by binary +1/-1 pulses, and the feedback loop incorporating the distant receiver (normally an integrator) helps in reducing the errors of quantization.

The binary AQ-PCM system seems to offer the optimum performance in coding and quantization. The coding circuit of AQ-PCM, however, includes translators, diode matrices, and stable logic circuits etc., and is highly complex. The coding efficiency of the AQ-PCM system is low at lower Pulse Repetition Frequencies (PRF). The Signal-to-Quantizing-Noise ratio [SNR(Q)] decreases with input signal level, and, therefore, results in a lower efficiency for the lower input levels. Thus the dynamic range is limited and the weaker signals have a poorer SNR. An artifice employed in AQ-PCM is to taper the signals before quantizing, so that the weaker signals are amplified more than the stronger ones, and hence, the dynamic range is extended.

AQ-PCM requires synchronous transmission but compensates for this by its insensitiveness to waveform and amplitude distortion of the pulses. By virtue of this independence, an intrinsic adaptation to high information rates can be achieved by minimising the amount of information on which the bit recognition is based. In the video transmission through cables, or in the analogue-to-digital conversions, an error free transmission is assured if the input SNR is more than a threshold of about 20 db. The theoretical bandwidth is half the bit rate, although a higher bandwidth is normally used in practice. AQ-PCM is accommodated readily by Amplitude Modulation (AM), Frequency Shift Keying (FSK), FM and PM schemes. Such compound modulation schemes are necessary for radio transmission and possess an additional property of channel noise suppression. Among the digital wideband modulation schemes, AQ-PCM-PM requires the least power for a certain communication, and the power and communication efficiencies of this system are the highest.

The Δ-M system, on the other hand, is very simple in circuitry and is comparable to the AQ-PCM system at lower PRF's. For a good grade of service, however, the required PRF is higher in the Δ-M than that in the AQ-PCM system. Moreover, the Δ-M system suffers in some other respects, e.g., it has a frequency distortion and a smaller overload and dynamic range. The frequency distortion makes the system unsuitable for many applications, although the coding characteristic matches the speech signal characteristics and a speech of commercial quantity can be processed. Zetterberg[11] has argued that the extension of the dynamic range by using a non-linear coding (for an equalised SNR) is not practicable in Δ-M system because of the limited message bandwidth handled by the system.

Inose et al have developed a modified Δ-M, called Δ-ΣM system (refer to Appendix B), which is also very simple in circuitry. The Δ-ΣM overcomes some of the shortcomings of the Δ-M, in the sense that it has no frequency distortion. The Δ-ΣM system is suitable for processing of wideband inputs, and the optimum SNR, although lower than that in Δ-M, is independent of the signal frequency in the message band. The coding efficiency, the power efficiency, and the communication efficiency of the Δ-M and Δ-ΣM systems are poorer as compared to those of the AQ-PCM system at higher PRF's. However, the use of Δ-M and Δ-ΣM in the analogue-to-digital conversion applications seems to be economical because of the simple circuitry. Other than the requirement of a slightly higher PRF in comparison to that in the AQ-PCM system, these two systems have a very slow improvement in SNR with an increase in PRF.

The overall limitations of the AQ-PCM, Δ-M, Δ-ΣM systems may then be summarised as:

(i) The channel capacity with binary coding is limited.

(ii) The coding efficiencies are low, and none of them approaches Shannon's ideal system.

(iii) The ΔQ-PCM is highly complex in circuitry, and though the Δ-M is simple, it has an appreciable frequency distortion and a lower SNR for practical applications.

(iv) The improvements in the Δ-ΣM over Δ-M system have been obtained at the cost of lower SNR.

(v) Inherently, all these systems produce more quantizing noise for the weaker signals.

(vi) By nonlinear coding of the input signals, the dynamic range in ΔQ-PCM is increased, but such a device has not been tried in the Δ-M system.

(vii) The improvement in SNR with an increase of PRF is slow in Δ-M and Δ-ΣM systems as compared to that in the ΔQ-PCM system.

(viii) The power efficiency and the communication efficiency of the Δ-M and Δ-ΣM systems are low.

1.2. SCOPE OF THE WORK.

The nature of the limitations of the various binary systems suggests that it would be worthwhile to explore the possibilities of a system using the ternary code. A ternary code, as has been mentioned earlier, has a larger channel capacity than that of the binary code, and it may be economical in the use of number of components and circuits. However, the practicability of such a system will depend upon (i) the availability of three-state devices that can be used for the coding, and (ii) the simplicity of the circuits so devised. So far, no devices are available which have three possible stable states, and therefore, the coding has to use only binary devices. An amplitude quantization of the signal waveform and a subsequent coding into three levels (+1, 0, -1) seem to be a reasonably attractive idea, but unfortunately, this will lead to complicated and elaborate circuitry. A Uni-digit system (somewhat similar to Δ-M), where the slopes of the signal waveform could be quantized into three level (+1, 0, -1) pulses, does, however, offer the advantages of the simple circuitry of Δ-M together with the improvement obtained in using a ternary instead of a binary code.

A new system based on the slope-quantization of signals has been developed in the laboratory by the present,author, and has been called here as "Ternary Slope Quantized-Pulse Code Modulation" (SQ-PCM) system[12]. In this system, the values of the slopes of the signal waveform at the sampling instants, sampling being done at a rather high rate, are quantized into +1, 0, -1 pulses by using two threshold devices in parallel. If the values of the slopes exceed the particular reference thresholds, either positive or negative, they are taken as +1 or -1 pulses, respectively. However, if the values of the slope are less than either of the thresholds, they are taken as 'no' pulses. The output thus consists of +1, 0, -1 pulses. The approximation of the input signal obtained by this straightforward but crude quantization is rather poor, though tolerable for a certain class of signals. A simple expedient of building up the above approximated signal and, then, comparing it with the original input in a negative feedback loop, improves the quality of approximation considerably. The feedback reduces the errors in the approximation and, thus, results in a better waveform matching between the input and the output. The Ternary SQ-PCM system with feedback has been found to be much superior to the Ternary SQ-PCM system without feedback.

Encouraged by the results obtained in the Ternary SQ-PCM system, the author has also investigated the possibilities of a similar binary coding, and has successfully developed a Binary SQ-PCM system[13]. The Binary SQ-PCM system uses 1/0 coding of the slopes of the signal waveform, and the refinements in the coding process over that of the Δ-M system are in the use of a threshold in the quantizer, and in the special feedback network used for signal approximation. Like the Ternary system (with feedback), the encoder compares the original signal with the approximated one built up by the feedback network, and tries to reduce the errors of quantization. The impulse response of the feedback network is such that it decays exponentially in a certain manner, and matches the positive slopes of the signal in the usual way. The negative slopes of the input waveform are matched by the cumulative negative slopes of the exponentially decaying pulses, and a +1 pulse is generated whenever the approximated slope exceeds the slope of the input waveform by more than an optimum threshold. Thus, the step by step modification of the cumulative positive and negative slopes matches the input waveform fairly well and a good approximation of the signal is obtained by 1/0 pulses only.

An endeavour has been made here to present in a connected way the details of the theory and the experimental results obtained for the three systems, viz., (a) Ternary SQ-PCM system without feedback, (b) Ternary SQ-PCM system with feedback, and (c) Binary SQ-PCM system. The performances of the systems have been experimentally evaluated for different types of input signals. The Ternary-coded SQ-PCM (with feedback) has been found to give a better SNR than the equivalent binary AQ-PCM system (on the basis of practical ranges of PRF). The circuit developed is quite simple and produces a negligible frequency distortion. The Ternary SQ-PCM system without feedback has been found to be suitable only for some special signals like narrow-band FM. Tests have shown that a tolerable quality of speech is also reproduced. The Binary SQ-PCM system has a better SNR (upto a certain PRF) than the equivalent (equivalent in PRF) AQ-PCM, Δ-M and Δ-ΣM systems, and the coding efficiency is higher. The frequency distortion is less than that in the Δ-M system. These systems, however, have the same inherent defect of producing more quantizing noise for the weaker signals. An instantaneous compandor has been successfully used to extend the dynamic range of both the Ternary (with feedback) and the Binary systems. But the improvement in SNR with an increase in the PRF remains slow like that in the Δ-M and Δ-ΣM systems.

A theoretical estimate of the transmission characteristics of the Ternary and the Binary SQ-PCM systems has been made. The SNR required at the input of the receiver for satisfactory communication, using either Video or Wideband modulated SQ-PCM systems, has been calculated. Different SQ-PCM-AM, SQ-PCM-FM and SQ-PCM-PM schemes have been proposed and it has been shown that the Ternary SQ-PCM-PM requires the least amount of transmitter power. The power and communication efficiencies have also been estimated, and again the digital phase modulation technique of transmission is found to be the most efficient. It has also been shown that the SQ-PCM systems can be multiplexed on the TDM basis in the same fashion as the AQ-PCM system.

In conclusion, a critical evaluation of the system characteristics and an overall comparison of these systems with the other existing digital systems has been made. From the results, discussed in the later Chapters, it will be shown that the Uni-digit Ternary-coded SQ-PCM system with feedback shows excellent promise for direct applications in communication and in other analogue-to-digital conversions, where the digital signal processing is advantageous. The Binary SQ-PCM system, though a grade inferior to the Ternary system at the same PRF, shows a considerable improvement over other similar systems, both for RF transmission and analogue-to-digital conversions.

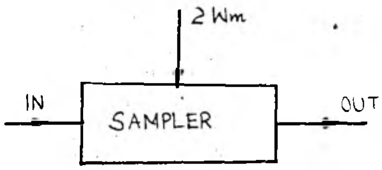


FIG. NO. 2·1 (a)

TIME QUANTIZER

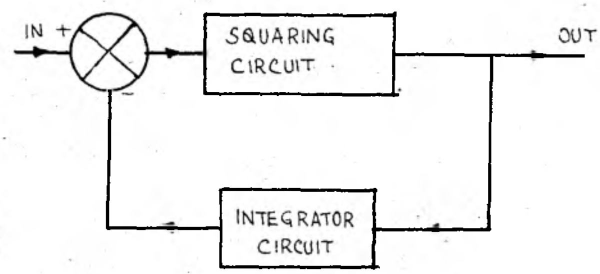


FIG. NO. 2·1 (b)

AMPLITUDE QUANTIZER

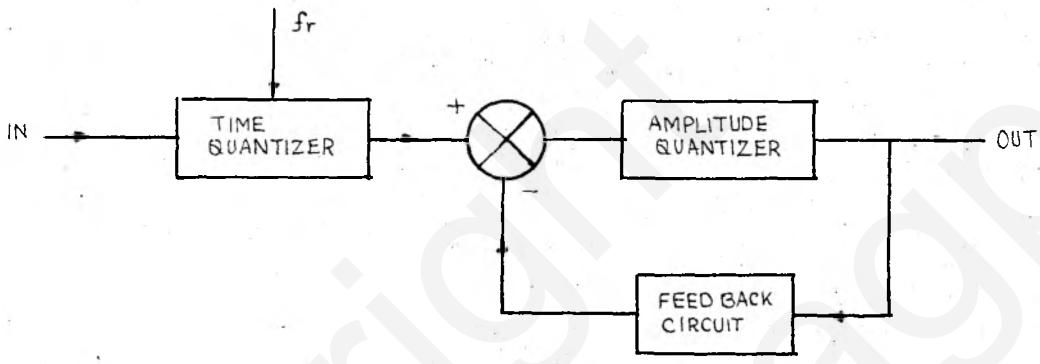
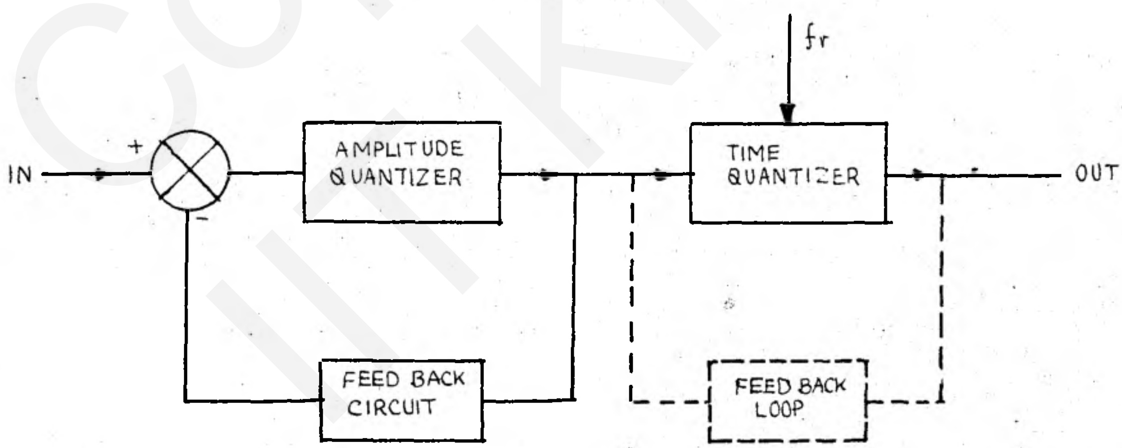


FIG. NO. 2·2

A MODEL OF CODED PULSE MODULATION



AN ALTERNATIVE MODEL OF CODING

FIG. NO. 2·3

the pulse-amplitude is quantized into q^n levels so that each sample is coded into 'n' q-ary digits. For q = 2 this is the familiar Binary AQ-PCM. If the time quantizer is after the amplitude quantizer as in Fig.2.3, the sampling frequency of the time quantizer has to be very high, as the bandwidth of the pulses at the output of the amplitude quantizer is large, say, of the order of 40 - 50 Kc/s. The sampling frequency then has to be of the order of 80 - 100 Kc/s to avoid the aliasing distortion. A separate feedback loop (shown dotted in Fig.2.3), across the time quantizer may correct some of the distortion introduced and thus improve the performance of the circuit considerably.

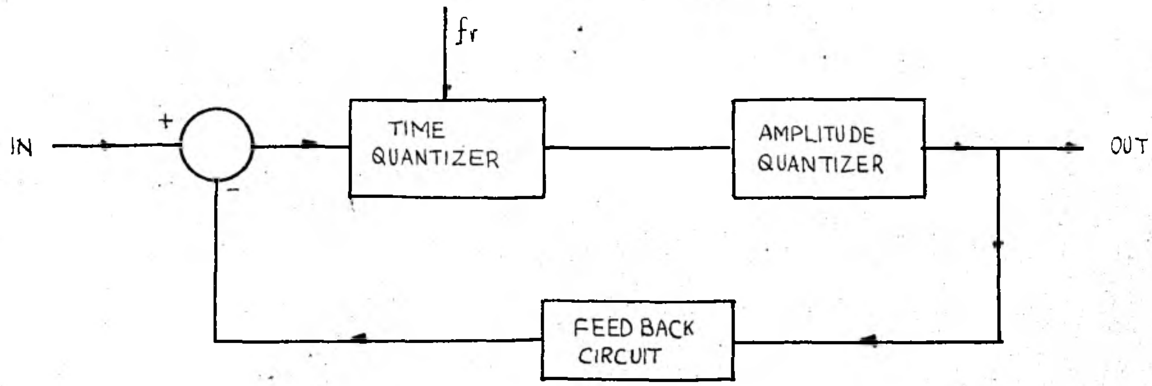
A rearrangement of the two basic blocks, to include both the quantizers in the feedback loop, is shown in Fig.2.4. The feedback circuit, therefore, has to be modified and generally it is a replica of the receiver at the distant end; the idea being that a comparison of the message signal with the reconstructed signal at the receiver would correct or reduce the error as much as possible. With proper time and amplitude quantization techniques, many systems can be built up based on this model. The sampling, for instance, can be done say at 2 W_m c/s with a subsequent n-q-ary-digit coding of each amplitude by the amplitude quantizer. The feedback circuit builds up the signal from the coder output into a staircase waveform which is fed to the comparator, and a consequent reduction of quantizing noise occurs. With a particular coding method employed, this is feasible and will lead to a PCM (AQ-PCM) system. For q = 2, it will be a Binary AQ-PCM system and with q = 3, it will be a Ternary AQ-PCM system.

However, the sampling may also be done at a much higher frequency ($\gg 2W_m$ c/s) and a q-ary amplitude quantization with uni-digit coding be used. The Δ-M system developed by de Jaeger[4] uses a binary +1/-1 Uni-digit coding where the feedback circuit (local receiver) consists of an integrator. A slightly different technique of binary +1/0 Uni-digit coding together with the shaped impulse response of the feedback circuit has been developed in the form of a Binary SQ-PCM system in the present work. With a similar sampling frequency, a Ternary +1/0/-1 Uni-digit coding, using an integrator in the feedback loop, has been found to give a much better performance and the Ternary SQ-PCM is another such system developed in this work.

An alternative arrangement of the time quantizer after the amplitude quantizer, with the integrating circuit included in the forward loop is shown in Fig.2.5 and was proposed by Inose[5] et al, called Δ-ΣM . The system is a Binary Uni-digit system and has certain advantages. In all the systems using the Uni-digit coding, the quantizing noise can be reduced by a proper shaping of the impulse response of the feedback network.

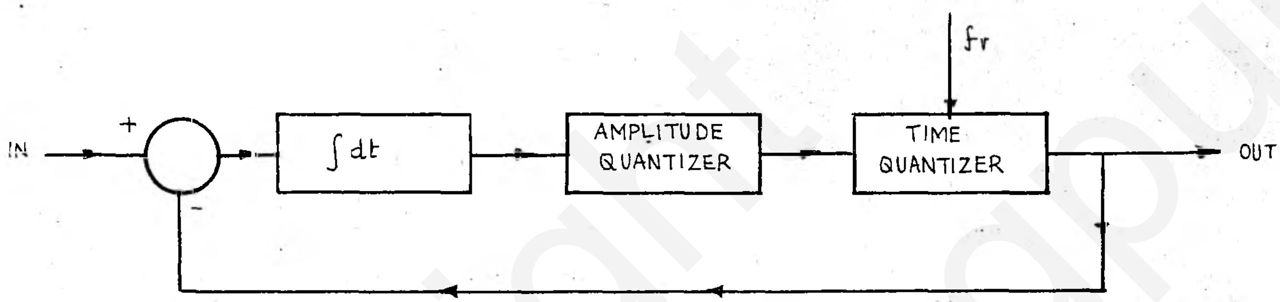
Thus it is seen that the majority of the Coded Pulse Modulation systems are built up by an interplay of the two basic blocks of time and amplitude quantizers and employ a feedback loop to reduce the errors of quantization as much as possible.

The Ternary coding of the Uni-digit pulse systems leads to some marked improvements over the Binary systems and Section 2.1



ANOTHER MODEL OF CODING USING A COMMON FEED BACK LOOP

FIG. NO. 2·4



MODEL FOR Δ-ΣM

FIG. NO. 2·5

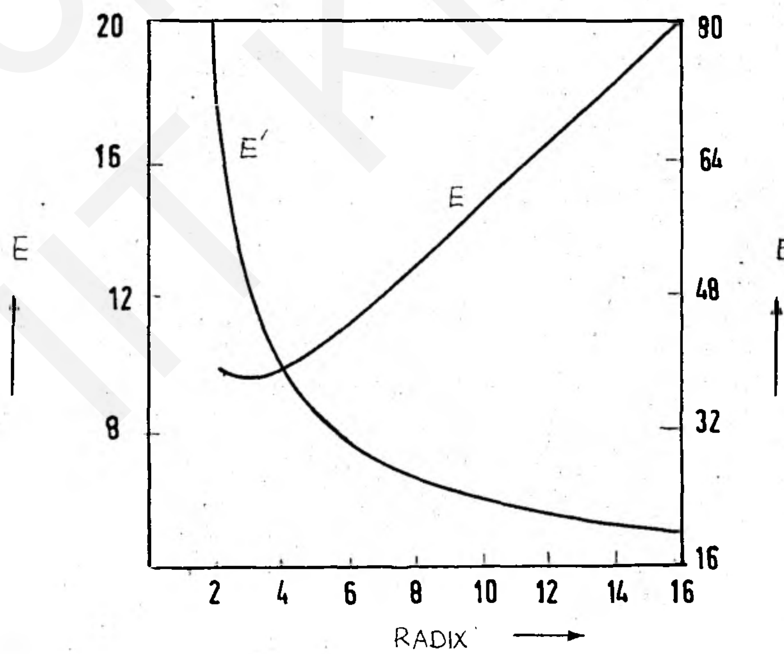


FIG. NO. 2·6

EQUIPMENT EFFICIENCY FOR DIFFERENT RADIX VALUES IN DIGITIAL SYSTEM discusses the advantages and disadvantages of employing a Ternary code. Section 2.2 gives briefly the principles of operation of the Ternary SQ-PCM system, while Section 2.3 discusses the principles of operation of the Binary SQ-PCM system with special reference to the impulse response of the feedback network. The theoretical calculation of the quantizing noise produced in the Ternary and the Binary SQ-PCM systems are given in Section 2.4. The principles of the compandors and the pros and cons of instantaneous vs syllabic compandors have been discussed in Section 2.5.

2.1. THE TERNARY CODE.

While discussing the different possible models of the coded pulse modulation systems, it was mentioned that the sampled and quantized signals could be encoded into a 2 level, 3 level or multilevel codes. The two level Binary code has been most commonly used both in communication systems and in digital computers. In fact, the Binary code with its associated Boolean Algebra has completely dominated the digital systems field so far, not only because it is simple, but also because all the electronic and magnetic devices so far known are two-state on-off devices. Recently there has been some realisation that a Ternary Computer[6] may prove more advantageous and economical. It has also been shown, in the present work, that a ternary-coded system offers certain advantages over a binary system. A theoretical study of the Binary and the Ternary codes has been made here and it is possible to show that the 3-level code (+1, 0, -1) is more economical in equipment and has a higher channel capacity.

Considering the general aspect of numbers, the common decimal system is based on ten-digit symbols, that is, its radix is 10. Almost as important as the radix are the idea of positional meaning of the various digits in a number, negative numbers and the concept of zero. The simplest number system is the binary system with a radix 2. It requires only two symbols 0 and 1 in all digit positions and all numbers could be expressed with this arrangement. A binary number may look like, 11 0 111, and represent number 55. The ternary number system requires 3 symbols (0, 1, 2 or 1, 0, -1) in all digit positions and its radix is 3. A ternary number may look like, 21120, and represent the number 104. A general expression for a number N in any base is

$$N = F_n R^n + F_{n-1} R^{n-1} \;..\;\;..\;\;..\; F_2 R^2 + F_1 R^1 + F_0 R^0$$

where R is the Radix and F represents the symbols or integers of the system. The highest integer, F_{max}, in any system must be smaller than the radix itself i.e.

$$0 \leqslant F_{max} \leqslant (R - 1).$$

As has been said before, all the devices so far used are suitable for the binary system, but it is possible to build up circuits for a higher radix by a combination of circuits operating on a lower radix. At the moment it is assumed that it is possible theoretically to use a different radix, different from a radix of 2. The question arises whether there is a particular radix that will lead to circuits that give greater economy in equipment without loss of speed. It is evident that the smaller radix-number system will require more number of digit positions to express a given number. If N is the largest number which has to be handled by the system, and if the radix used is R with n digit positions required to express the number, then $N = R^n$. If the amount of equipment E is taken proportional to the digit capacity of the system, then,

$$E = Rn$$

$$\text{or} \quad E = R\frac{\log N}{\log R} \qquad \ldots (2.1).$$

Now the radix for the minimum amount of equipment required for handling N can be obtained by minimising the equation (2.1), i.e. by putting $dE/dR = 0$. This gives the minimum radix as $R = \epsilon = 2.718$. Since R must be a whole number, the nearest integer is 3. Therefore, purely from the theoretical view point, the ternary radix is the most economical.

From a different consideration, supposing it was possible to have devices which exhibit 3, 4, 5, 6 etc. stable states, then one can choose the device to correspond to the radix chosen. The number of components or the amount of equipment E' required to handle the number N would be just proportional to n only, thus,

$$E' = \frac{\log N}{\log R} \qquad \ldots (2.2).$$

If the number N is chosen as 10^6, the amount of equipment E and E' is calculated from eqn.(2.1) and (2.2) and is plotted in Fig.2.6.[6] For both the conditions it is seen that the amount of equipment, compared to the Binary, is less in the Ternary system.

2.1.1. CHANNEL CAPACITY.

It will be further shown that the Ternary coded pulses have a higher channel capacity in transmission although they will require about 4.2 db more power than the binary coded pulses to achieve the ideal channel capacity. Considering first the case of a noiseless channel, the information rate in a bandwidth W is given as[15],

$$H(x) = C = 2W \log_2 b \qquad \ldots (2.3).$$

where $H(x)$ = Entropy of the source

C = information rate or the channel capacity in bits/sec.

and b = no. of equally possible levels of the samples.

In the Binary system $b = 2$, as the two possible levels are 1 and 0, and therefore

$$C_{BINARY} = 2W \text{ bits/sec.} \qquad \ldots (2.4).$$

In the Ternary system $b = 3$, as the three equally possible levels are +1, 0, -1, and therefore

$$C_{TERNARY} = 3.17\, W \text{ bits/sec.} \qquad \ldots (2.5).$$

Thus ideally, the channel capacity of the Ternary coded systems is more (in the ratio of 3.17 to 2) than the Binary coded system. Now supposing that noise is present in the channel and the effect of noise is to cause errors in the identification at the receiver. The rate of transmission of information is less now, and evidently, if no information is to be lost, the receiver must remove all uncertainty as to what was sent. The information and the noise are both statistical processes. The entropy of signal output (x) of the transmitter is H(x) and the entropy of signal input (y) of the receiver is H(y). In the absence of noise

$$H(x) = H(y)$$

If the conditional entropy, $H_x(y)$, is the entropy of the receiver input when the transmitter output is known, the conditional entropy $H_y(x)$ is the entropy of the transmitter output when the entropy of the receiver input is known, and if H(x,y) is the joint entropy of the transmitter output and receiver input, then

$$H(x,y) = H(x) + H_x(y) = H(y) + H_y(x)$$

$H_y(x)$ is the uncertainty of the sent message when the received message is known, and has been called Equivocation by Shannon[8]. Equivocation is the residual remaining uncertainty when the received signal has been interpreted. Consequently the actual rate of transmission is :

$$C' = H(x) - H_y(x) \qquad \ldots (2.6).$$

The equivocation $H_y(x)$ can be calculated with the help of the following formula :

$$H_y(x) = -\sum_i P_i \sum_j p_{ij} \log p_{ij} \qquad \ldots (2.7)$$

where P_i = probability of the symbol i

p_{ij} = probability that j will be the received message when i is transmitted.

The equivocation for a frequency of error of 1 in 10^3 has been calculated from formula (2.7) for the Binary and Ternary systems as:

$$\left[H_y(x)\right]_{BINARY} = 0.011396 \text{ bits/digit.} \qquad \ldots (2.8)$$

$$\left[H_y(x)\right]_{TERNARY} = 0.020370 \text{ bits/digit.} \qquad \ldots (2.9).$$

and the entropy of the source is :

$$\left[H(x)\right]_{BINARY} = 1 \text{ bits/digit.} \qquad \ldots (2.10).$$

$$\left[H(x)\right]_{TERNARY} = 1.58496 \text{ bits/digit.} \qquad \ldots (2.11).$$

Therefore, the actual channel capacity or the information rate for the binary system from eqns.(2.8) and (2.10) is :

$$\left[C'\right]_{BINARY} = 0.988604 \text{ bits/digit.} \qquad \ldots (2.12).$$

The actual channel capacity for the ternary system from eqns.(2.9) and (2.11) is :

$$\left[C'\right]_{TERNARY} = 1.56459 \text{ bits/digit.} \qquad \ldots (2.13).$$

It is seen from eq.(2.12) and (2.13) that the actual channel capacity for the ternary system is higher than that for the binary system.

Shannon[8] has further shown that the ideal channel capacity in a noisy channel is :

$$C = W \log_2 (1 + P/N) \qquad \ldots (2.14).$$

where P = mean signal power

N = mean noise power.

It means that for a certain P/N in the channel, the ideal system will transmit at the rate given by eq.(2.14), although this ideal system will be very complicated in coding and would require large delays.

It has been shown in Appendix A, that the actual channel capacity for a binary system operating above threshold is,

$$[C']_{BINARY} = W \log_2 \left(1 + 12S/K^2 N\right) \qquad \ldots (2.15).$$

where K is a constant such that $K\sigma$ is the separation between the adjacent levels to provide adequate noise margin; σ is the rms noise. Then $C' = C$ if $S = K^2 P/12$ and as K is approximately 10, the power in the Binary system has to be increased by 9 db to make the actual information rate equal to that of the ideal system.

As has been shown later in Section 5.1, the threshold of the slicing circuit in the Ternary system is at 1/2 the amplitude of the pulses (for the minimum error rate), the separation between adjacent levels is $K'\sigma$ where K' = 2K. Proceeding along similar lines as in Appendix A, the actual channel capacity of the ternary system is :

$$[C']_{TERNARY} = W \log_2 \left(1 + 12S/K'^2 N\right)$$

$$\text{or} \quad = W \log_2 \left(1 + 3S/K^2 N\right) \qquad \ldots (2.16).$$

From eqns.(2.16) and (2.14), therefore, the channel capacity of the Ternary will be equal to that of the ideal system if,

$$S = \frac{K^2}{3} P$$

i.e., the signal power is increased by 15 db. But in the above derivation for the Binary, the peak power and the average power are the same because Bipolar pulse transmission was assumed. In the Ternary, however, the average power is 1.8 db below the peak power. The increase in average power is, therefore, only 13.2 db rather than the peak 15 db for achieving the ideal channel capacity in the ternary system.

2.1.2. <u>COMMENTS</u>.

Thus, it is seen that the Ternary-coded system will use less amount of equipment, and the channel capacity of the Ternary system is more than that of the Binary system. The Binary system requires 9 db more power to obtain the channel capacity of the ideal system, whereas the Ternary system requires about 4.2 db more power than the Binary for the ideal capacity. It has been said[7] that a multi-level transmission may be preferable in some cases, as the channel capacity is much larger for the same bandwidth. The disadvantage of the signal-power increase for an almost error-free transmission may be offset by the gain in the channel capacity with a much lesser bandwidth.

2.2. <u>TERNARY SQ-PCM</u>.

The ternary code seams to offer some advantages over the binary code in terms of saving of equipment and larger channel capacity. To explore these possibilities, a Uni-digit Slope-quantized system has been developed in the present work which uses a 3-level quantization and is a Ternary system. It will be shown that the Ternary SQ-PCM is a less complex and more stable system compared to a Binary system for the same grade of performance. The theoretical basis for the Ternary coding and its relevance to the system noise and other parameters is discussed below.

A continuous function of time f(t), limited to a bandwidth of W_m c/s, can be represented by samples taken at discrete times at a rate not less than $2W_m$ c/s. The representation as derived from Shannon's Sampling theorem may be expressed as :

$$f(t) = \sum_n f\left(\frac{n}{2W_m}\right) \frac{\sin\pi(2W_m t - n)}{\pi(2W_m t - n)} \quad \ldots (2.17).$$

The interpolating function sinx/x in the above equation may be replaced by a simple rectangle of height $f(n/2W_m)$ and width $1/2W_m$. This is equivalent to an approximation by steps, or in other words, this gives the staircase approximation of the signal shown in Fig.2.7(a) and, therefore,

$$f(t) \cong \sum_n f(t_n) \cdot \Delta t \quad \ldots (2.18)$$

where $t_n = n/2W_m$

and Δt = length of each step.

This approximation (eq.2.18) can be written in a slightly different form as,

$$f(t) \cong \sum_n \Delta B_n \cdot U(t - t_n) \quad \ldots (2.19).$$

where $\Delta B_n = f_n - f_{n-1}$

and $U(t - t_n)$ = unit step at $t = t_n$

The first derivative of this approximation will be :

$$f'(t) \cong \sum_{n} \Delta B_n . \delta(t-t_n) \qquad \ldots (2.20).$$

where $\delta(t - t_n)$ = Unit impulse at $t = t_n = n/2W_m$.

This derivative of the approximation f'(t), is shown in Fig.2.7(b). The height of ΔB_n's is proportional to the slopes between the sample points and naturally the slopes can be positive, negative or almost zero. If the slopes are now quantized with a Ternary code, one obtains a further approximation of f(t) as is used here in the Ternary SQ-PCM, given by,

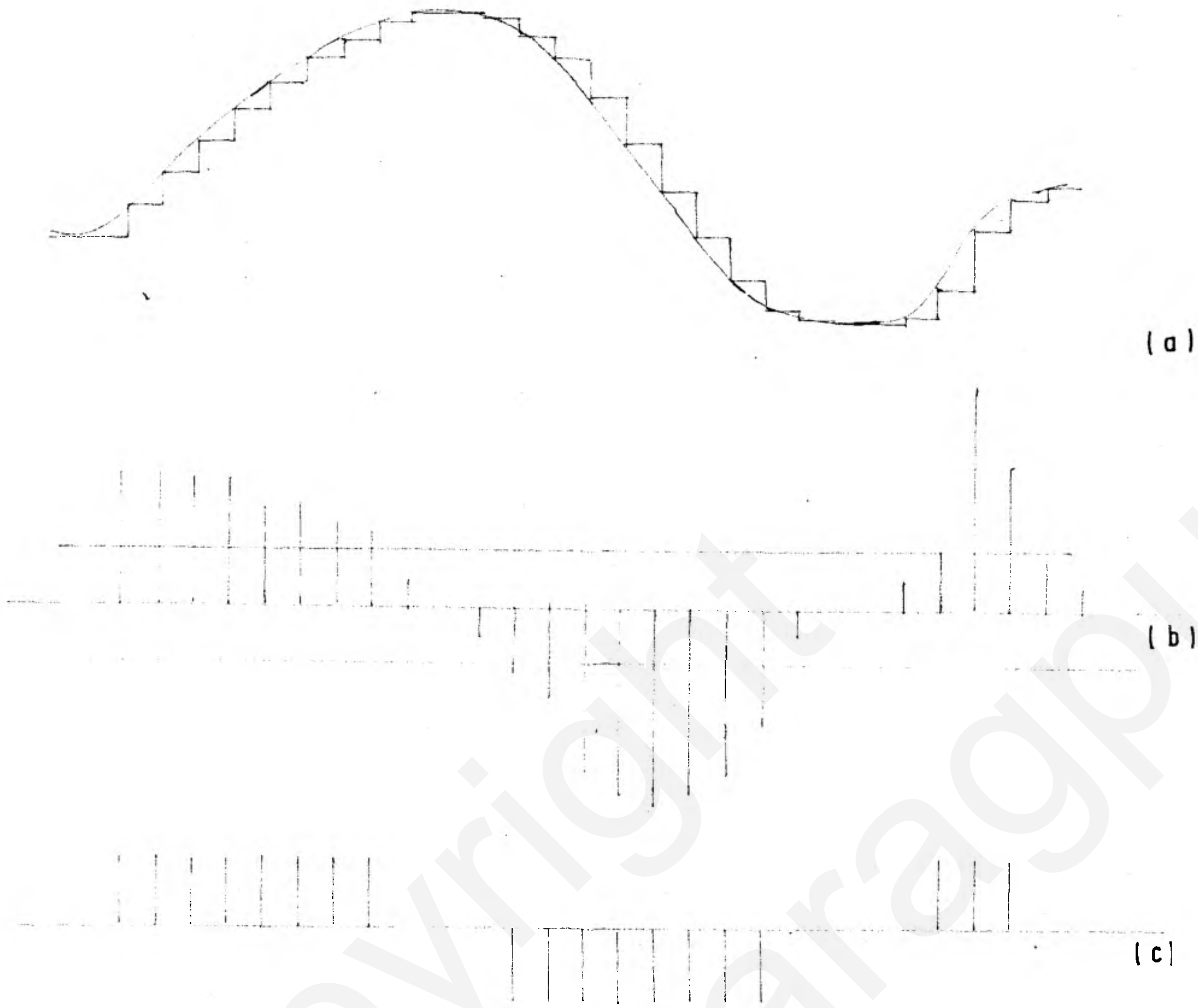
$$f'(t) \cong \sum_{n} B_o . \delta(t-t_n) = \widetilde{f'(t)} \qquad \ldots (2.21).$$

where B_o takes the discrete values of +1, 0, -1 according as:

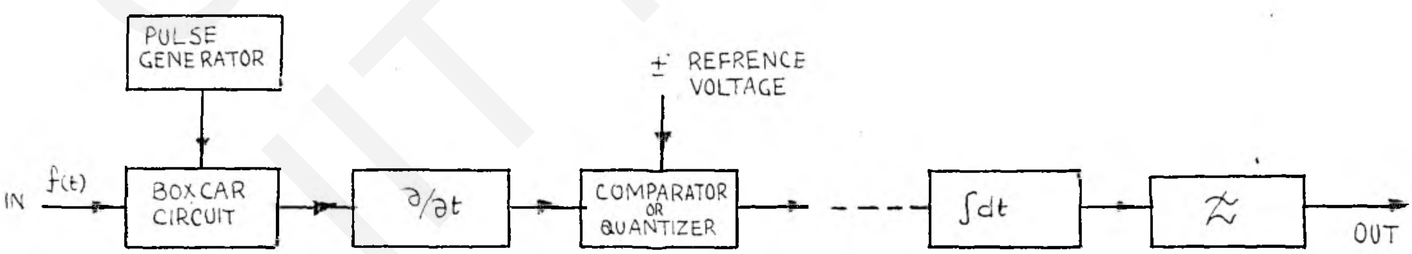
$$B_o = \begin{cases} \pm 1 , & \text{if } |\Delta B_n| \geqslant |\text{rms } \Delta B_n| \\ 0 , & \text{if } |\Delta B_n| < |\text{rms } \Delta B_n| \end{cases}$$

The coded pulses in SQ-PCM therefore will be as shown in Fig.2.7(c). Thus the basic mechanism of approximation in this method is that, the discrete waveforms, synthesised with 'n' ternary pulses at a higher sampling rate, n times that of the Binary AQ-PCM, are matched with the segments of the signal waveform between the ideal sample points at $1/2W_m$ seconds interval.

A block diagram of the system working on the above principle



SIGNAL APPROXIMATION BY SAMPLING AND QUANTIZATION FIG. NO. 2·7



A BLOCK SCHEMATIC OF TERNARY SQ-PCM (WITHOUT FEEDBACK) FIG. NO. 2·8 of approximation is given in Fig.2.8. The boxcar circuit converts the continuous-signal waveform into a staircase approximation of it. The differentiator performs the operation of eq.2.20 to produce narrow pulses of heights depending upon the slopes of the waveform. The comparator next produces a '+1' pulse if the slope exceeds a particular positive reference value, a '-1' pulse if the slope is negative and is larger than a particular negative reference value, and a 'no' pulse if the slope is small and in between the positive and negative reference values. At the receiver, the inverse process of integration of the pulses gives back an approximated staircase waveform, which, when passed through a low pass filter, will give the approximated signal $\tilde{f}(t)$.

In this process of quantization, it is seen that the weightage of the larger pulses and that of the smaller pulses as compared to the average has been lost. This may be demonstrated by comparing the $\tilde{f}(t)$ given by the coded pulses with an original message signal. As an example, Fig.2.9(a) shows a complex signal,

$$f(t) = 15 \sin (2\pi \times 20000\, t + 0^{\circ}) + 10 \sin (2\pi \times 1000\, t - 36^{\circ}) + 10 \sin (2\pi \times 500\, t - 36^{\circ}) + 10 \sin (2\pi \times 3{,}3200t - 90^{\circ}).$$

which is passed through the system shown in Fig.2.8. The pulse output of the comparator is shown in Fig.2.9(b), and these pulses at the receiver will build up the staircase signal, also shown in Fig.2.9(a). The reconstructed signal at the receiver is seen to have some large error and level compression.

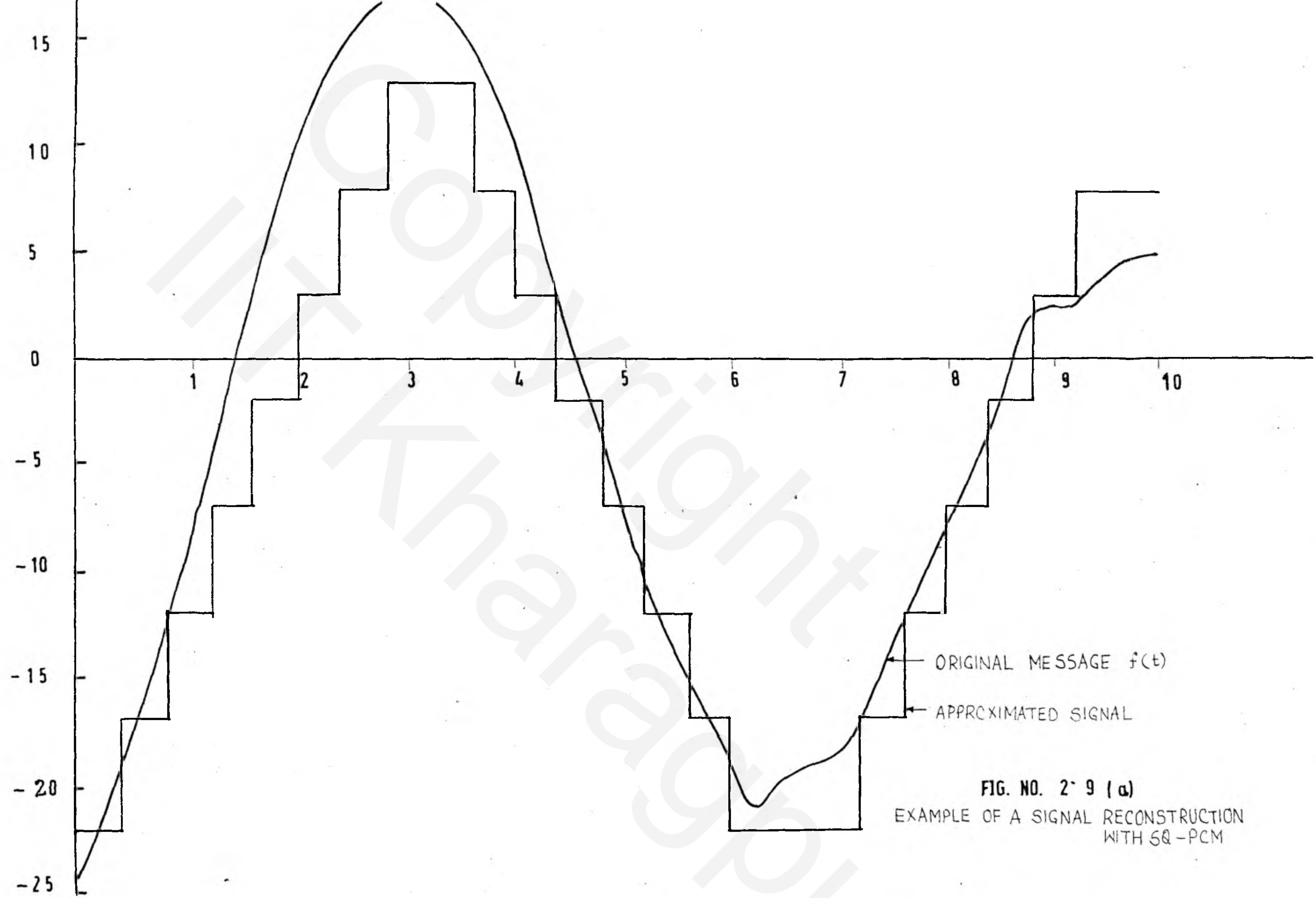
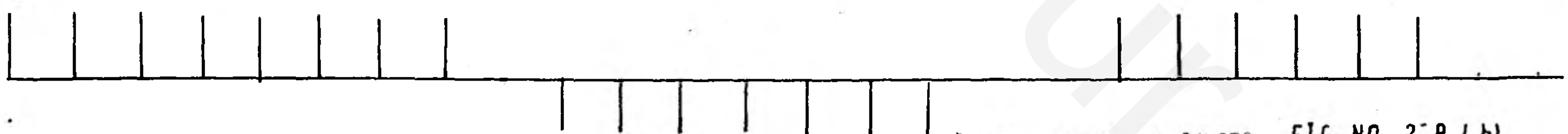


FIG. NO. 2·9 (a)
EXAMPLE OF A SIGNAL RECONSTRUCTION WITH SQ-PCM



DESIRED TERNARY PULSES FIG. NO. 2·9 (b)

The level compression so obtained in the system is a serious drawback for many applications. A feedback from the output to the input may be provided to improve the matter and it is found that the feedback actually improves the quality of matching of the input with the output by a continuous comparison between the signal and its approximation. The feedback loop, necessarily therefore, contains a local receiver. A block schematic diagram of the system with feedback is shown in Fig.2.10. Since the feedback loop includes an integrator, the total effect of the system becomes equivalent to a double differentiation. The dynamic range of the lower frequencies at the input is then reduced considerably if the reference level of the comparator is set for the optimum threshold at the higher frequencies. Also, the distant receiver is generally a single integrator and the net output, therefore, is the differential of the input. To compensate for this effect an equaliser at the input is necessary. The equaliser has a frequency response falling off with higher frequencies, and with the inclusion of this network the distant receiver can be a simple integrator again.

Since the transmitter system includes in the forward loop both an integrator and a differentiator, it is evident that they can be dispensed with. The problem of generating the bidirectional samples is solved by using a Bi-symmetrical sampling circuit and the block diagram of such a system is shown in Fig.2.11. Functionally, the circuit is the same as the one in Fig.2.10, as the integrator in the feedback loop is equivalent to a differentiation in the forward loop and the output of the distant-end receiver-integrator reproduces the original signal approximately. The

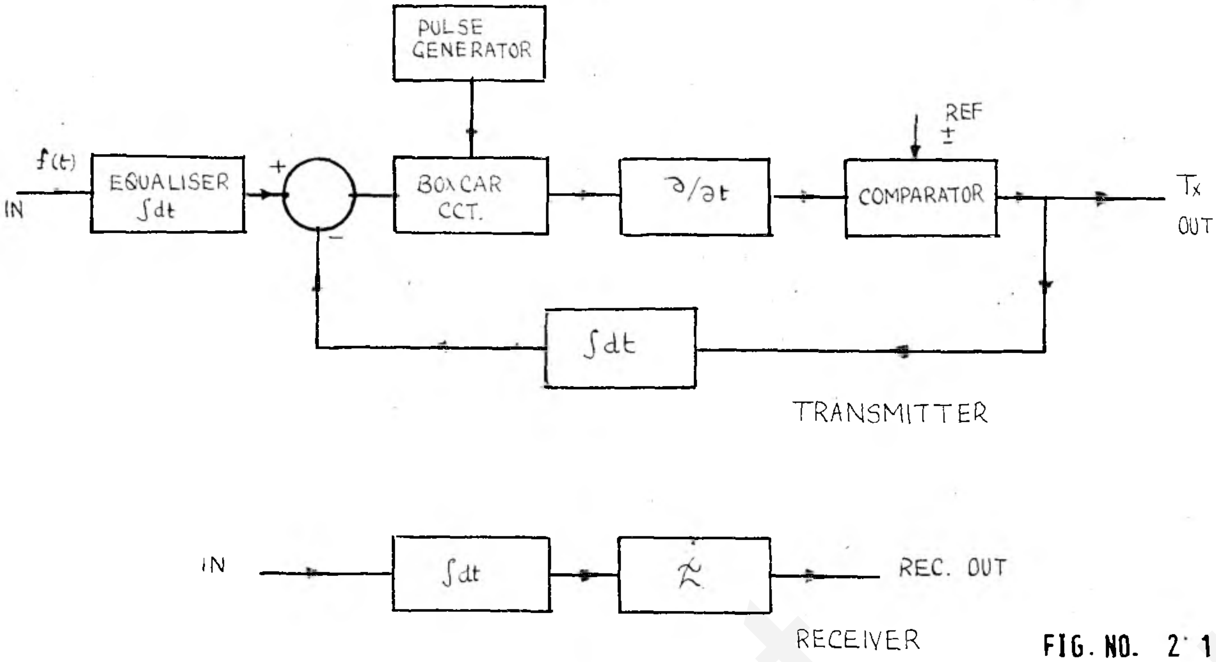


FIG. NO. 2.10

A BLOCK SCHEMATIC OF TERNARY SQ-PCM WITH FEEDBACK

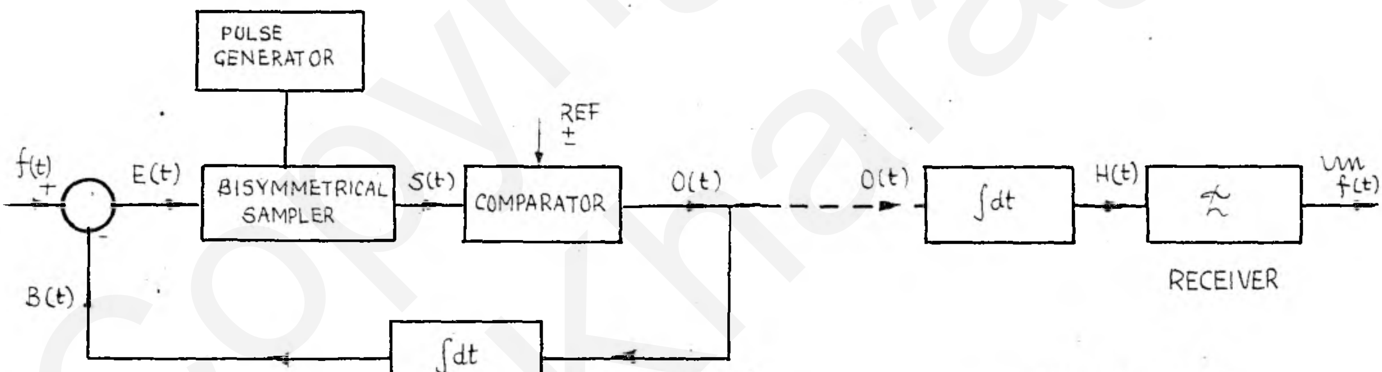


FIG. NO. 2.11

ALTERNATIVE SCHEME FOR TERNARY SQ-PCM WITH FEEDBACK

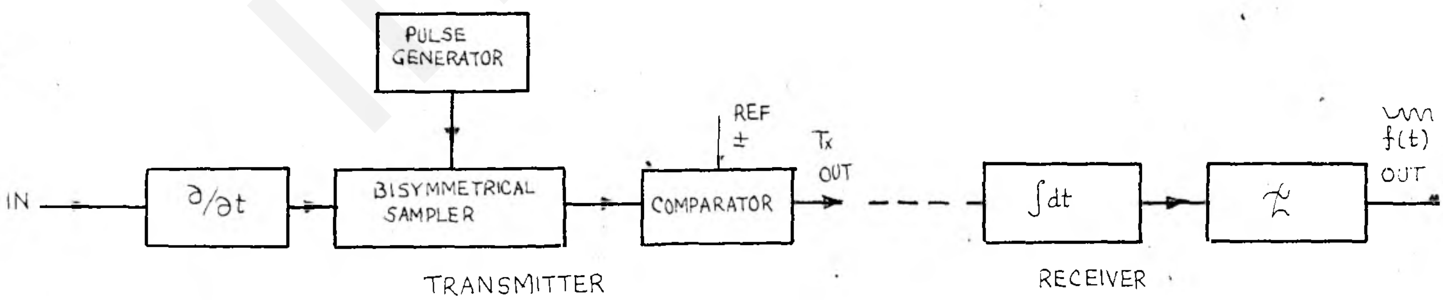


FIG. NO. 2.12

ALTERNATIVE SCHEME FOR TERNARY SQ-PCM WITHOUT FEEDBACK circuit of Fig.2.11 is certainly simpler and more economical, although both the circuits have been tried experimentally and gave similar performances.

In Fig.2.11, the error signal E(t) is sampled with a Bisymmetrical sampling circuit to give an output S(t). The comparator quantizes S(t) into +1, 0, -1 pulses O(t). This is fed back degeneratively through an integrator whose output B(t) is compared with f(t), the input signal, to produce the difference or error signal E(t). At the distant-end receiver, the sequence of ternary pulses are just integrated and filtered to give the approximated signal. The feedback coding can be demonstrated with reference to Fig.2.13, where the reconstruction and quantization of the same signal, as that of Fig.2.9, is done step by step, while comparing, at each sample point, with the original signal. It is seen that ~~with~~ larger groups of positive and negative pulses build the approximated signal to larger heights, thus maintaining a linear relation between f(t) and $\tilde{f}(t)$. With very small inputs, alternate positive and negative pulses with zero pulses in between do not allow the receiver output to be built up to any large value, and $\tilde{f}(t)$ follows f(t) closely. For zero signal input, the stationary pattern, then, becomes a sequence of +1, 0, -1 pulses. Comparisons of Figs.2.9 and 2.13 shows that the feedback increases the dynamic range of the signals which can be handled by the system and also reduces the errors to the minimum possible under the given conditions.

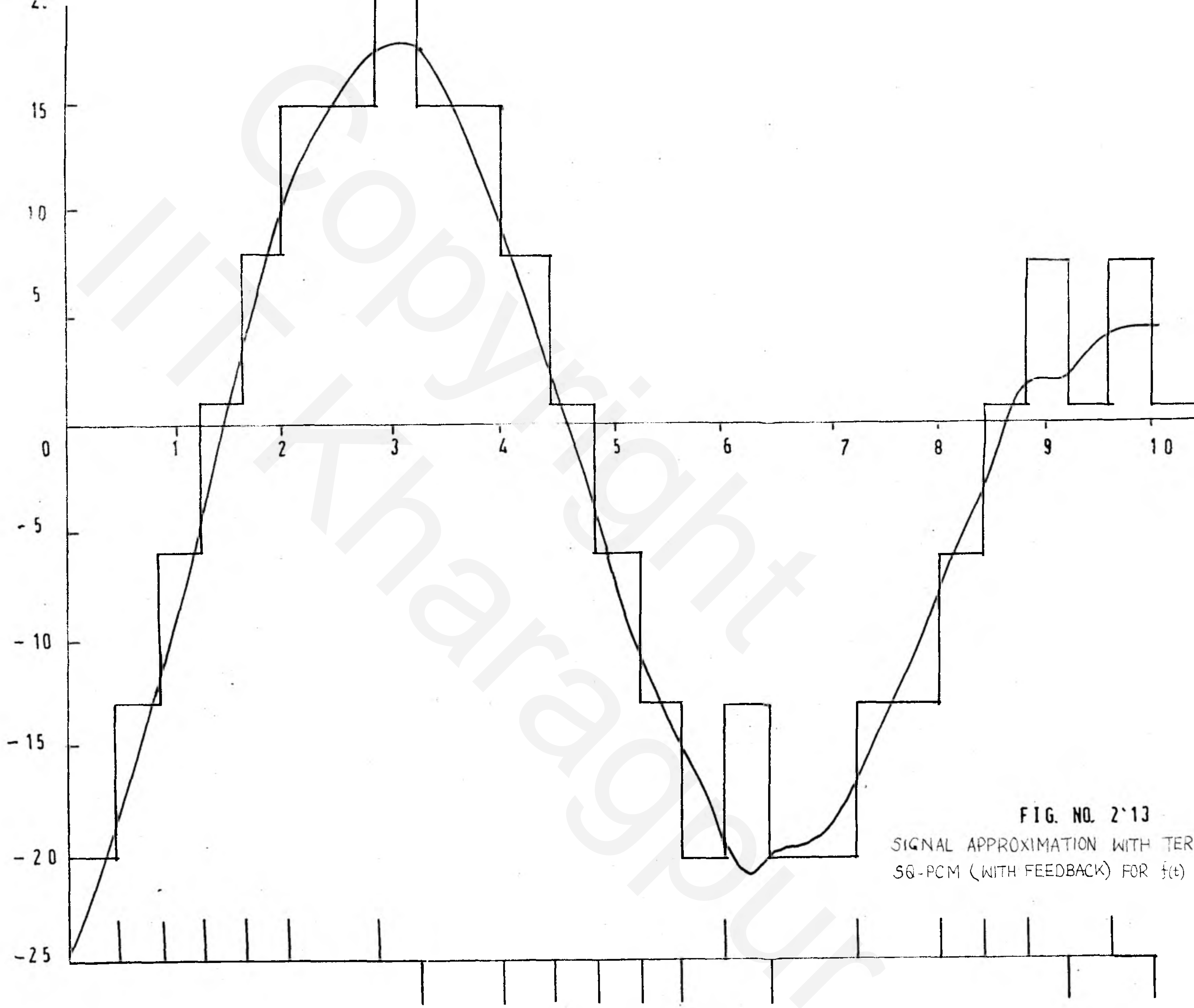


FIG. NO. 2·13
SIGNAL APPROXIMATION WITH TERNARY SQ-PCM (WITH FEEDBACK) FOR f(t) OF FIG 2·9

2.3. BINARY SQ-PCM.

A method for approximation of signals has been proposed in Section 2.2, where the positive and negative slopes of a waveform are approximated by +1, 0, -1 pulses occuring at a very high rate, and the interpolating function used is of $Si(x)$ form leading to a staircase approximation. With the same interpolating function and a Binary 1/0 approximation of the slopes of the signal, one can only match the positive and zero slopes of the waveform, and to approximate the changes of negative slopes, one has to use some special interpolating function. The idea used for the ternary coding resulting in an approximation of the type given in eq.2.20, therefore, will not apply here. For the quantized 1/0 system, the signal will have to be sampled to give only positive samples, and the negative changes of the slopes will have to be matched by the cumulative effect of some decaying interpolating function. Gabor[16] has shown in his generalised Sampling theorem that the interpolating functions other than $Si(x)$ are equally satisfactory for representing a signal, and in fact, it will be seen that an exponentially decaying type of interpolating function leads to a sufficiently good approximation.

The generalised Sampling theorem states that a band-limited continuous signal f(t), even after differentiation, may be represented as,

$$f'(t) \cong \sum_n C_n \cdot r'(t - t_n) = \widetilde{f}'(t)$$

where $$C_n = \frac{1}{2W_m} \int_{-W_m}^{+W_m} \frac{f(\omega)}{R(\omega)} \cdot e^{-j\pi n\omega/W_m} \cdot d\omega$$

$r(t - t_n)$ = impulse response of the interpolating filter to impulses occuring at $t = t_n$

and, $f(w)$ and $R(w)$ are the frequency domain characterisation of the signal and the filter respectively. By making the sampling intervals much smaller than the Nyquist interval of $1/2W_m$, and using a wider spread in the filter response, it is possible to approximate $f'(t)$, even with quantized values of the sample amplitudes C_n (for Uni-directional signal i.e. with d.c. bias in the sampled output) and the approximate $f'(t)$ will be given by :

$$\widetilde{f'(t)} \cong \sum_n C_o \cdot r'(t - t_n) \qquad \ldots (2.22).$$

and $$\widetilde{f(t)} = \sum_n C_o \cdot r(t - t_n) \text{ after integration}$$

where $$C_o = \begin{cases} +1, & \text{if } |C_n| \geqslant |\text{rms } C_n| \\ 0, & \text{if } |C_n| < |\text{rms } C_n| \end{cases}$$

The above equation is similar to the eq.2.20 and the signal processing may also be done in a way similar to that shown in Fig.2.11, by using an integrating type of network in the feedback path.

Fig.2.14 illustrates the idea of reconstruction of a waveform from some assumed pattern of 1/0 pulses. The impulse response of the interpolating filter is assumed and is shown in Fig.2.14(a).

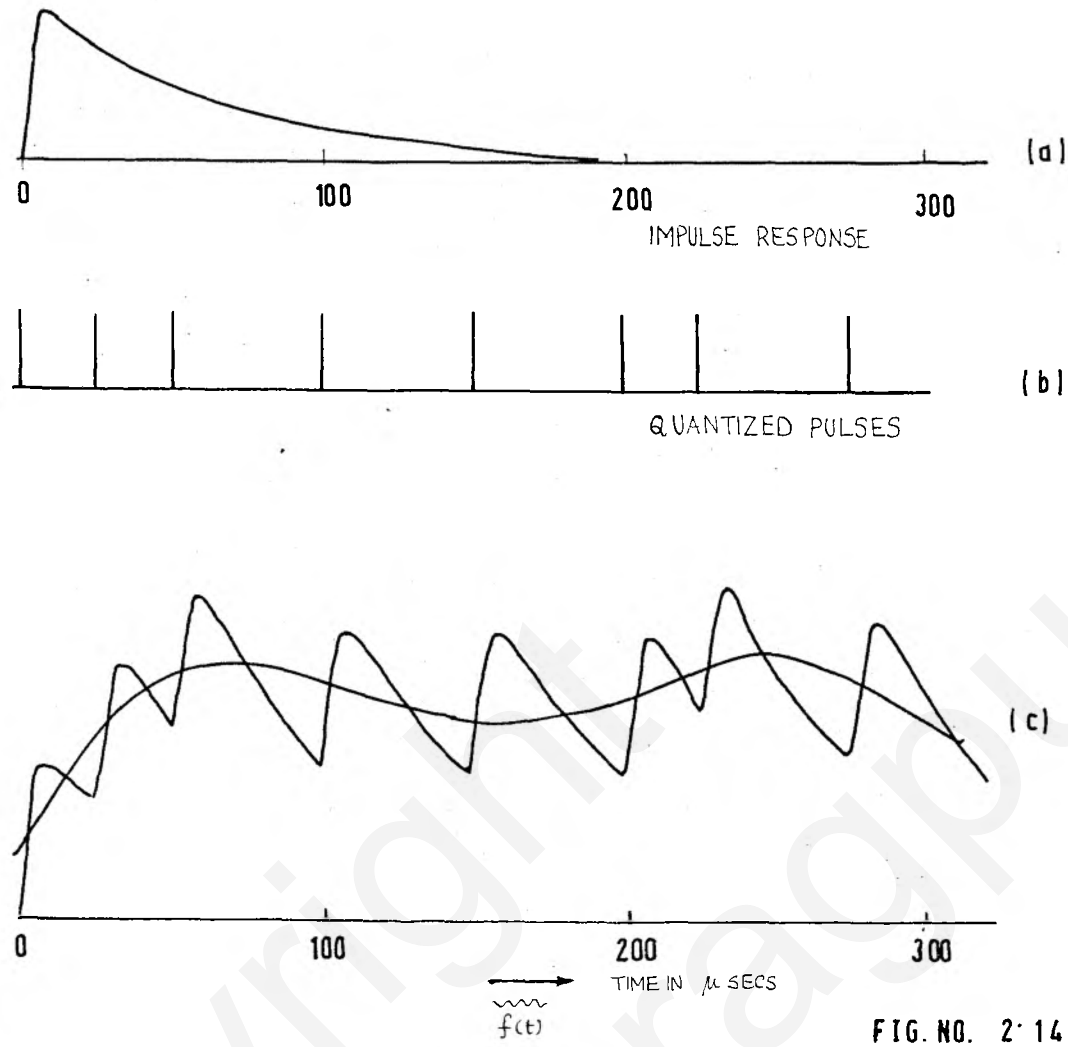
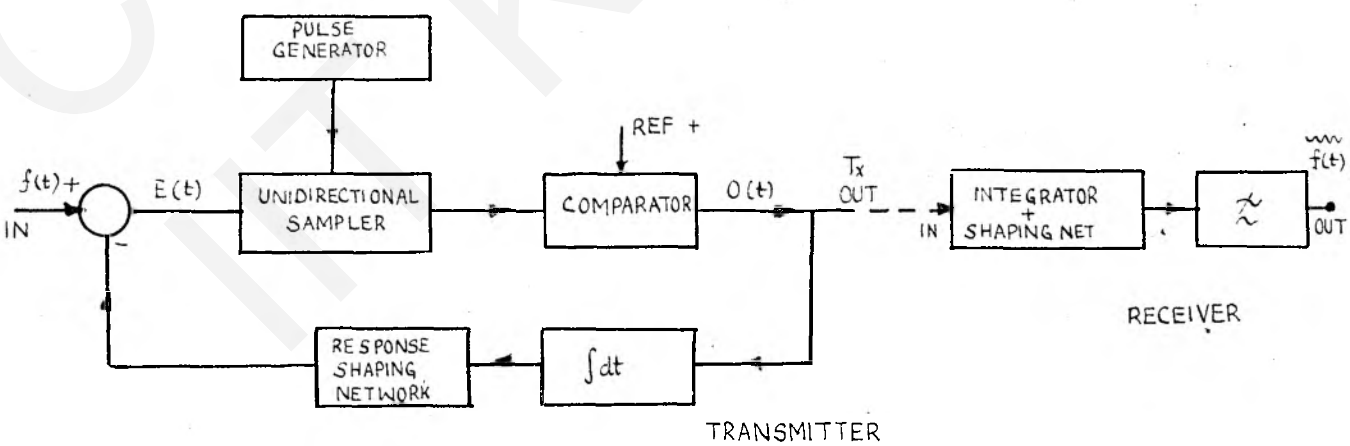


FIG. NO. 2·14

APPROXIMATION OF A WAVEFORM WITH 1/10 PULSES



BLOCK SCHEMATIC OF BINARY SQ-PCM

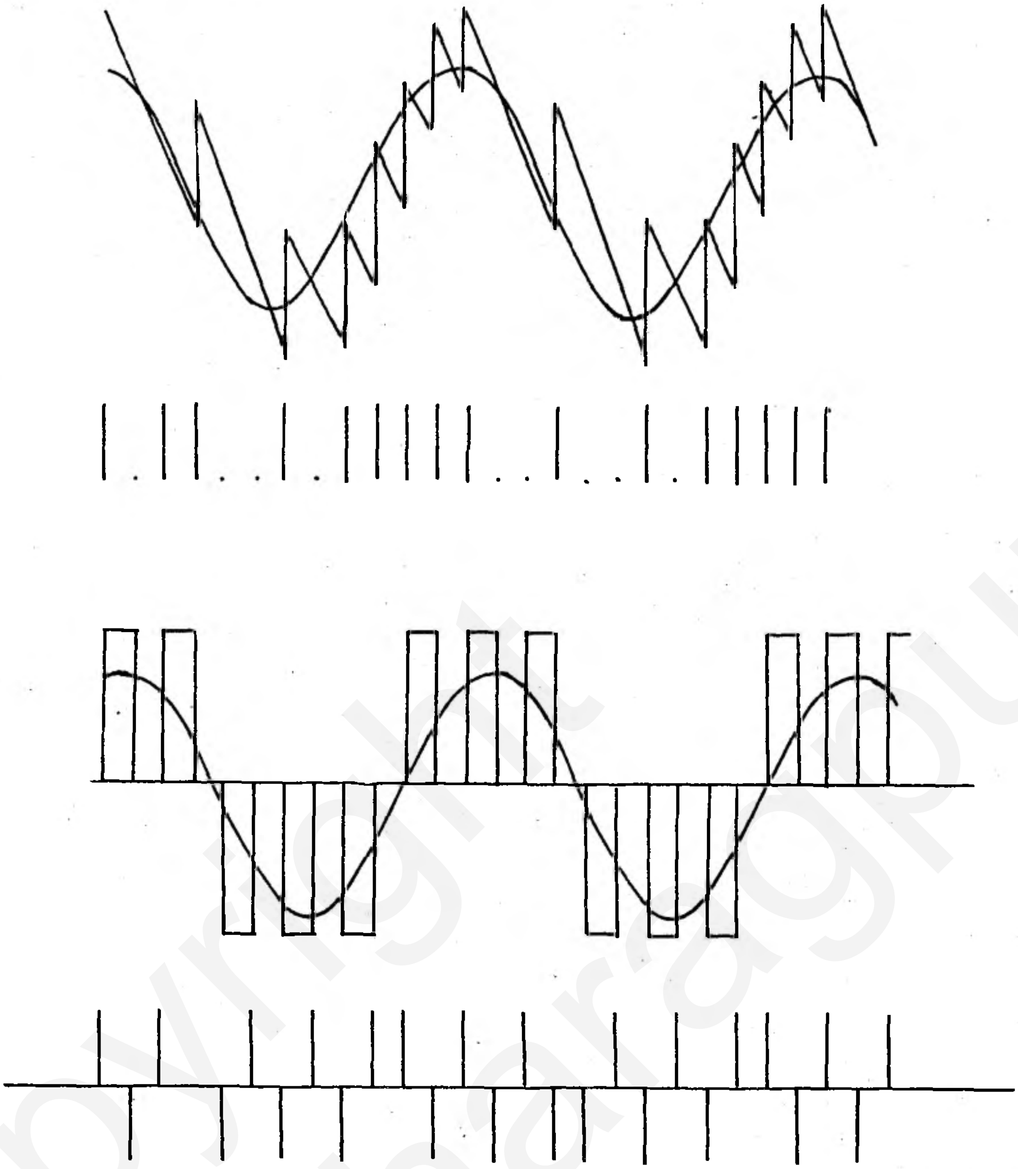
FIG. NO. 2·15

The signal built up by the group of the assumed pulses of Fig. 2.14(b) is indicated by the smoothed curve of Fig.2.14(c). It is seen that there is a strong correlation between the occurances of pulses and the slope of the smoothed curve. This means that if the smoothed curve is compared with the one built up by the pulses in a feedback loop, a pulse is seen to occur whenever the amplitude difference between the two curves exceeds a certain value. One of the important points is therefore the pulse response of the interpolating network, the response being such that its cumulative effect match the negative slope of the signals. The second important thing is the reference level below which the difference amplitudes are ignored.

The actual processing of the signal to obtain 1/0 pulses may be done in a simple circuit shown in Fig.2.15. The error signal E(t) is sampled at the desired PRF and compared with an optimum reference level to produce 1/0 pulses O(t) at the output. The feedback loop consists of an integrator and a response-shaping network. The output pulses passed through this feedback circuit will build up the input signal approximately as B(t). The approximated signal is then compared with the input signal f(t) to give rise to the error signal E(t). The optimum reference of the comparator and the feedback network response are so adjusted that the error signal is minimised. The distant receiver is a replica of the network in the feedback circuit and hence the output of the receiver f(t) approximates the input fairly well. As in the Ternary Sq-PCM the composite modulator, here also, operates on the successive differences between the input f(t) and the feedback signal B(t), and the successive approximations match the slope of f(t), thus giving rise to a Binary Slope-quantized PCM system. It may be noted that the feedback network should be such that the negative slopes of f(t) are matched by the cumulative negative slopes of B(t) when no pulses are present at the transmitter output.

The block schematic circuit of Fig.2.15 bears some resemblance to that of the Δ-M system but there are some significant differences between the two systems. The process of coding in Δ-M discussed in Appendix B, is such that there is always either a positive or a negative pulse output depending on the polarity of the error waveform at the sampling instants, and the pulses of either polarity after integration try to match approximately the corresponding signal waveform. By using a somewhat modified technique of quantizing the signal slope in the Binary SQ-PCM system, the signal slope is directly matched with the slope of B(t) to produce a positive pulse only if the difference exceeds a certain threshold. The negative slopes are matched by the combined negative slopes of B(t) as has been explained above. A special wave-shaping network in the feedback loop is therefore included.

It is further felt that the absence of a desirable threshold in the pulse modulator of the Δ-M system results in some extraneous pulses, either positive or negative, which do not contribute to the minimisation of the error waveform and tend to give some extra error at the output. That the error will be less in SQ-PCM system compared to that in the Δ-M system can be explained with the help of Fig.2.16. The build up of signals and the generation of +1/-1 pulses in the Δ-M are shown in Fig.2.16(a) where an



(b)

(a)

COMPARISON OF SIGNAL APPROXIMATION
BY,(i),BINARY SQ-PCM AND (ii),Δ-M SYSTEMS

FIG. NO. 2·16

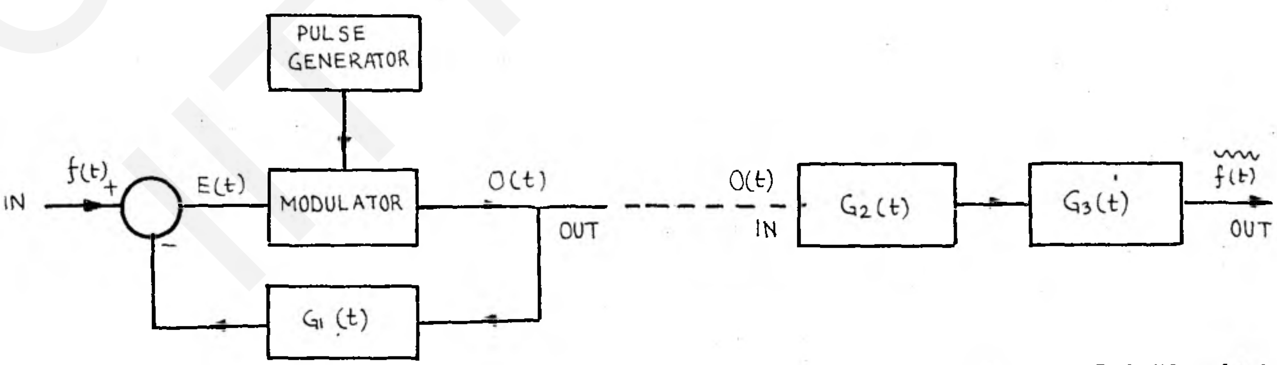


FIG. NO. 2·17

TRANSMITTER RECEIVER

A SYMPLIFIED MODEL OF SQ-PCM

ideal integrator in tha feedback loop has been assumed for the purpose of the demonstration. It is seen that if the difference between the steps and the original signal is positive, there is a positive pulse at the output, and if the difference is negative, there is a negative pulse at the output. In the signal build up of the SQ-PCM system, shown in Fig.2.16(b), the cumulative slope of the response of the feedback network to the 1/0 pulses is such, that either it matches the slope, or is higher than the slope of the input waveform. The free fall of the response continues till the difference between it and the signal waveform is positive and above the threshold, when a positive pulse will be given out by the comparator. The two possible approximations of the signals indicate clearly that the error in the SQ-PCM is going to be less than that in Δ-M; although the figure 2.16 has been exaggerated a little to bring out the differences.

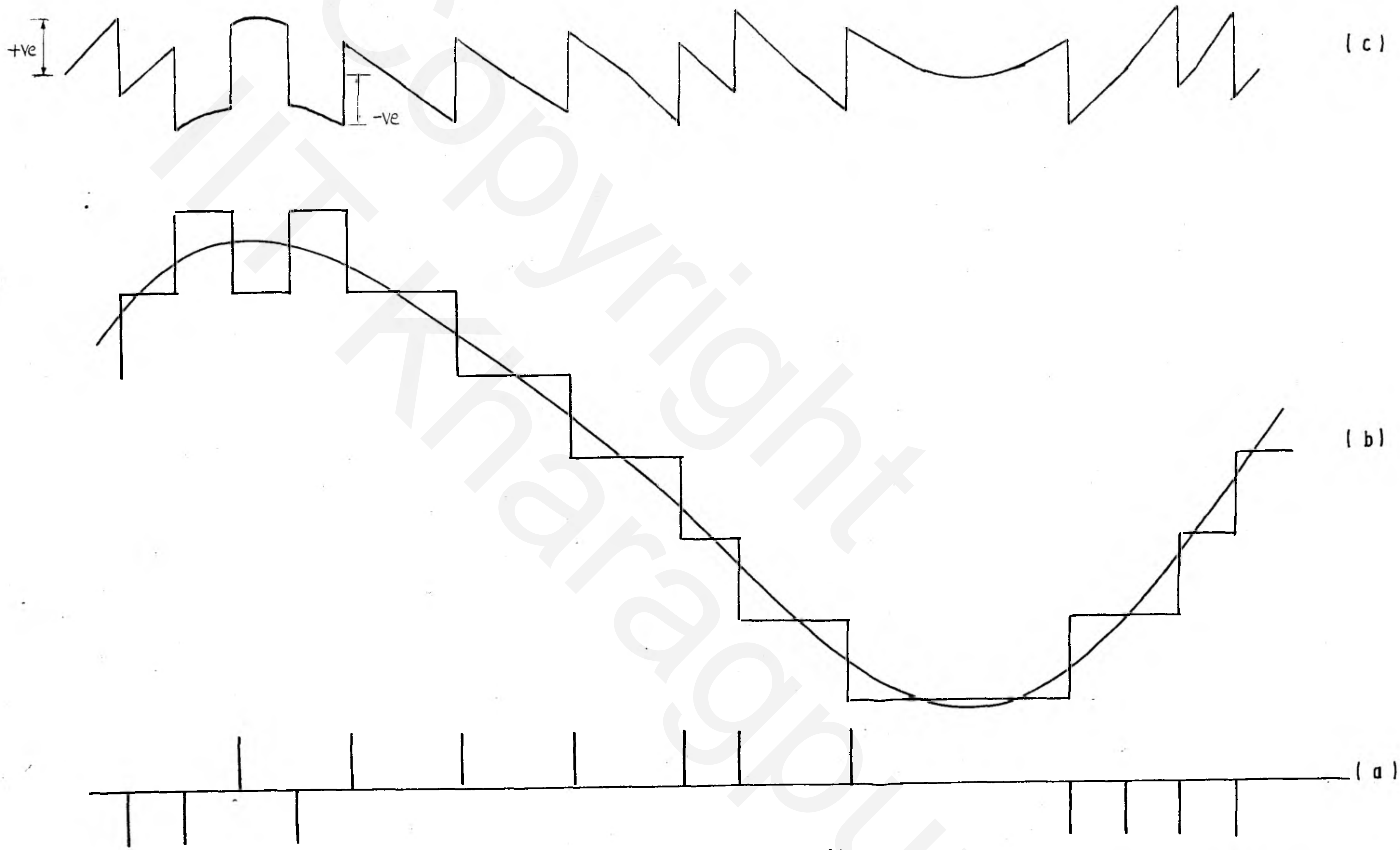
2.4. SIGNAL TO NOISE CHARACTERISTICS.

It has been shown in earlier Sections that the message signal may be approximated either by plus and minus steps (Section 2.2) or by exponentially decaying pulses (Section 2.3). In both cases there is an error between the original signal and the approximated signal, and attempts have been made to reduce the error as much as possible. Since the distant-end receiver is a copy of the feedback network, the output of the receiver will have an error. This error arises because of the quantization in the processing of the signal and will, henceforth, be called the quantizing error. The total quantizing error in a system is quite large and spreads out over a large band of spectrum, but fortunately the portion of the error falling in the passband of the lowpass filter at the receiver is within reasonable limits.

A general and simplified block diagram of the Ternary SQ-PCM system is shown in Fig.2.17, where f(t), B(t), E(t) and O(t) are the message signal, the approximated signal, the error signal and the output pulse signal of the 3-level quantizer respectively. The gain function of the feedback network is $G_1(t)$ and the gain function of the receiver and the lowpass filter is $G_2(t)$ and $G_3(t)$ respectively. As in all approximations, the residual error in the approximated function $\widetilde{f(t)}$ will also be somewhat random[17] when the input signal is semi-random, e.g., speech. For simplicity of analysis, an almost sinusoidal signal is assumed as f(t) and the consequent error signal together with pulse output of the system is shown in Fig.2.8. The error waveform consists of approximate triangular pulses of random amplitudes varying between the limits $\pm V_e$, where V_e is the amplitude of the pulses at the comparator output O(t) (the gain of $G_1(t)$ is normalised to give the same peak value for B(t)).

To calculate the signal-to-quantizing-error characteristics of the system, shown in Fig.2.17, the frequency domain characteristics of quantities like f(t), E(t), O(t), $G_1(t)$, B(t), $G_2(t)$, $\widetilde{f(t)}$, and $G_3(t)$ are taken as f(w), E(w), O(w), $G_1(w)$, B(w), $G_2(w)$, $\widetilde{f(w)}$, and $G_3(w)$ respectively. The equations governing the system in the frequency domain can be written as follows:

$$E(w) = f(w) - B(w)$$



NATURE OF THE WAVEFORM IN TERNARY SQ- PCM
(a) THE TERNARY PULSES, (b) RECONSTRUCTED WAVEFORM,
(c) ERROR WAVEFORM

FIG. NO. 2.18

$$B(w) = O(w) \times G_1(w)$$
$$N(w) = f(w) - \widetilde{f(w)}$$

where N(w) is the noise power density in the message band. Now,

$$E(w) = f(w) - O(w).\ G_1(w)$$

or $E(w).\ G_3(w) = f(w).\ G_3(w) - O(w).\ G_1(w).\ G_3(w)$ ... (2.23).

If the receiver filter network is assumed to have a brickwall characteristics with cut-off frequency at f_o, then

$$G_3(\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \omega_0 \\ 0, & \text{for } |\omega| > \omega_0 \end{cases}$$

and $f(w).\ G_3(w) \equiv f(w)$ in the pass band.

If $G_1(w)$ and $G_2(w)$ are assumed to be similar networks, the equation (2.23) is modified as :

$$E(w).\ G_3(w) = \left[f(w) - \widetilde{f(w)}\right] = N(w)$$

as, $O(w).\ G_1(w).\ G_3(w) = \widetilde{f(w)}$.

The comparator has an abrupt nonlinearity with a positive and a negative threshold and the error waveform is triangular in nature as assumed above. The power spectral density of this random error waveform varying between the limits $\pm V_e$ can be found by resolving the fluctuating waveform into a product of two components. One component is a regular triangular waveform of constant height which is multiplied by the other amplitude component varying randomly about a mean value of zero and between the peak limits of $\pm V_e$. The amplitude component varies randomly around zero with a probability defined by $p(y)$. The correlation function of this oscillating function is $\overline{v^2}$, where $\overline{v^2}$ is the mean square amplitude of the pulses for the probability distribution assumed. The power density spectrum is then given as[18]:

$$N_E(\omega) = \frac{\overline{v^2}}{T} \left| E_T(j\omega) \right|^2 \qquad \text{... (2.24).}$$

where $\left[E_T(j\omega)\right]$ = Fourier transform of the regular triangular pulse.

and $T = 1/f_r$; where f_r is the PRF

As
$$\left|E_T(j\omega)\right|^2 = \frac{\tau^2}{4}\left[\sin\frac{\omega\tau}{4} \Big/ \frac{\omega\tau}{4}\right]^4$$

for a triangular pulse of unit height, the power density spectrum is

$$N_E(\omega) = \frac{\overline{v^2}\,\tau}{4}\left[\sin\frac{\omega\tau}{4} \Big/ \frac{\omega\tau}{4}\right]^4 \qquad \text{... (2.25).}$$

Therefore, the noise power N in the message band is :

$$N = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} N_E(\omega)\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} \frac{\overline{v^2}\,\tau}{4} \cdot \left[\sin\frac{\omega\tau}{4} \Big/ \frac{\omega\tau}{4}\right]^4 \cdot d\omega.$$

$$\cong \frac{\overline{v^2}}{2\pi} \cdot \int_{-\omega_0}^{\omega_0} \frac{\tau}{4} \cdot d\omega \qquad \left[\begin{array}{l}\text{Since } \sin x/x \cong 1 \\ \text{for } \omega \ll \omega_0 \\ \text{and } \omega_0\tau \ll 1\end{array}\right]$$

$$\text{or } N = \frac{\overline{v^2}}{2} \cdot \frac{f_0}{f_r} \qquad \text{.... } [2.26(a)]$$

and the rms noise voltage is :

$$N_v = \left(\frac{\overline{\upsilon^2}}{2}\right)^{1/2} \cdot \left(\frac{f_o}{f_r}\right)^{1/2} \qquad \ldots (2.26(b)).$$

If the frequency of the message signal is f_m, then f_r/f_m number of pulses occur during a full period of the input signal, and ideally only half of these i.e. $f_r/2f_m$ pulses are responsible for building the signal from negative peak to the positive peak. Hence in the SQ-PCM systems, the ratio $f_r/2f_m$ (=n) gives the number of pulses necessary to build the maximum signal and 'n' is designated here as length of the 'word', in a manner similar to that in the AQ-PCM system where 'n' is the number of digits used in coding. Each pulse of height V_e builds the signal by an equal amount V_e and therefore 'n' pulses will build up a peak-to-peak signal of nV_e. With varying signal, this built up signal will be KnV_e where $K \leq 1$, as has been discussed in Section 2.2. The rms value of the signal voltage in the Ternary SQ-PCM is then given by:

$$S_v = \frac{K n V_e}{2\sqrt{2}} = \frac{K V_e}{2\sqrt{2}} \cdot \frac{f_r}{2f_m}. \qquad \ldots (2.27).$$

Therefore, the rms signal-to-noise ratio for the Ternary SQ-PCM is:

$$\left[S_V/N_V\right]_{\text{TERNARY}} = \frac{K V_e}{4\sqrt{\overline{\upsilon^2}}} \cdot \frac{f_r^{3/2}}{f_o^{1/2} \cdot f_m} \qquad \ldots (2.28).$$

If it is assumed that all error amplitudes are equally probable between $\pm V_e$, i.e. the probability distribution of the amplitudes $p(y)$ of the error waveform is rectangular, the mean square amplitude $\overline{\upsilon^2}$ is found to be $V_e^2/3$ and from eq.2.28, the SNR becomes

$$\left[S_v / N_v \right]_{\text{TERNARY}} = 0.433 \frac{K f_r^{3/2}}{f_o^{1/2} . f_m} \quad \ldots (2.29).$$

The error entropy now is maximum and the SNR at the receiver is minimum.

The use of an optimum threshold and a proper feedback network, however, reduces the error entropy by decreasing the number of large-amplitude error pulses. This gives rise to a peaky probability distribution for the error voltage and the optimised distribution would approach a truncated Normal distribution[19] for a complex f(t). The minimum error power may then be calculated by assuming the average crest factor of four, as is used for random noise and speech signals. It has been shown in Appendix C that if in the Normal distribution of amplitudes, a restriction is imposed on the maximum amplitude such that it does not exceed a particular value of $\pm V_e$ for more than, say, 3 in 10^5, this truncated Normal distribution will have approximately the same rms value as the Normal distribution[20]. For such a truncated Normal distribution, the mean square amplitude $\overline{v^2}$ is found to be $V_e^2/16$ and therefore, from the equation 2.28, the SNR is :

$$\left[S_v / N_v \right] = \frac{K f_r^{3/2}}{f_o^{1/2} . f_m} \quad \ldots (2.30).$$

The system SNR obtained will normally be within the limits of the values given by eq.2.29 and 2.30. For example if f_o = 3.4 Kc/s;

f_m = 1 Kc/s and f_r = 40 Kc/s then according to equation 2.29,

$$\left[S_V/N_V\right]_{\text{TERNARY}} = \frac{(40)^{3/2} \times 0.433}{(3.4)^{1/2} \times 1} = 35.6\,\text{db}, \quad \text{for } K = 1.$$

and according to eq.2.30,

$$\left[S_V/N_V\right]_{\text{TERNARY}} = 42.8\,\text{db}, \qquad \text{for } K = 1$$

The two equations 2.29 and 2.30 show that the SNR is directly proportional to $(f_r)^{3/2}$ and inversely proportional to the frequency f_m of the message signal. If the input level is more than the optimum, which occurs for K = 1, the receiver output cannot increase proportionately and the follow up of the signal by the feedback voltage is lost. The coder, then, becomes equivalent to the one without feedback shown in Fig.2.8, and produces saturation and harmonics in f(t), thus giving lower SNR for large inputs above the overload point. As the feedback voltage decreases with the increase of f_m, this overload will occur at lower input levels for higher signal frequencies, and there will also be frequency distortion in the receiver output. An ideal integrator with a break point at a frequency lower than the lowest frequency in the message signal has been assumed for calculation. In practice (for speech like signals only) it is possible to use, however, a partial integrator with a break point near 1000 c/s when the frequency dependence of SNR will not be so pronounced, as is shown by the experimental results given in Chapters 3 and 4.

The feedback network, so far considered, has been a single integrator, but a double integrator could also be used. As has been shown in Appendix B, equation (B.11), the use of a double integrator gives an SNR which is directly proportional to $(f_r)^{5/2}$ and inversely proportional to $f_m^{\,2}$. No doubt this will give a better SNR at lower input frequencies, but the higher signal frequencies will have a very low overload point. Since the frequency response of the feedback network is changing at the rate of 12 db/octave the SNR and the output will be very poor, at the higher end of the band. A modification of the system, such that the receiver uses only a single integrator and the transmitter feedback loop retains the double integrator, has been proposed by De Jager and others. The system then is stable and suitable for speech signals where the mean power spectrum falls in a manner similar to that of a single integrator. However, as the interest here has been to develop a system with the least amount of frequency distortion, consistent with a good SNR, it is found that a coding with double integration will not be suitable.

The SNR calculations for the Binary SQ-PCM system will also be similar to the one given above for the Ternary system. The modulator in Fig.2.17 will now be a 2 level quantizer and the feedback network will include a response shaping network as has been explained before. The error waveform and the built-up signal by the Binary 1/0 pulses are shown in Fig.2.19. The error signal can again be assumed to consist of triangular pulses with random heights, having either rectangular or truncated Normal distribution for the random amplitudes. The signal built up by the 1/0 pulses is, however, approximately half of that in the Ternary case, because of the particular impulse response of the feedback network and the absence of negative pulses. Hence

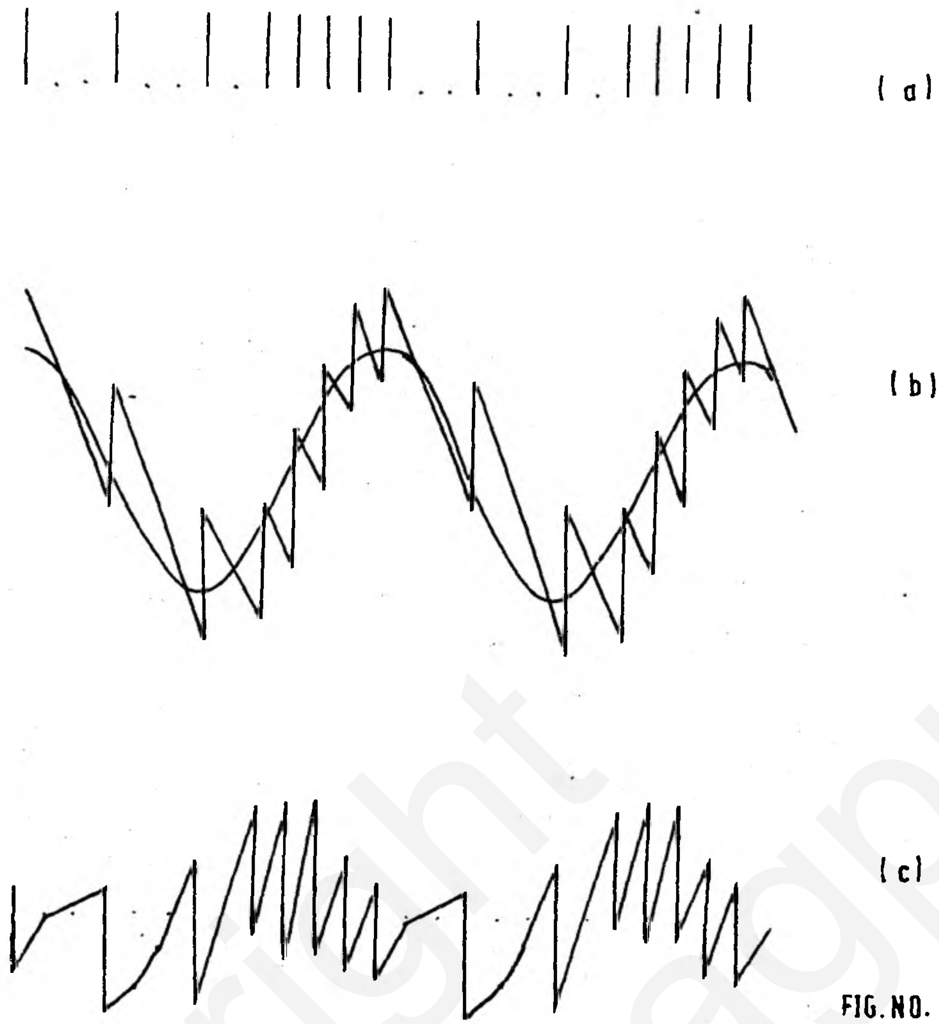
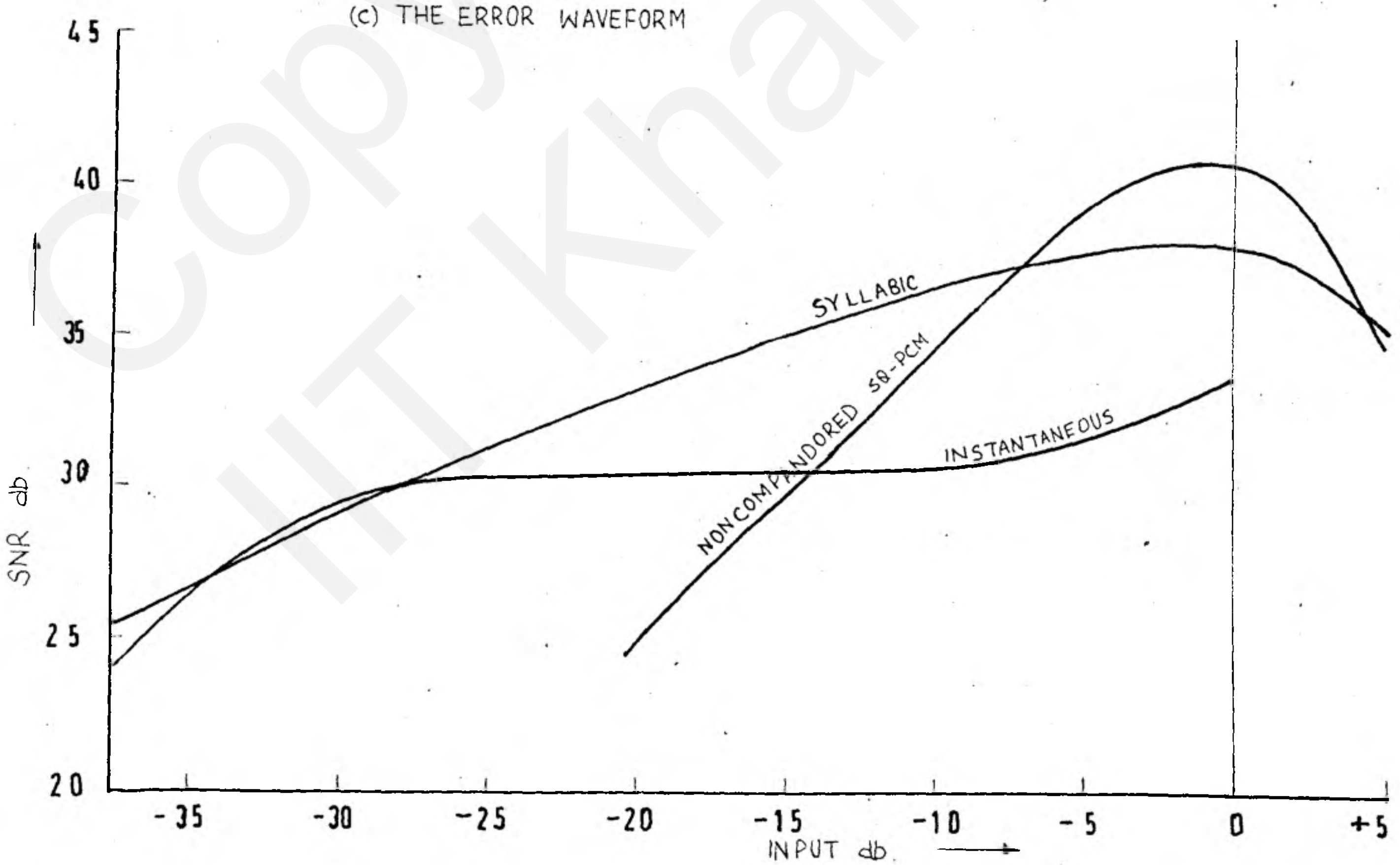


FIG. NO. 2·19

NATURE OF ERROR WAVEFORM IN BINARY SQ-PCM SYSTEM
(a) BINARY PULSES, (b) RECONSTRUCTED WAVEFORM, AND
(c) THE ERROR WAVEFORM



SNR CHARACTERSTICS WITH
COMPANDORS IN SQ- PCM SYSTEM

FIG. NO. 2·20

proceeding along lines similar to eqs.2.24, 2.25 etc.,

$$N_v = \left(\frac{\overline{v^2}}{2}\right)^{1/2} \cdot \left(f_o / f_r\right)^{1/2} \qquad \ldots (2.26(b)).$$

and

$$S_v = \frac{kV_e}{4\sqrt{2}} \cdot \frac{f_r}{2 f_m} \qquad \ldots (2.31(a)).$$

Therefore, the SNR is :

$$\left[S_v / N_v\right]_{BINARY} = \frac{kV_e}{8\sqrt{\overline{v^2}}} \cdot \frac{f_r^{3/2}}{f_o^{1/2} \cdot f_m} \qquad \ldots (2.31(b)).$$

For a rectangular distribution of the random error amplitudes, the SNR is

$$\left[S_v / N_v\right]_{BINARY} = 0.22 \; \frac{k\, f_r^{3/2}}{f_o^{1/2} \cdot f_m} \qquad \ldots (2.32).$$

and for a truncated Normal distribution of the amplitudes, the eq.(2.32) is modified as,

$$\left[S_v / N_v\right]_{BINARY} = 0.5 \; \frac{k\, f_r^{3/2}}{f_o^{1/2} \cdot f_m} \qquad \ldots (2.33).$$

The equations (2.32) and (2.33) give the limits of the SNR in the Binary SQ-PCM system. The results of the Binary SQ-PCM are poorer by 6 db compared to those of the Ternary SQ-PCM for the same PRF and message signal frequency.

The noise power density spectrum given by eq.2.25 has nulls at 2 nf_r and continuous spectrum in between. This assumed that the noise pulses have uniform basewidth of τ , which is only an approximation for the typical noise waveform. As is seen in Fig.2.18 and 2.19, the noise waveform will have triangular pulses of varying base approximately between $\tau/3$ and 3τ for a complex input. Thus, the noise spectrum will not show any definite null at 2 nf_r, but will have dips only at nf_r, as has been experimentally verified (refer Fig.3.38). This will also effect the values of SNR in eq.2.30 and 2.33, as the lower values of the bases will give higher SNR. On an average, however, the calculated SNRs will be the practically realizable values.

2.5.0. COMPANDORS.

In similarity with AQ-PCM, discussed in Appendix A, the SNR characteristics obtained for the SQ-PCM systems in Section 2.4 has been shown to be dependent upon the amplitude of the signal, with the quantizing noise remaining constant. The effect of such a situation is that the SNR deterioration for the lower amplitudes of the signal ~~and~~ will become unacceptable for low inputs, e.g., in speech, the total dynamic range is of the order of 40 db and the SNR at a level of -40 db below the peak is of the order of 5 db only as shown in Fig.3.8. Naturally an attempt is to be made to improve the SNR at the lower amplitudes so that a large range of inputs is carried at an acceptable SNR. In the AQ-PCM system, a linear quantizer, which divides the total range of input levels into equal steps is normally replaced by a nonlinear quantizer of unequal steps, so that, the size of the quantum steps is small for weaker signals. This is achieved in practice by tapering the signals in such a manner that the weaker signals are spread over a considerable number of quantum steps and a combination of compressor and expandor, known as Compandor, is used for the purpose. In SQ-PCM systems also, a similar compandor, which allows the weaker signals to be approximated like stronger signals with accompanying good SNR, will offer certain advantages. In practice a compandor - a compressor at the input and an expandor at the output of the system - may be arranged either to equalise the SNR (quantizing) for the useful dynamic range, or to improve the SNR at the lower inputs only.

Generally two types of compandors, Syllabic and Instantaneous, have been used for combating channel noise. But they have entirely different fields of application, although they serve essentially the same purpose. Syllabic compandors have been used extensively in providing a considerable noise advantage in telephone channels. Instantaneous compandors, where the effective gain varies in response to instantaneous values of the signal, are found to be suitable for pulsed signals and have been used in many TDM pulse modulation systems. In the analysis of these compandors, it is generally assumed that the channel noise during the quiet intervals are most disturbing and the SNR improvement is maximum for the quiet intervals, but decreases with larger signals present.

In case of quantizing noise, however, it is seen that the most disturbing noise occurs only when the signal is being processed, and other noises during the quiet intervals are negligible in the coded pulse modulation systems. To be able to show that the same compandor scheme improves the SNR characteristics for both quantizing and channel noise, one observes that the quantizing noise may be considered as an added noise at the output of an ideal quantizer producing no noise at all. Referring to equation 2.26(a) for noise power in SQ-PCM and to equation A.2 for AQ-PCM, it is seen that for sufficiently larger number of quantizing steps used and for the input signal below the overload point, the quantizing noise is independent of the signal input and the noise spectrum is essentially flat over the signal band. Spang[21] and others, thus, justify the use of a simplified model of a moderately efficient quantizer, and represent the coder-quantizer as a device which adds signal-independent white noise to the signal. The overall coder-decoder system, therefore, consists of an ideal coder, an ideal decoder, and a noise source placed in between them which is equivalent to the channel noise present in a practical transmission system. The compandor now has to operate on this quantizing noise in the same way as it does on the channel noise, and most of the analysis is similar for both cases.

2.5.1. SYLLABIC COMPANDOR.

It is known that the syllabic compandor offers a good improvement in SNR performance for speech signals without increasing the bandwidth and the peak power of the signals. The advantages of the syllabic compandor are due to two separate contributions[22], (a) an increased SNR for the weaker signals due to the compressor action, and (b) a quieting of the channel during silent intervals due to expandor action. If the compression ratio in the compandor is 'n', and the noise is assumed to be introduced only after the compressor, then the matched expandor will reduce the noise level to $-nN_1$ db, where the noise during the no signal condition is N_1 db below the reference. This reduction in the output noise depends on the noise level at the input to the expandor and the noise improvement decreases linearly with the increase of the input noise level. Considering only the noise during the quiet intervals to be effective, Lawton[23] has shown that the noise N_2 at the compandor output is given by :

$$N_2 = N_1 + M(1 - 1/n) \quad \ldots\ldots \quad \text{for } N_1 \leqslant -M/n$$

but,

$$N_2 = (n - 1) N_1 + N_1 \quad \ldots\ldots \quad \text{for } -M/n \leqslant N_1 < 0$$

$$= nN_1$$

where

N_2 = noise output in db below the compandors 0 db reference level.

N_1 = noise input in db below the reference.

M = compression/expansion range of the compandor in db.

n = compression/expansion ratio.

Thus it is seen that the noise improvement is constant $[I = M(1 - 1/n)]$ if the noise input is below the expansion range, but varies as $(n - 1) N_1$, if the noise input is within the expansion range of the compandor. From practical considerations, the value of n is generally chosen as 2, although a higher value of n will give a higher noise improvement, but has the disadvantage of introducing more variations in channel loss and other system instabilities. With $n = 2$ and $M = 40$ db, the average noise improvement in syllabic compandors is about 20 db for $N_1 < -20$ db, but the improvement is only N_1 db for $N_1 > -20$ db.

When the speech signals are present, however, the noise level will be reduced by varying amounts depending upon the effective loss in the expandor required to restore the original volume relations and a gradually diminishing improvement for signals of increasing level will occur till there is no improvement for 0 db signal level. Thus for the with-signal condition where **S** is the signal level in db, the output noise is :

$$N_2 = N_1 + S/n, \quad \text{for} \quad -M < S < 0 \text{ db and } N_1 \leq -M/n$$

and,

$$N_2 = N_1 + M(1 - 1/n), \text{ for } S < -M,\ N_1 \leq -M/n \qquad \ldots (2.34)$$

The input vs. signal-to-noise curve in SQ-PCM is seen to be linearly rising with input level within the useful range and the net effect of the compandor would then be to spread the SNR curve because of the proportional improvement in S/n, as is shown in Fig.2.20. Because of the compressor distortion and the nonlinear expansion at higher input levels, the SNR for large signals will now be slightly lower than the above ideal values. Thus, the syllabic compandor would be useful in combating quantizing noise, specially for speech-like signals, where the useful energy levels would be processed by the system with higher output SNR.

2.5.2. INSTANTANEOUS COMPANDOR.

To estimate the SNR characteristics and the noise susceptibility of the instantaneous compandor, one may study the output of an expandor when the input is a signal plus noise voltage (as in the case of quantizing noise). Assuming the characteristic of the expandor to be as shown in Fig.2.21, an input of E volts results in an output of V volts. The same characteristic with input-output axes interchanged will apply for the compressor. If a noise voltage ΔE is also present at the input, the new input will be $E + \Delta E$ and the new output will be $V + \Delta V$ volts. At the output of the expandor the instantaneous signal, when there is no noise, is :

$$S = a V$$

where 'a' is a constant of proportionality. The instantaneous noise voltage, if it is present at the input, is :

$$N = a . \Delta V$$

$$= a . \Delta E . \Delta V/\Delta E$$

$$\text{or } N = a . \Delta E . M.$$

where M is defined as the noise susceptibility $= \Delta V/\Delta E$. Therefore, the output SNR is :

$$S/N = V/\Delta E.M. \qquad \text{... (2.35).}$$

In a non-compandored system with M = 1 the SNR is $V/\Delta E = E/\Delta E$. The equation (2.35) shows the improvement in SNR by a factor of $1/M$ provided a compandor is used. Since the characteristic of the expandor is non-linear, M varies with the amplitude of the signal and is a function of the slope of the characteristic.

Many non-linear single valued characteristics for the compandor are possible and the choice ultimately is in favour of

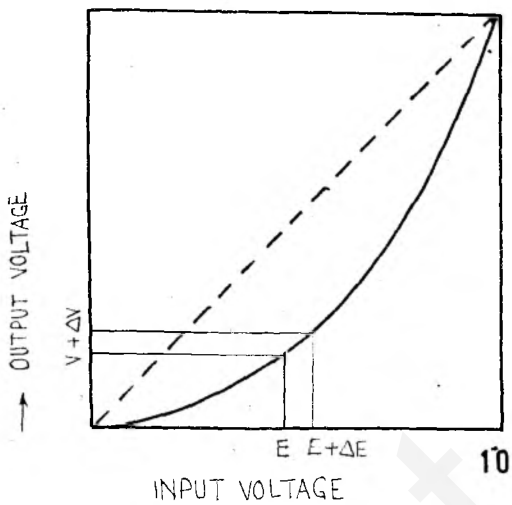


FIG. NO. 2.21

AN EXPANDOR CHARACTERSTICS

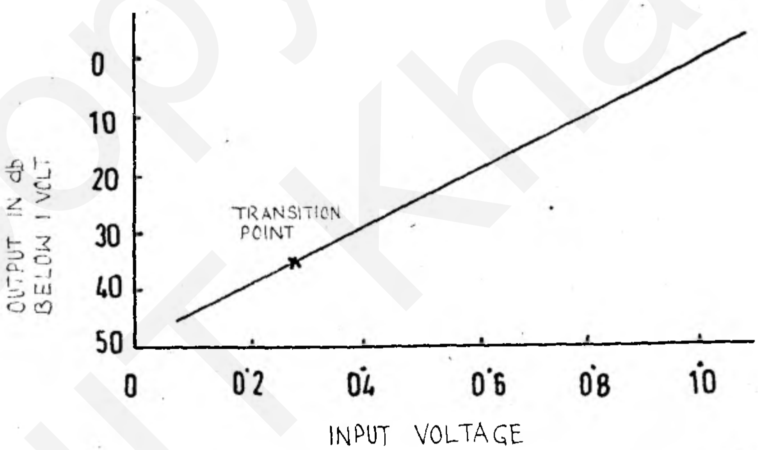


FIG. NO. 2.22

EXPANDOR CHARACTERSTIC
SHOWING THE TRANSITION POINT

a logarithmic one[24,22]. In a logarithmic compandor, the output voltage of the compressor is a logarithmic function of the input voltage, and conversely, the output voltage of the expandor is an exponential function of the input voltage, e.g.,

$$V = ce^{bE}$$

where c and b are **arbitrary** constants. It is evident that the characteristic cannot follow the exponential law at small values of input voltages, because, if the relationship is exponential, V is not zero when E is zero. This difficulty is avoided by using a characteristic which is linear for input voltages below a given value and exponential for input voltages above this value. A characteristic of this type is shown in Fig.2.22[24]. The point at which the characteristic changes from a linear to an exponential relationship is referred to as the transition point.

The exponential portion of the characteristics is given by

$$V = \epsilon^{(E-1)/E_t} \qquad \dots (2.36).$$

provided, $V = 1$ when $E = 1$ and provided, $M = \Delta V/\Delta E = V_t/E_t$ at the transition point. V_t and E_t are the output and input voltages respectively of the expandor at the transition point and from Eq.(2.36),

$$V_t = \epsilon^{(E_t - 1)/E_t} \qquad \dots (2.37).$$

If the "expansion ratio" K is defined as the ratio of V_m/V_t and E_m/E_t, where V_m and E_m are the maximum values of the expandor output and input voltages, then,

$$K = E_t/V_t \qquad \dots (2.38).$$

(Since it was assumed that $E_m = V_m = 1$). Substituting the value of V_t from eq.(2.37),

$$K = E_t \, \epsilon^{(1-E_t)/E_t} \qquad \ldots (2.39).$$

If E_t is replaced by V_t in the above expression then, K will define the "compression ratio". The equations (2.37) and (2.39) define completely the relation between K, V_t and E_t. If any one of them is specified, the other two can be calculated and the complete expandor characteristic is known.

If the input voltage is within the exponential region, the SNR can be calculated by finding the noise susceptibility M and substituting it in eq.(2.35). By differentiating eq.(2.36) with respect to E,

$$\Delta V/\Delta E = (1/E_T) \cdot \epsilon^{(E-1)/E_t} = V/E_t = M$$

This shows that the noise susceptibility is directly proportional to the magnitude of the signal. Putting this value of M in eq. (2.35).

$$[S/N]_{\text{INSTANTANEOUS}} = V/\Delta E.M = E_t/\Delta E \qquad \ldots (2.40).$$

In the system without compandor, the SNR obtained will be $E_{max}/\Delta E$ and since $E_t < E_{max}$, the SNR with the compandor included will be less, which agrees with the physical reasoning given earlier. The eq.2.40 shows that the ratio of instantaneous signal to instantaneous noise voltages at the output of the compandored system is independent of the amplitude of the input signal within the exponential range of the expandor. This is a definite advantage of the logarithmic compandor. Since the quantizing noise in PCM systems has a uniform power density, it is more convenient to use the rms noise voltage E_n in eq.(2.40). Hence,

$$S_{INSTANT}/N_{rms} = E_t/E_n \qquad \dots (2.41).$$

Eq.(2.41) gives a very simple method of evaluating the SNR in an instantaneous compandor. E_t is estimated from the amount of the compression or expansion to be used and E_n is the total rms noise at the input of the expandor. The above derivation of eq.2.41 applies to the sampled PAM pulses, but in the present case, it can also be used if E_n is defined as the total quantizing noise contributed by the distorted and compressed signal.

If the input voltage is within the linear region of the characteristic, the noise susceptibility is V_t/E_t or is equal to 1/K, i.e., it is inversely proportional to the expansion ratio. Thus it is seen that the improvement due to the compandor is due to two reasons. First of all, the input to the quantizer is so changed by the compressor that the weak signals are quantized with much better SNR, and secondly the expandor, while correcting the action of the compressor, makes the output SNR independent of the input in the exponential region of its characteristics. The net result is a flat, though lower, SNR characteristic for almost the entire dynamic range of the input, as is shown in Fig.2.20.

A sample calculation of the SNR for the compandored SQ-PCM system is given below for illustrating the use of Fig.2.22 and eq.(2.41). It is assumed that a range of 35 db below 1 volt has to be handled by the exponential region of the compandor characteristic, and inputs below 35 db are to be handled by the linear region. This means that the exponential range at the output of the expandor extends from 1 volt to 35 db below 1 volt. The transition-point E_t for the input is therefore fixed at 0.283 volts i.e. 11 db below 1 volt as is seen in Fig.2.22, and hence an input change of 11 db results in an output change of 35 db in the expandor. From Eqs.(2.37) and (2.39) V_t and K are -22.5 db and 11.5 db respectively. The complete expandor characteristic is therefore specified.

To calculate E_n of eq.2.41, one has to determine the total rms noise at the input to the expandor. The instantaneous compressor distorts the signal and if a sinusoidal signal input is assumed, the 3rd, 5th, 7th etc. harmonics apart from the fundamental are produced. The levels of the harmonics generated at the output with different input levels are given in Table I, for the compressor characteristic shown in Fig.2.24.

TABLE I.

Input level.	Fundamental.	3rd Harmonic.	5th Harmonic	7th Harmonic
0 db ref.	0 db ref.	-14 db.	-20 db.	-24 db.
-10 db.	-2.5 db.	-17 db.	-24 db.	-29 db.
-20 db.	-5 db.	-22 db.	-32 db.	-45 db.
-30 db.	-12 db.	-36 db.	-50 db.	-
-40 db.	-21 db.	-	-	-

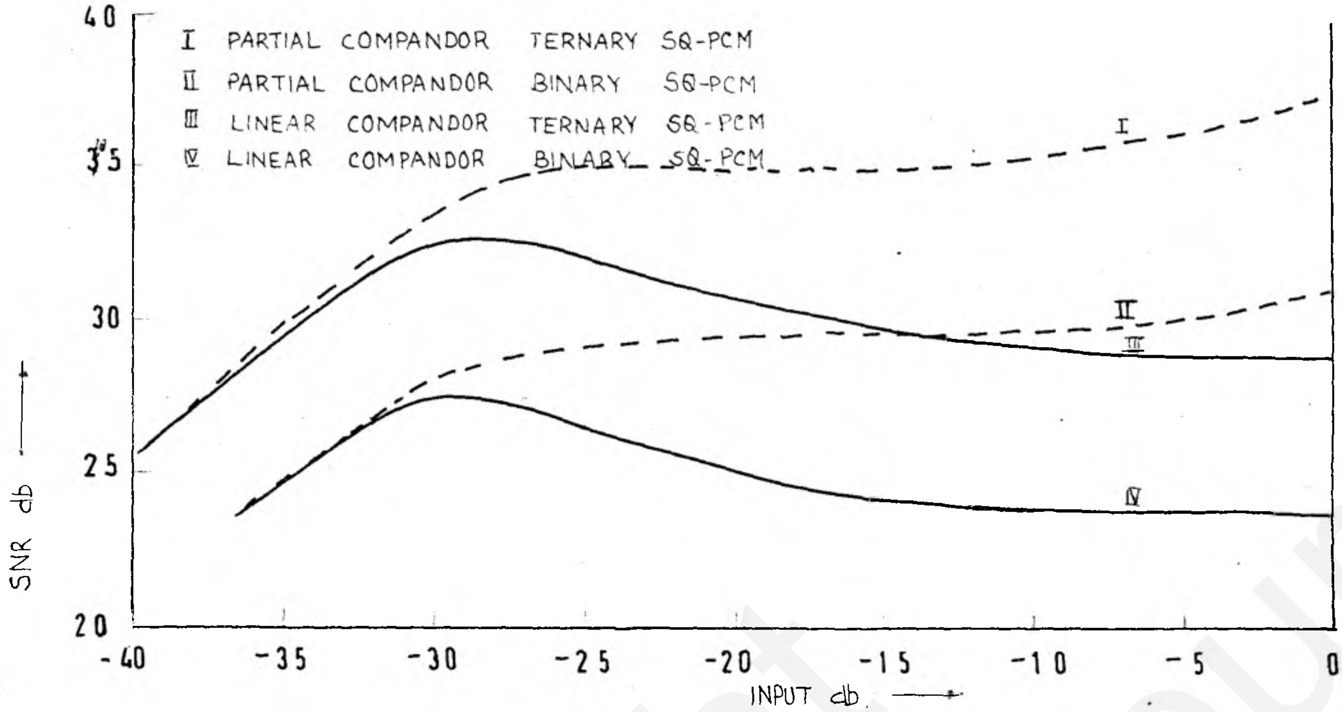
Each harmonic, at the above levels, will pass through the SQ-PCM system and be reproduced together with a quantizing noise. If the variation of SNR (Quantizing) with input levels in the SQ-PCM system is assumed to be linear, then the contribution of quantizing noise due to each harmonic is known. In the Ternary SQ-PCM system at 40 Kc/s, for instance, the quantizing noise is 45 db below the signal for a 0 db (reference) input. Hence the noise contribution due to the fundamental is 0.0056 V for 1 volt (= 0 db reference) input signal to the compressor. For a 3rd Harmonic of -14 db, the SNR is again -45 db (equal to 31 db below the harmonic level) and the noise is 0.0056 V, and similarly for others. The rms value of the sum of the noise voltages due to the fundamental, 3rd, 5th, and 7th harmonic is equal to 0.01 volt for 0 db input level. In the same way, the rms noise for other input levels is calculated. Now from eq.(2.41),

$$S_{INST.}/N_{rms} = E_t/E_n = \frac{0.283}{0.01} = 29 \text{ db}$$

(for 0 db input level).

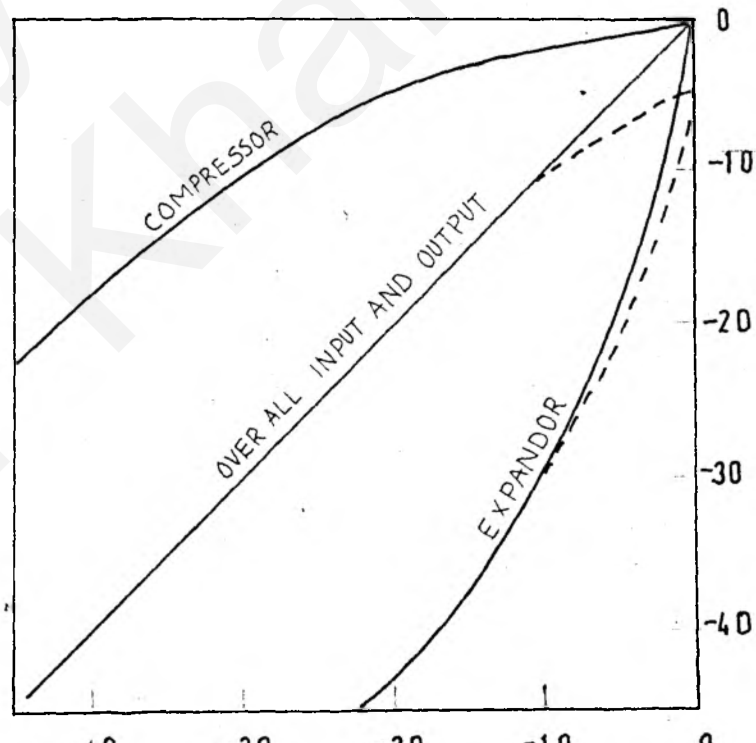
Fig.2.23 (solid curve II) shows a plot of SNR vs. input level of the compandored Ternary SQ-PCM system at a PRF of 40 Kc/s. The figure (solid curve IV) also shows the theoretical values of SNR with input level (calculated in a similar manner as in the Ternary system) in case of the Binary SQ-PCM system at a PRF of 40 Kc/s.

The usual input-output characteristics of the compressor and the expandor are shown in Fig.2.24. The overall input-output relation of the compandored system is generally linear as shown in the same figure. However, it is experimentally found that in



EFFECT OF COMPANDORS ON THE SNR CHARACTERSTICS OF THE SQ-PCM SYSTEMS

FIG. NO. 2·23



INPUT CHARACTERSTICS OF THE COMPRESSOR AND THE EXPANDOR

FIG. NO. 2·24

certain applications e.g. in speech processing, it is not necessary to have this characteristic absolutely linear. A certain amount of compression (at the higher input levels) in the output of the system may be tolerated. For this class of signals, therefore, a partial compandor with the input-output relation shown in Fig.2.24 (dotted) can be used. The SNR calculation is now modified slightly and it is found that this SNR characteristic is better than that of the matched compandor. The SNR vs. input curves for the Ternary and Binary SQ-PCM systems using partial compandor are also shown in Fig.2.23 (dashed curves).

Thus it is seen that the instantaneous compandor can provide an almost flat SNR for an input dynamic range of about 35 db. The main advantage of the instantaneous compandor, lies in the practicability of compressing and expanding instantaneous amplitudes of sampled signals, as are present in TDM pulse modulation systems.

2.5.3. COMMENTS.

An important difference between the syllabic and the instantaneous compandor is the bandwidth requirement. Syllabic compandors can transmit signals in a frequency band not significantly larger than that of the original signal. The distortion of the signal by the compressor may virtually be restricted to a change in loudness only. Instantaneous compandors used on time-division systems, also do not require any extra bandwidth. On the message signal, however, they will introduce heavy distortion and the bandwidth required after the compressor will be quite large compared to the message bandwidth. Lozier[25] has shown that theoretically, the bandwidth can be restricted to the message bandwidth provided the compressed signal is sampled at a frequency which is precisely the double of the message bandwidth transmitted in the channel and recovered at the receiver by a synchronous sampling again at $2W_m$. The instantaneous compandors thus do not work satisfactorily in systems having a bandwidth of the order of the signal bandwidth, the additional transmission requirements being very severe if the bandwidth is so restricted.

Suitability or otherwise of a particular type of compandor can be assessed with reference to the type of message signals at the input, the bandwidth of the SQ-PCM systems, and the cost considerations in time-multiplexing of the number of channels. It is well known that in speech signals, the maximum amount of energy is concentrated around 10-15 db of the rms input signal. The SNR of the SQ-PCM systems is quite acceptable for this range of input signals, but the weaker signals will be reproduced with poor SNR. Syllabic compandors, by compressing the total volume range of the speech input, will allow the processing of the message at a level where the SNR is good. The instantaneous compandor, on the other hand, will improve the SNR at the low levels at the cost of the SNR at higher input levels. The overall SNR characteristic will be flatter though at a much lower level as shown in Fig.2.20.

One of the disadvantages of the syllabic compandor is that in a TDM system, each channel will require a compressor and an expandor. Whereas the instantaneous compandor, working on the instantaneous values of the sampled signal, can be made common, by first sampling and multiplexing all channels and then compressing this multiplexed signal by one compressor. Similarly, at the receiver, after decoding, the instantaneous values are expanded before being separated into individual channels. The SQ-PCM system like the AQ-PCM system can also be multiplexed on TDM basis and a single compandor used, as will be shown in Section 5.5. The considerable economy in the equipment and the valuable noise reducing properties of the instantaneous type of compandor make it the most obvious choice in TDM coded-pulse modulation systems.

2.6. DISCUSSION.

The two block diagrams of Fig.2.3 and 2.4 are the basic units of any uni-digit quantized pulse modulation system and the systems only differ in the details of arrangement of filters, equalisers and feedback circuits. A three level quantizer directly leads to a Ternary system and it is seen that the Ternary coding offers many advantages over the Binary coding. A two level quantizer basically gives Binary SQ-PCM, Δ-M and Δ-ΣM systems. The signal-to-noise characteristic of the SQ-PCM has been worked out in Section 2.4, with the assumption that the feedback network is an integrator and a replica of the receiver. The signal-to-noise characteristics of two of the other arrangements possible have been worked out in Appendix B. The improvement in the approximation by the Binary SQ-PCM could be attributed to the use of a threshold in the quantizer and an optimum response of the feedback network. The approximation in Δ-M with +1/-1 pulses and hence no threshold, does not allow the optimization of the amplitude distribution of the error pulses. The SNR at the output (eq.B.8), therefore, is equivalent to the worst case in the Binary SQ-PCM (eq.2.32). The SNR, however, depends upon the message signal frequency f_m and also on the signal amplitudes KnV_e; hence, SNR deteriorates at the higher frequencies and at lower signal levels. The use of an integrator at the receiver also leads to cumulative errors due to channel noise. The second model in Appendix B (Δ-ΣM) has an SNR (eq.B.22) independent of f_m and does not use an integrator at the receiver. Jager[4] has reported that a double integration in the feedback loop of the coder of Δ-M helps to improve the SNR at sufficiently high PRF's. At the PRF's which have been used here, it has been found that the SNR using double integration in the feedback are poorer and there is a tendency of the circuit to be unstable.

In similarity to AQ-PCM, it has been possible to use instantaneous compandors in the Ternary and Binary SQ-PCM systems, and the theoretical calculations show a satisfactory SNR at the output for a total input range of about 40 db. Theoretically, the SNR properties of the Ternary SQ-PCM at 40 Kc/s PRF are very similar to those of a 7-digit AQ-PCM and it has been shown later that the two practical systems are almost equivalent in performance. Similarly, the Binary SQ-PCM at 40 Kc/s PRF is found to be equivalent to a 6-digit AQ-PCM in actual performance.

CHAPTER III.

CHARACTERISTICS OF THE TERNARY SQ-PCM.

3.0 GENERAL.

The coding scheme of Section 2.2 has been worked out experimentally in the laboratory. It has been shown there that the input signal is differentiated, sampled and then passed through a 3 level quantizer which converts the variations of the slopes of the signal into unit pulses of +1, 0 or -1. An integrator at the receiver builds up a staircase approximated signal which, when filtered, gives the signal back. It is also seen that a continuous comparison of the input signal with the approximated signal, obtained by using a feedback loop incorporating the local receiver, gives rise to a minimisation of error in quantizing. The use of a threshold in the comparator results in an optimization of the distribution of error amplitudes, as has been shown in Section 2.4. The present Chapter is intended to give the detailed circuit diagrams and performance characteristics of the Ternary SQ-PCM systems - with and without feedback - developed in the laboratory.

The performance characteristics like the SNR, the linearity and the frequency distortion etc. of the Ternary SQ-PCM systems has been tested with different types of input signals such as sinusoidal signals, FM signals, Noise signals, and Speech signals. The sinusoidal signals, though in themselves not ideal signals for testing any system, are used here extensively as test signals because the simplicity and ease of measurement outweigh the disadvantages. Test results of sinusoidal signals are also a realistic and convenient basis of comparison with other systems. On the other hand, the signals which one may come across in practice are more complex and semi-random in nature. A noise signal test, therefore, gives a more correct picture of the actual performance under practical conditions. At the same time, in some applications like Telemetry etc. the signals are frequency modulated with no, or little, amplitude variations, and the behaviour of the system for this special class of input is also worth investigating. More often than not, the input signal is a speech signal which is also random like noise, but more redundent as far as intelligibility is concerned. The speech signals can be transmitted intelligibly by systems which may have no linearity in input vs. output relation, and whose frequency response is not uniform.

As has been mentioned in Section 2.2, the circuit configurations of Figs.2.10 and 2.11 have been worked successfully in the laboratory, but the configuration given in Fig.2.11 is simpler and leads to slightly improved overall results, and, therefore, the discussion and the performance of this latter system only will be given here. The removal of the feedback loop and insertion of a differentiator (AF band) in the circuit before the sampler will convert the circuit to a SQ-PCM-without feedback system. The receiver in both cases is the same.

Section 3.1 deals with the Ternary SQ-PCM system with feedback. The detailed circuit diagrams are given in Section 3.1.0

and the system characteristics are given in Section 3.1.1. Sections 3.1.2 and 3.1.3 discuss the stability and the quantizing noise spectrum respectively, and Section 3.1.4 gives a summary of the results. Section 3.2 deals with the Ternary SQ-PCM system without feedback, it's circuit details and the performance characteristics. Section 3.3 concludes the Chapter with a discussion of the results obtained for the Ternary SQ-PCM systems.

3.1.0 TERNARY SQ-PCM WITH FEEDBACK.

A functional diagram of the Ternary SQ-PCM system with feedback has been shown in Fig.2.11 and a full block schematic diagram of the system is shown in Fig.3.1. The input signal is fed through an amplifier and cathode follower to the difference circuit, to which is also fed the approximated signal obtained through the feedback circuit. The error signal at the output of the difference circuit is amplified and sampled at the desired PRF by the Bisymmetrical sampling circuit with symmetrical positive and negative pulses from the pulse generator. The samples are next separated by the positive and the negative clipper circuits. The positive error samples are quantized by the Schmitt trigger, with a reference bias so adjusted that the error amplitudes larger than the threshold are converted into a positive unit pulse, and error amplitudes less than the threshold produce no pulse. Similarly, the negative error samples are quantized by another Schmitt trigger into a negative unit pulse, or a 'no' pulse, depending upon the amplitudes of the error samples being below, or above, a reference threshold. The output of the Schmitt triggers are differentiated and amplified to remove any

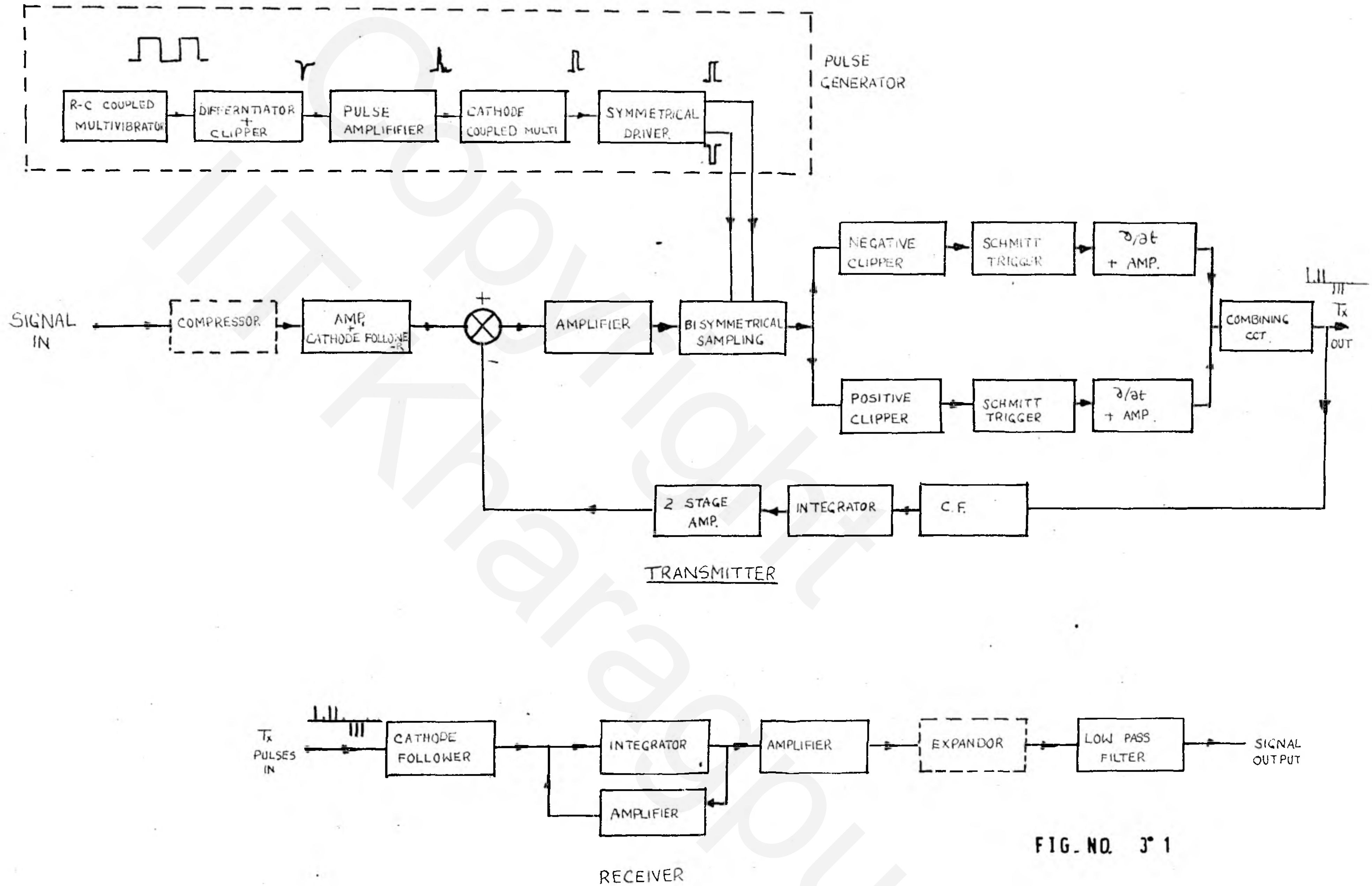
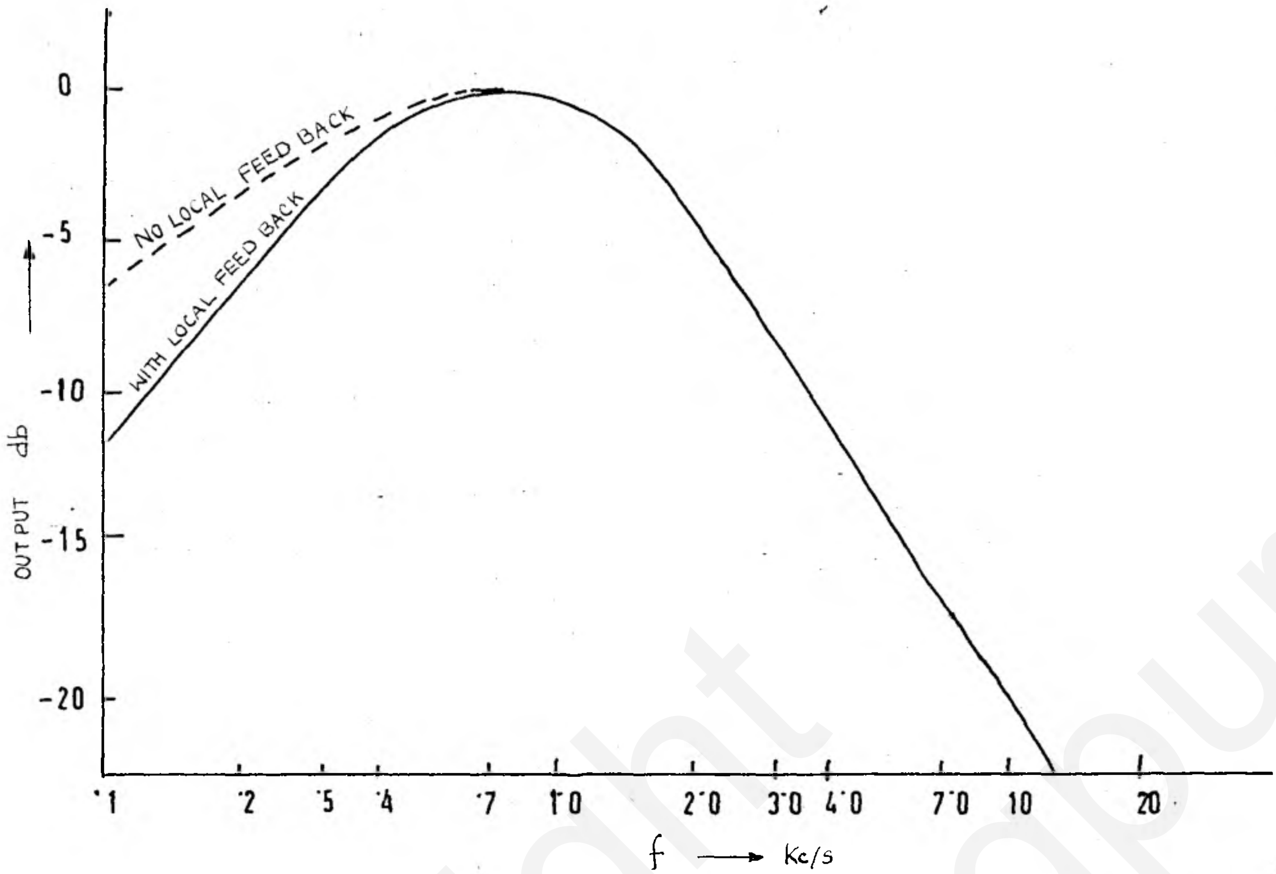


FIG. NO. 3·1

BLOCK SCHEMATIC DIAGRAM OF THE TERNARY SQ-PCM SYSTEM WITH FEEDBACK width variation and combined in an adder circuit to form the transmitter output. A part of the output is taken through a cathode follower, integrator and 2 stage amplifier (with gain control) to form the feedback loop. The integrator builds up the signal in positive, or negative, steps depending upon the polarity of the transmitter pulses. The pulses are therefore converted into a continuous signal which is amplified by the 2 stage amplifier and fed into the difference circuit to complete the loop. Two stages of amplifiers are necessary to make the feedback signal 180 degrees out of phase with the input signal.

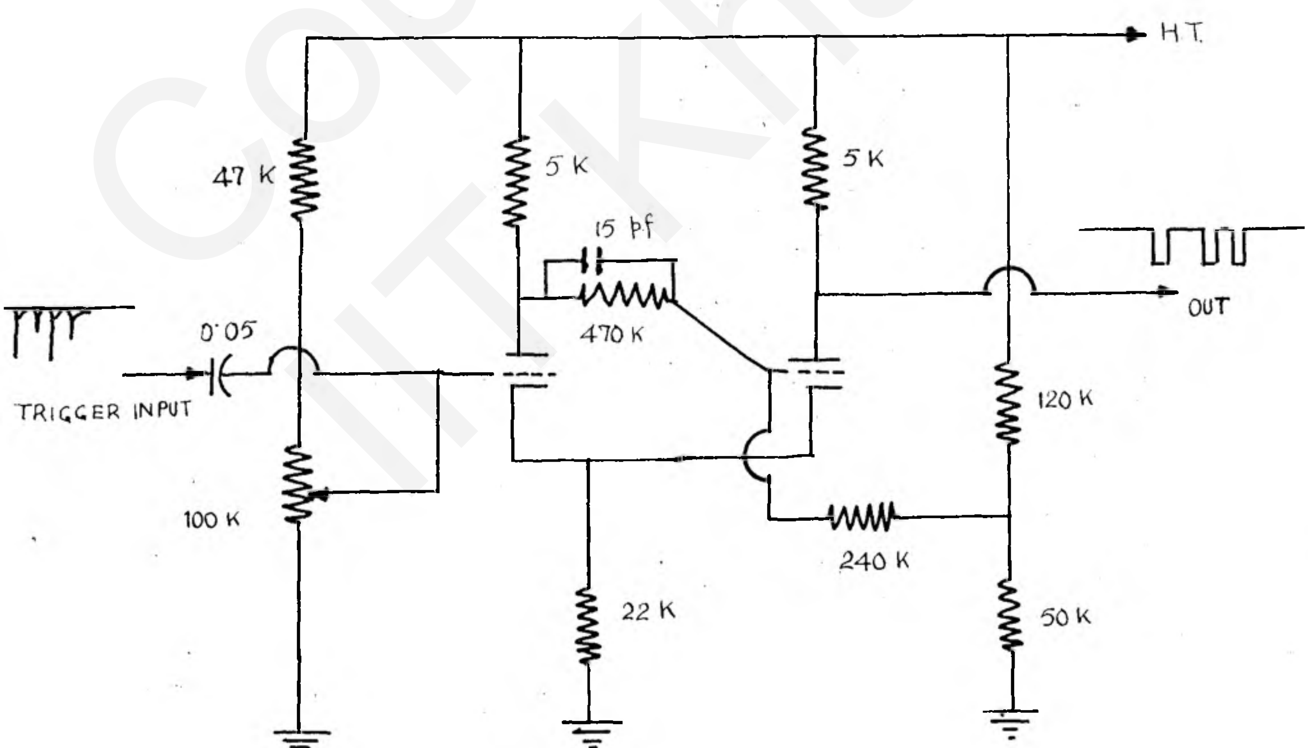
The receiver consists of a cathode follower, followed by an integrator, amplifier and a bandpass filter (300 c/s to 3400 c/s passband), to give the approximated output of the system. The integrator has a break point at 800 c/s and its response is further modified by a local feedback across it. The purpose of this local feedback across the integrator is to reduce the output of the integrator at low frequencies. This has the effect of reducing the output of the system at lower frequencies and thus equalising frequency response to some extent. It also improves the SNR at higher frequencies by suppressing the periodic and distortion noise produced at the lower end of the spectrum. A plot of the variation of the output amplitude with frequency for the integrator-amplifier combination in the receiver is shown in Fig.3.2.

Most of the circuits in the transmitter and the receiver are of the conventional type. The pulse generator in Fig.3.1 consists of the RC-coupled multivibrator whose repetition frequency is controllable by varying the common grid bias. The pulses obtained are differentiated and the positive half clipped to give nega-



THE VARIATION OF THE OUTPUT WITH FREQUENCY OF THE RECEIVER CIRCUIT

FIG. NO. 3·2



THE MODIFIED NEGATIVE PULSE SCHMITT TRIGGER

FIG. NO. 3·3

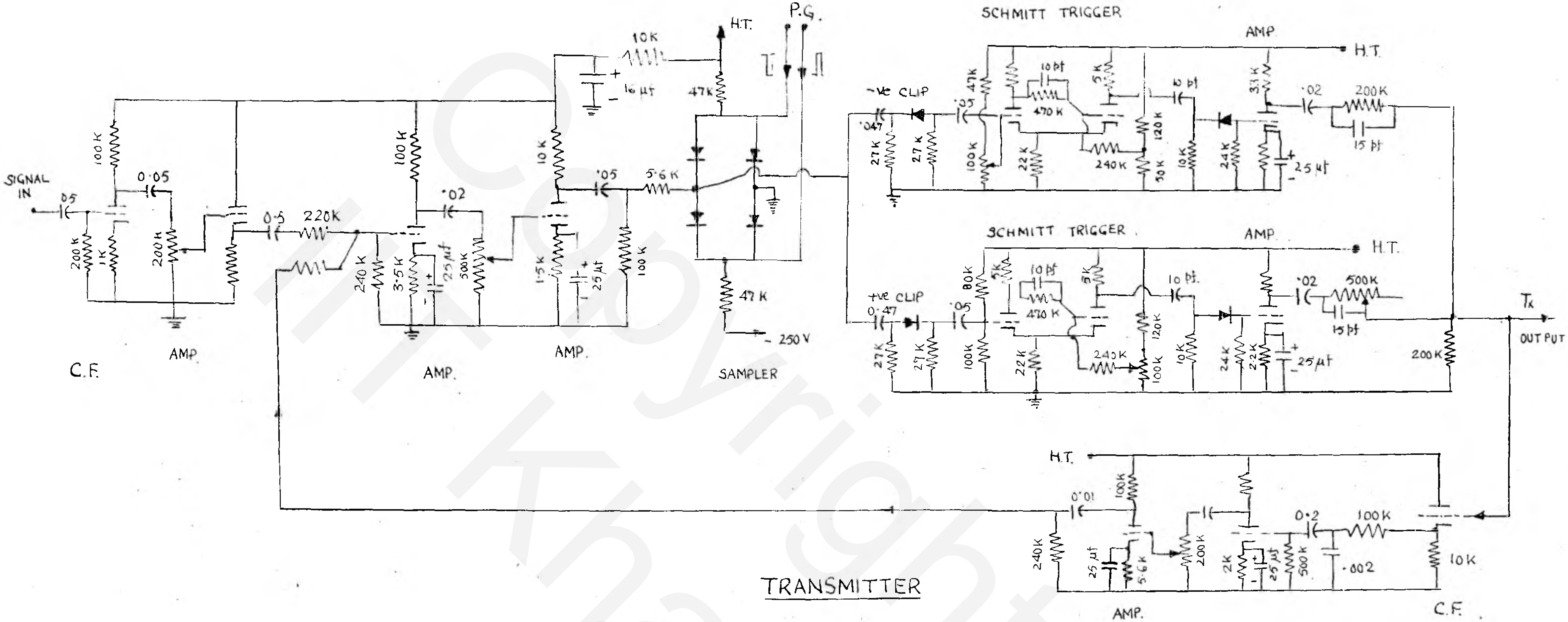
tive spikes which are amplified by a pulse amplifier. The trigger pulses so generated are used to trigger a cathode-coupled one-shot multivibrator which produces pulses of uniformly large amplitude and constant width. The output of the multivibrator is split into symmetrical positive and negative pulses which are used in the Bisymmetrical sampling circuit. The positive pulse Schmitt trigger is a conventional circuit, but the negative pulse Schmitt trigger, shown in Fig.3.3, is a slight modification of the conventional circuit. Here the negative trigger pulses give a negative pulse output, and the bias arrangement is sensitive enough to make the circuit trigger for input amplitudes larger than a particular value. A complete circuit diagram of the transmitter and the receiver is given in Fig.3.4.

The compandor - compressor at the input and the expandor at the output - is shown dotted in Fig.3.1, the circuit details being given in the Fig.3.5(a) and (b). The diodes used are either OA79 or OA85. The input-output characteristics of the compressor and the expandor have been determined experimentally and are shown in Fig.3.6. These characteristics agree fairly well with the theoretical curves in Section 2.5. The partial compandor uses the same compressor but uses only one expandor to give the input-output relation shown dotted in Fig.3.6.

3.1.1. <u>SYSTEM CHARACTERISTICS</u>.

(a) <u>Sinusoidal Signal Test</u>.

Quantizing-noise obtained with the variation of input



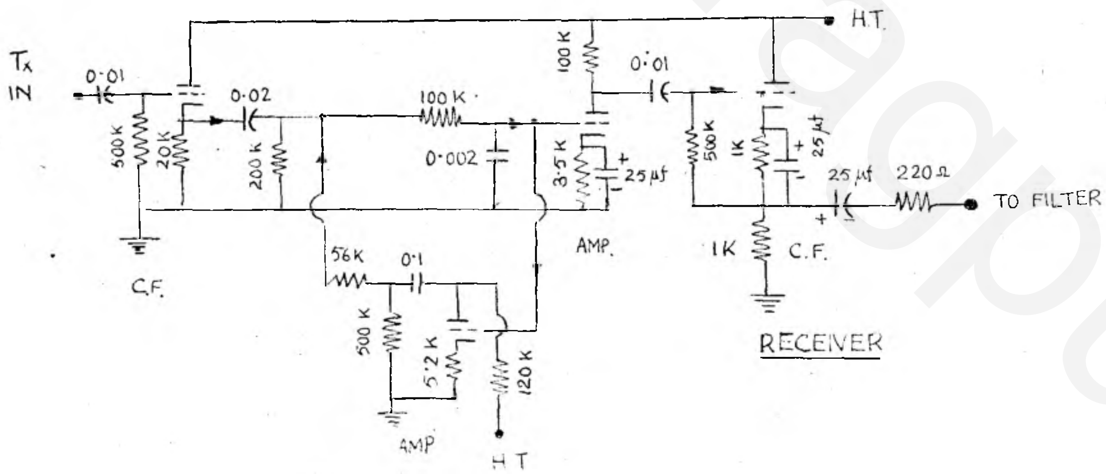


FIG. NO. 3.4

COMPLETE CIRCUIT DIAGRAM OF THE TERNARY SQ-PCM SYSTEM WITH FEEDBACK

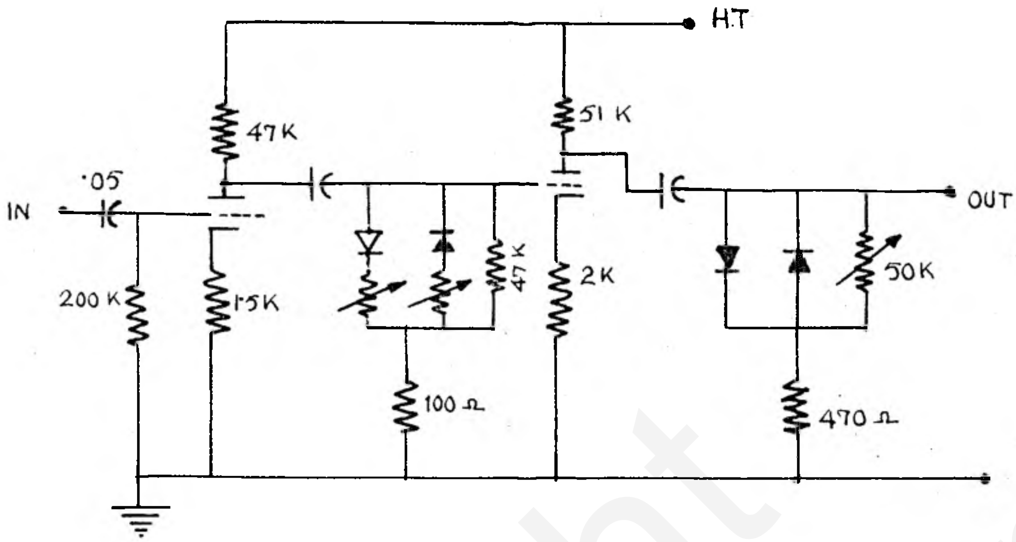


FIG. NO. 3·5(a)

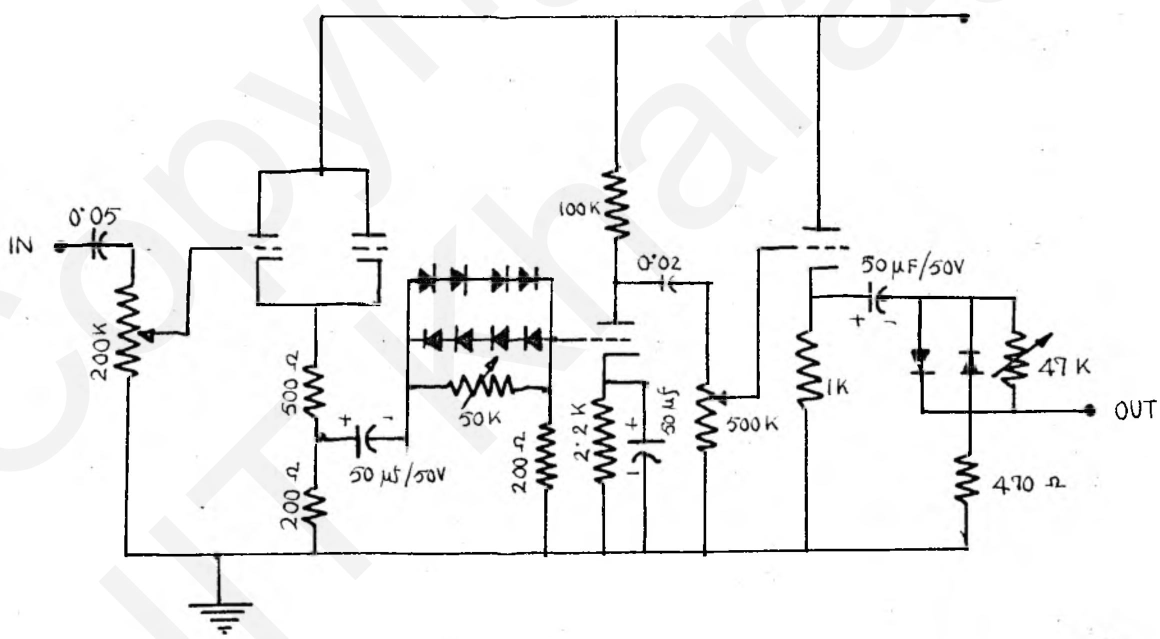
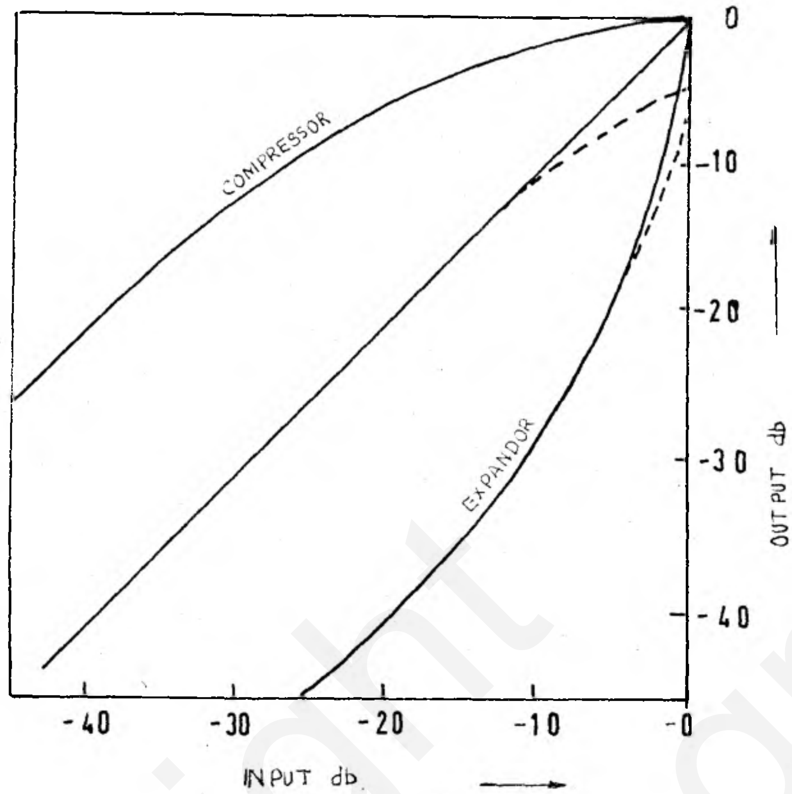


FIG. NO. 3·5(b)

THE CIRCUIT DIAGRAM OF THE COMPANDOR
(a) THE COMPRESSOR, AND
(b) THE EXPANDOR

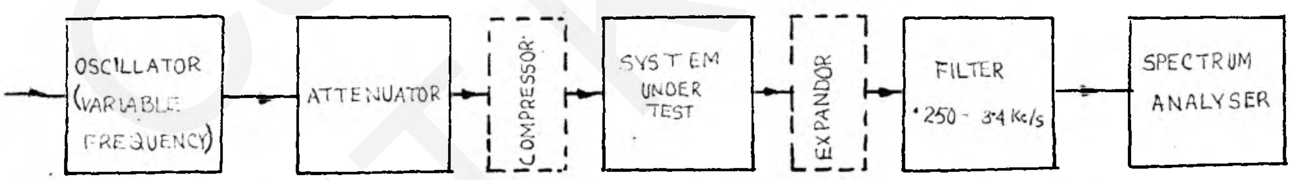
signal amplitude, and frequency, is one of the fairly important characteristic in any coded system. The measurement of signal-to-quantizing-noise (SNR(Q)) ratio will involve a measurement of the signal amplitude, and the noise in the passband (excluding the signal). Two types of equipment have been utilised for this purpose. A visual-display audio frequency Spectrum Analyser (Panoramic Sonic Analyser Model LP-1.a), shows the complete frequency spectrum of the output signal together with the amplitudes of each of these components. The average value of the noise part of the spectrum is displayed as a patch on the frequency-amplitude (db) scale. The signal-to-noise ratio is therefore directly read out. The calibration of this equipment is done with the other type of measurement which utilises a Noise-distortion Analyzer (Hewlett Packard Model 330 B). In this measurement, assuming that the SNR is high, the signal plus noise is measured, and then the signal is suppressed by a tunable filter. The resulting noise alone is measured by the average meter calibrated in terms of rms value, and the ratio of the two gives the SNR. It has been found that the measurements made by the spectrum analyser and the distortion analyser agree fairly well. The test results of the SNR etc. of the SQ-PCM systems reported here have been taken by either one of the two methods, depending on the convenience of measurement.

The experimental observations, like variation of SNR(Q) with input signal magnitude, frequency, and PRF etc., for the sinusoidal signals, have been taken with a laboratory set up as given in Fig.3.7. First of all, the compressor and the matching expandor are not used, and the system is tested with signals in the band



THE INPUT- OUTPUT CHARACTERSTICS OF THE COMPRESSOR AND THE EXPANDOR; THE DOTTED CURVE INDICATES THE RELATION FOR THE SINGLE EXPANDOR

FIG. NO. 3·6

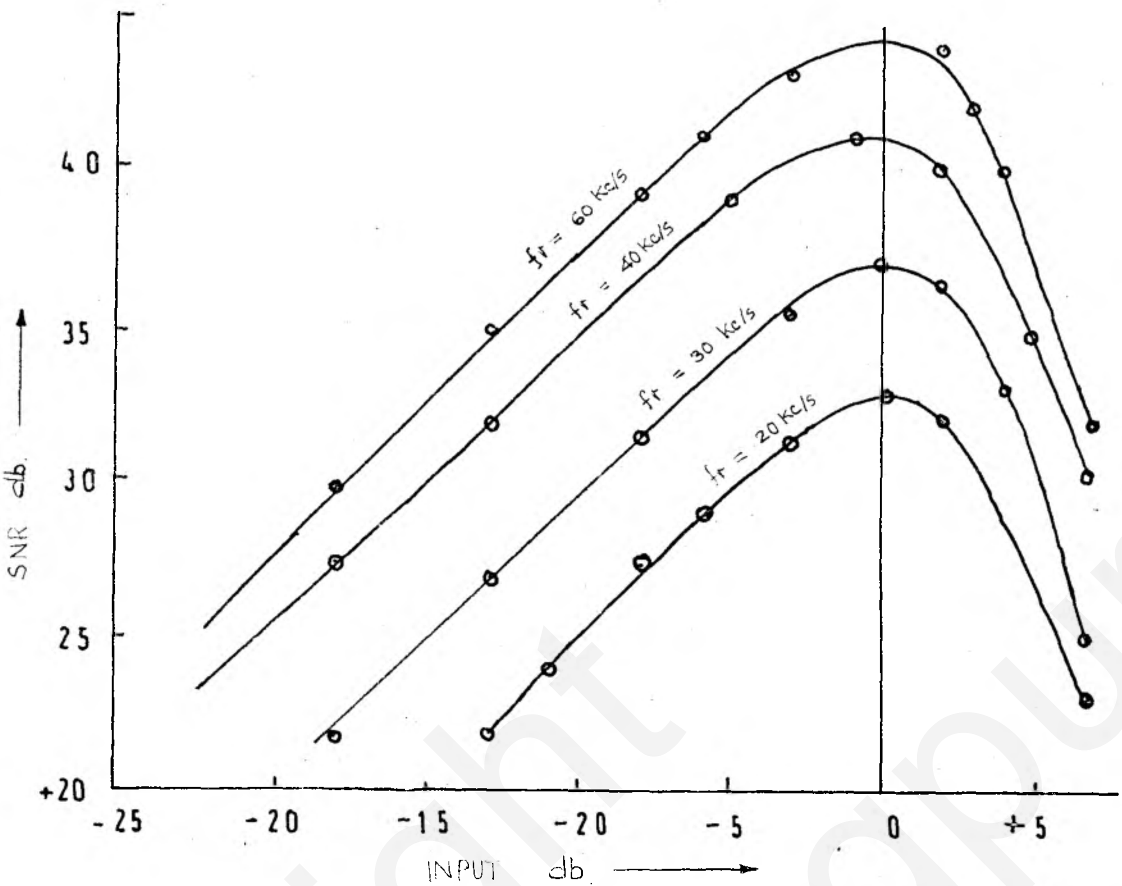


AN EXPERIMENTAL SET-UP FOR THE MEASUREMENTS WITH SINUSOIDAL SIGNALS

FIG. NO. 3·7

of 300 c/s to 3400 c/s frequency. The variation of SNR with input signal level is shown in Fig.3.8 for 4 different PRF's of 60, 40, 30 and 20 Kc/s. The signal frequency is 1 Kc/s and the best SNRs are 44, 41, 37 & 33 db at PRF's of 60, 40, 30 and 20 Kc/s respectively. The SNR is above 25 db for a dynamic range of 31, 29, 22 and 16 db for the same four PRF's respectively. The SNR falls sharply above the optimum input level of 0 db because of the overloading effect, discussed in Section 2.4. The feedback voltage is built up by $f_r/2f_m$ number of positive and negative pulses, each pulse changing the signal by one unit. The signal amplitude built up, therefore, is limited by the number of pulses which is again fixed by f_r and f_m. Any increase of the input signal amplitude beyond the reference level will thus result in clipping at the output of the system. Harmonics will be produced due to clipping, and Fig.3.9 shows an experimental measurement of the amplitudes of the 2nd and 3rd harmonic when the input signal is above the overload point. A +5 db increase in the signal results in a -38 db 2nd harmonic and a -35 db 3rd harmonic level, below the fundamental. A further increase to +7.5 db input results in a -37 db 2nd harmonic and a -31 db 3rd harmonic level, below the fundamental.

In similarity with AQ-PCM and Δ-M systems, a periodic noise[17,5] in the form of beat notes occurs for the discrete values of the signal frequencies. Fig.3.10 shows the nature of variation of the periodic noise in the SQ-PCM system with feedback, where the periodic noise output below the fundamental is plotted for a signal frequency of 2700 c/s. The frequency of the periodic



THE VARIATION OF SNR WITH INPUT LEVEL FOR fr OF 60, 40, 30, AND 20 Kc/s
fm = 1 Kc/s

FIG. NO. 3·8

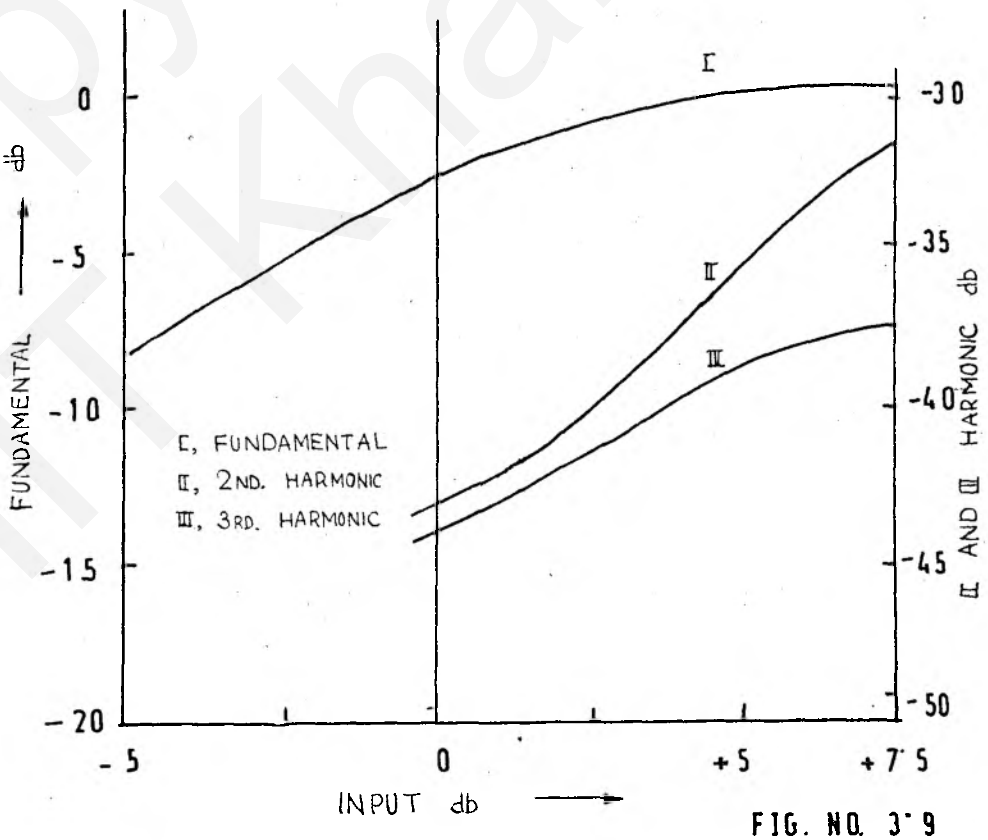
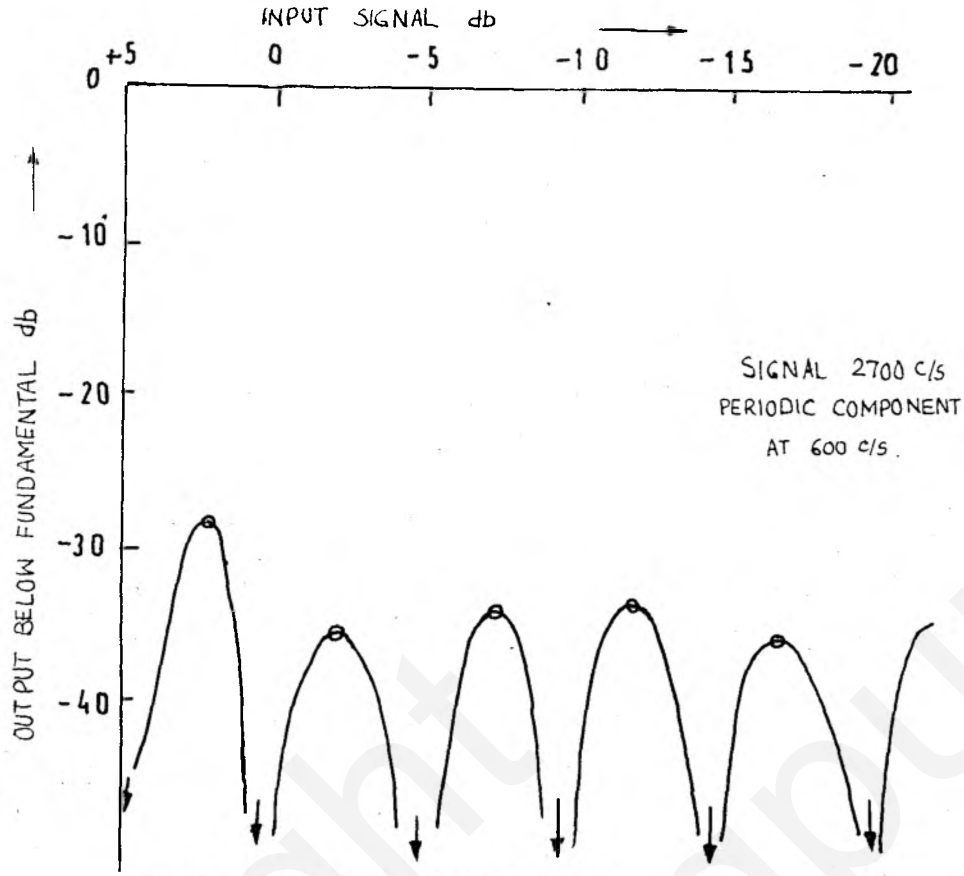


FIG. NO. 3·9

AMPLITUDE OF HORMONICS DUE TO OVERLOAD



VARIATIONS OF THE PERIODIC NOISE WITH INPUT LEVEL; f_r = 40 Kc/s; f_m = 2700 c/s. AND THE PERIODIC NOISE AT 600 c/s

FIG. NO. 3·10

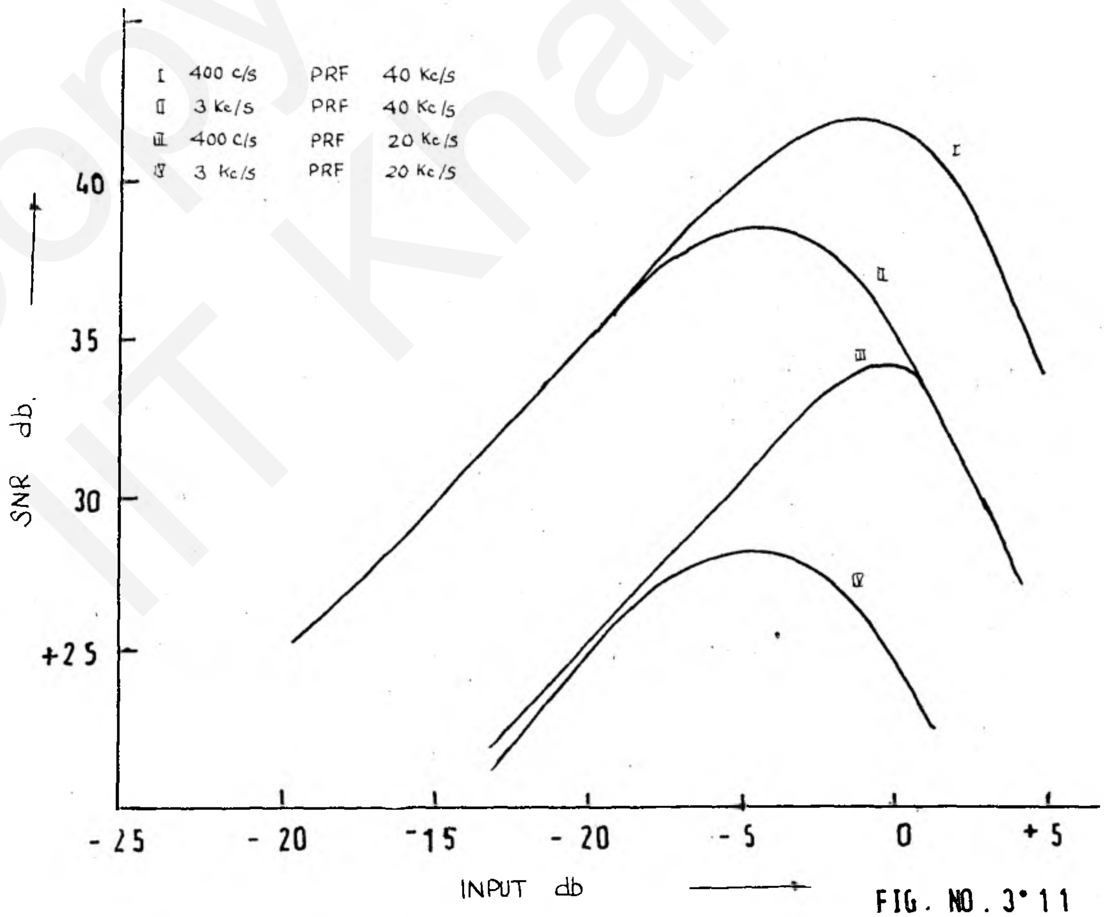


FIG. NO. 3·11

VARIATION OF SNR WITH INPUT LEVEL FOR DIFFERENT f_m AND f_r

component is 600 c/s in this particular measurement. The points shown as circles are the measured values and the arrows indicate the input levels for which the periodic component is negligibly small; the continuous curves have been extrapolated from these.

Fig.3.11 shows the variation of the SNR with input level for signal frequencies of 400 c/s and 3 Kc/s. These curves have been drawn only for two representative PRF's of 40 Kc/s and 20 Kc/s as the results at other PRF's have a similar nature. The SNR at higher signal frequency deteriorates by about 6 db at 0 db input level for 40 Kc/s PRF and by about 8 db for 20 Kc/s PRF. In Fig.3.12 the variation of SNR with signal frequency is shown for different input levels and it is seen that, for the -5 db input level (ref.0 db), the SNR at the higher frequency falls only by one db, but at still lower input levels it is constant for all frequencies. It is found that the feedback network is not able to build up enough feedback voltage from the fewer transmitter output pulses at the higher frequencies, as explained earlier, and therefore, the feedback is not effective for input levels above -3 db. Around this input level, the decreased signal amplitude becomes comparable to the feedback voltage and the feedback starts functioning so that a linear input-output relation obtained. The same effect is also seen in the output vs. sig-l frequency curve shown in Fig.3.13. There is a frequency dis-rtion at higher input levels, and the 3 Kc/s output, at 0 db put level, is about 3 db below the output of the 1 Kc/s signal. wever, at -3 db input the frequency distortion is negligible,

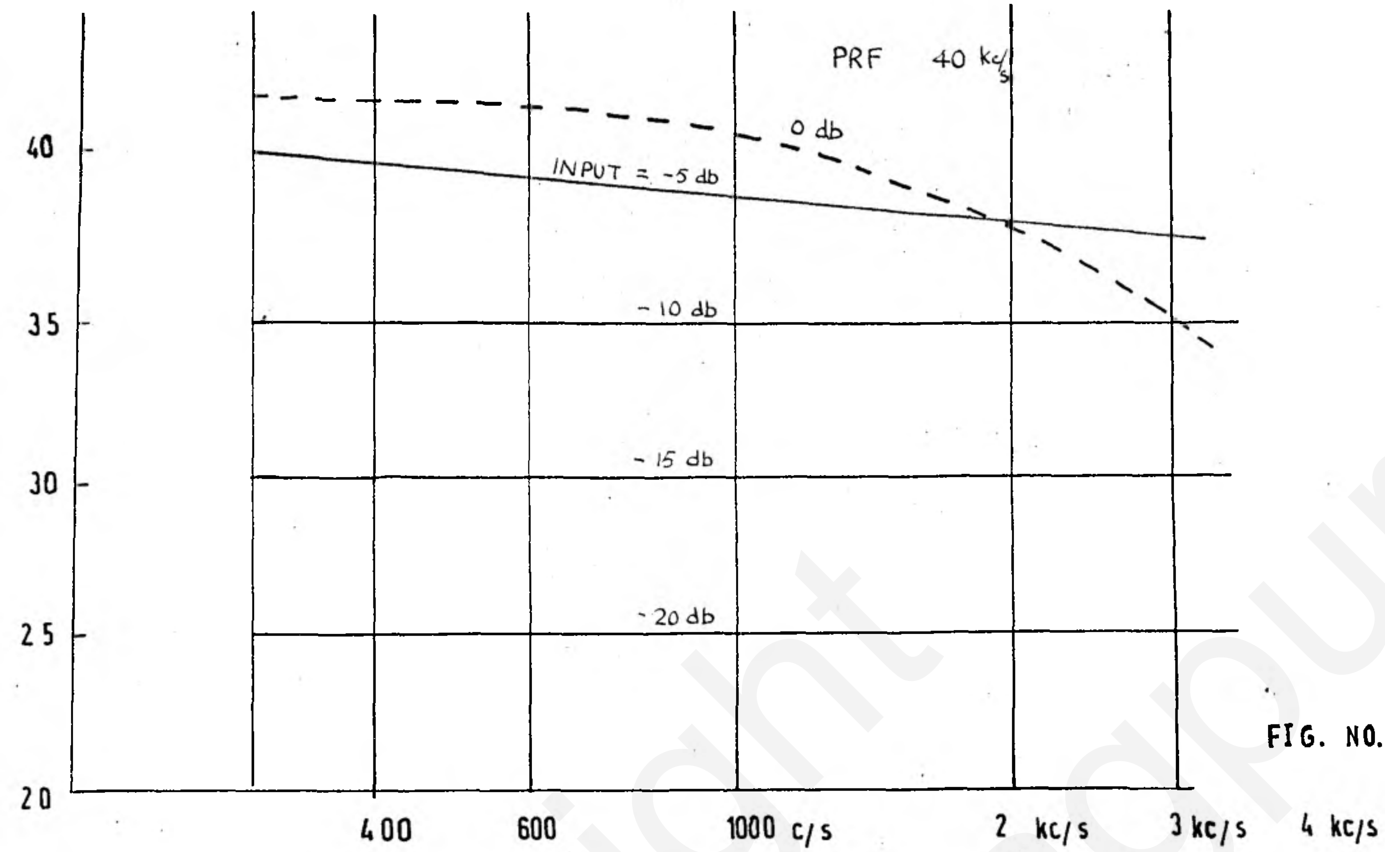
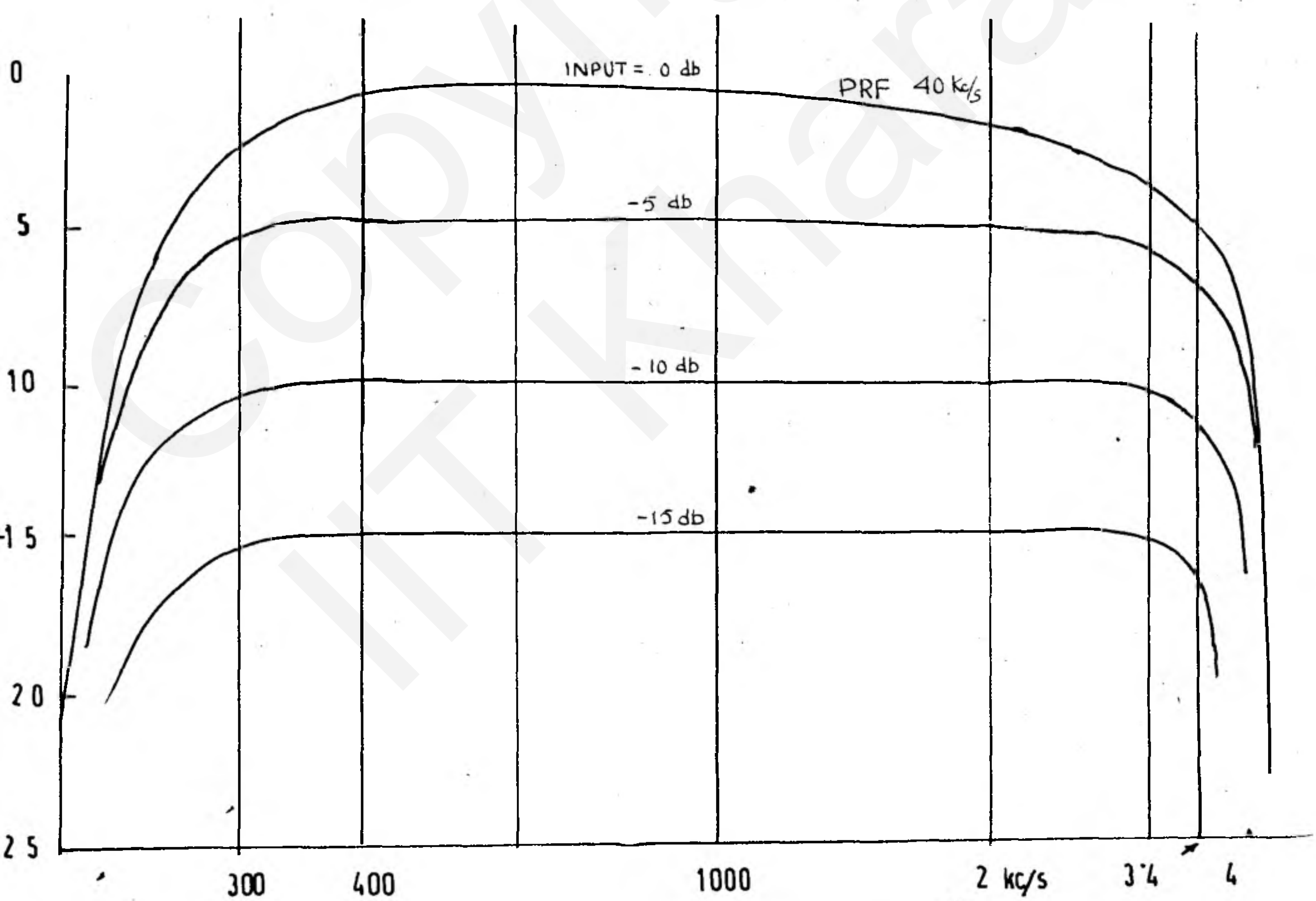


FIG. NO. 3·12

VARIATION OF SNR WITH f_m FOR DIFFERENT INPUT LEVELS



VARIATION OF THE OUT PUT WITH f_m FOR DIFFERENT INPUT LEVELS

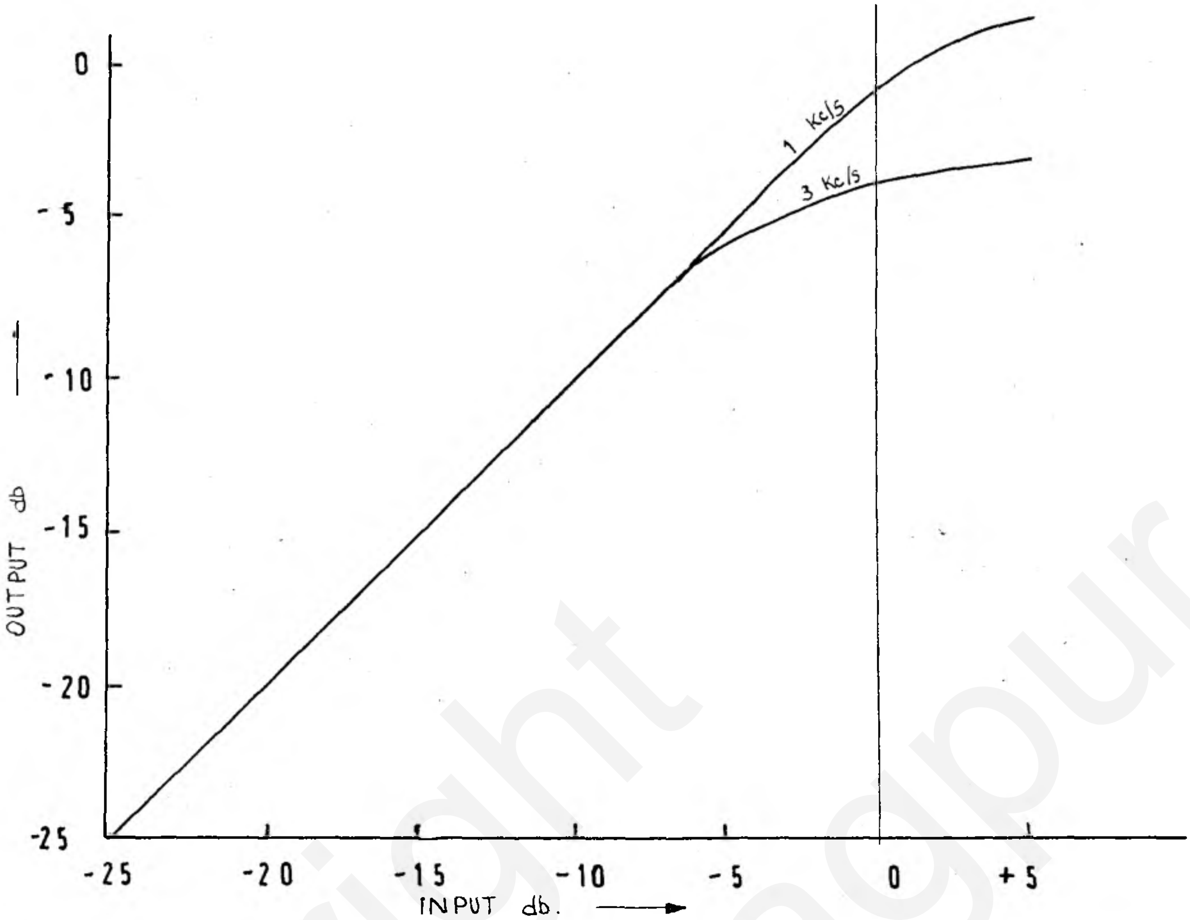
FIG. NO. 3·13

and at still lower inputs the output is constant with change in frequency. The input-output voltage relation is shown in 3.14 for 1 Kc/s and 3 Kc/s signals. All the figures (Figs.3.11, 3.12, 3.13, and 3.14), therefore, show the effect of the overloading in the form of slightly poorer results for the higher frequency signals at comparatively higher input levels.

Fig.3.15 shows the variation of the optimum SNR with PRF. Referring to the equation (2.30) for the SNR, it is seen that the SNR is dependent upon the PRF to the power 3/2, i.e., the SNR increases by 9 db per octave change of PRF. The slope of the experimental curve is about 8.0 db/octave and agrees fairly well with the theoretical equation.

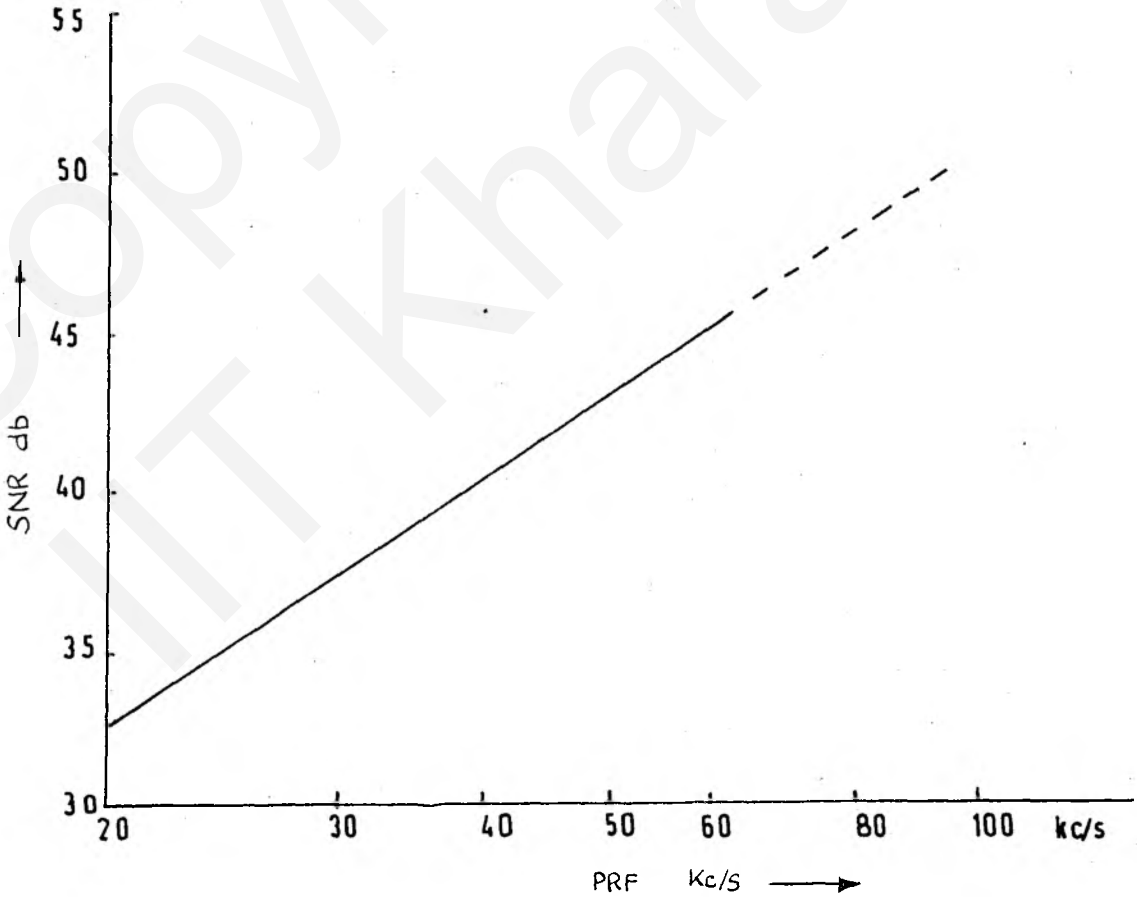
A photograph of the output pulses of the transmitter is shown in Fig.3.16(a) and the sinusoidal signal output at the receiver, after the bandpass filter, is shown in Fig.3.16(b). In Fig.3.16(c) the input and the output waveform of the system is shown for a triangular signal. The output waveform shown is before the filter and the PRF is 40 Kc/s. It can be seen that the waveform is reproduced without any visible distortion.

To improve the dynamic range of the input voltage, the compressor and the expandor are introduced and similar tests performed. If a matching expandor (double expansion) at the output is used, it will give rise to the linear compandor. The variation of SNR with input level at 60, 40, 30, 20 Kc/s PRF, for a signal frequency of 1 Kc/s, is shown in Fig.3.17. The dynamic range (range for which the SNR is above 25 db), with the compandor included, is



VARIATION OF OUTPUT VOLTAGE WITH INPUT VOLTAGE
fr = 40Kc/s

FIG. NO. 3·14



VARIATION OF OPTIMUM SNR WITH PRF

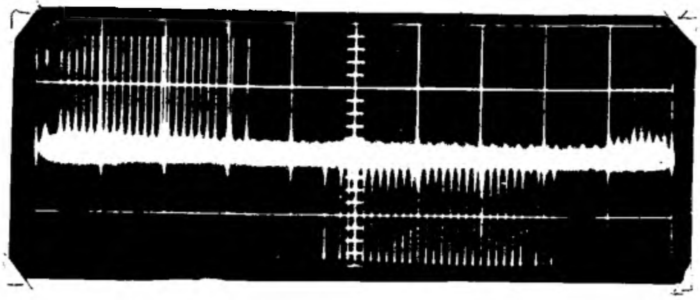
FIG. NO. 3·15

45 db and 40 db, at 60 Kc/s and 40 Kc/s PRF, respectively. Also the SNR is above 31 db and 27.5 db for a range of input of 41 db and 37 db, at PRF's of 60 and 40 Kc/s, respectively. The SNR at 30 Kc/s is above 24 db for an input range of 32 db, and at 20 Kc/s PRF, the SNR is above 20 db for an input range of about 27 db. The variation of SNR with input level for the signal frequency of 3 Kc/s is also shown in Fig.3.17 (dotted curves). The SNR at 3 Kc/s signal frequency is about 3 db below the SNR for 1 Kc/s for almost the entire dynamic range. The SNR variation with the input for the signal frequencies below 1 Kc/s is almost identical with the 1 Kc/s curve.

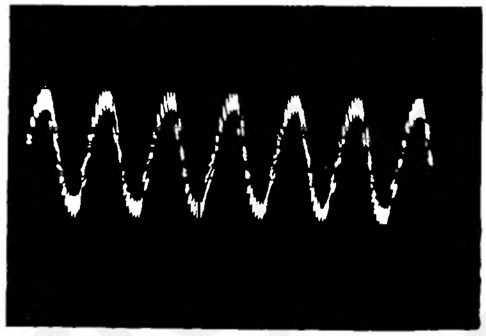
If, just one expandor is used at the output without disturbing the compressor, the output remains partially compressed at the higher input levels. This has been called the Partial compandor and the SNR variation with input level has been plotted in Fig.3.18 for the PRF's of 60, 40, 30 and 20 Kc/s, at a signal frequency of 1 Kc/s. It is seen that the dynamic range reduces a little bit and for a SNR of 25 db and above, the new dynamic range is 42 db, 40 db, 34 db and 26 db respectively, for the 4 PRF's. The advantage of the partial compandor is that the SNR within the dynamic range is better than those in the linear compandor. The optimum SNR in the dynamic range is 34, 29 and 27 db, for the 60, 40, 30 and 20 Kc/s PRF respectively. The variation of SNR with input level for different signal frequencies is shown in Fig.3.19, where it is seen that the SNR at higher frequency falls by about 2 db, compared to 1 Kc/s signal. The SNR at frequencies lower than 1 Kc/s is similar to it and has not been drawn. Fig.3.20

FIG 3.16

(a)

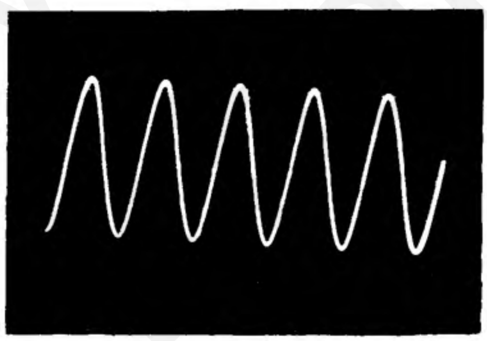


TRANSMITTER PULSES OF TERNARY SQ-PCM.



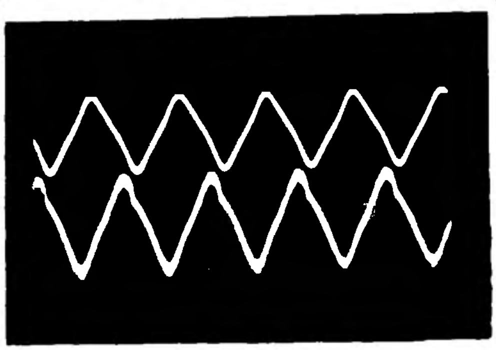
RECONSTRUCTED SIGNAL AT THE RECEIVER

(b)

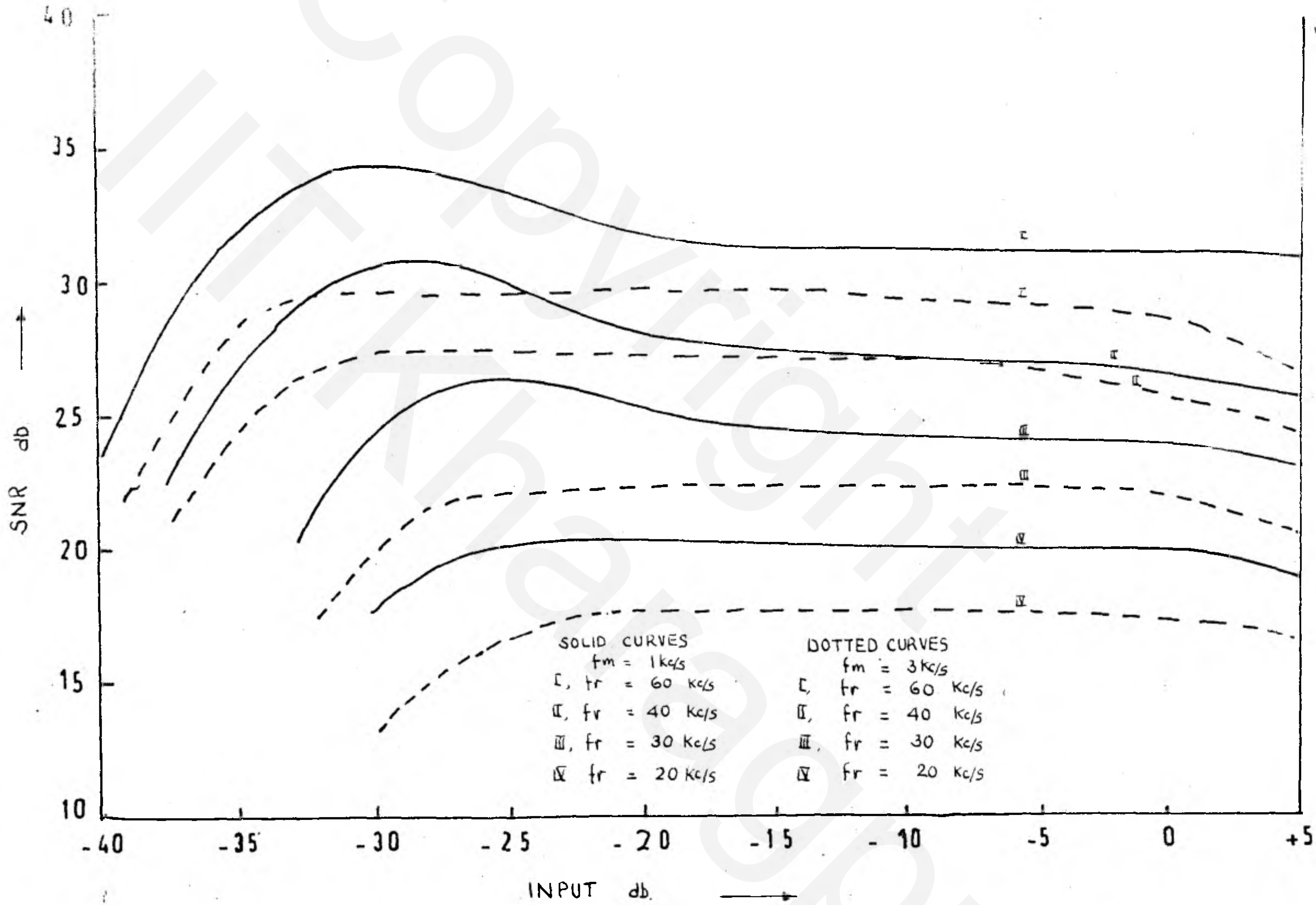


FILTERED SIGNAL AT THE RECEIVER OUTPUT

(c)

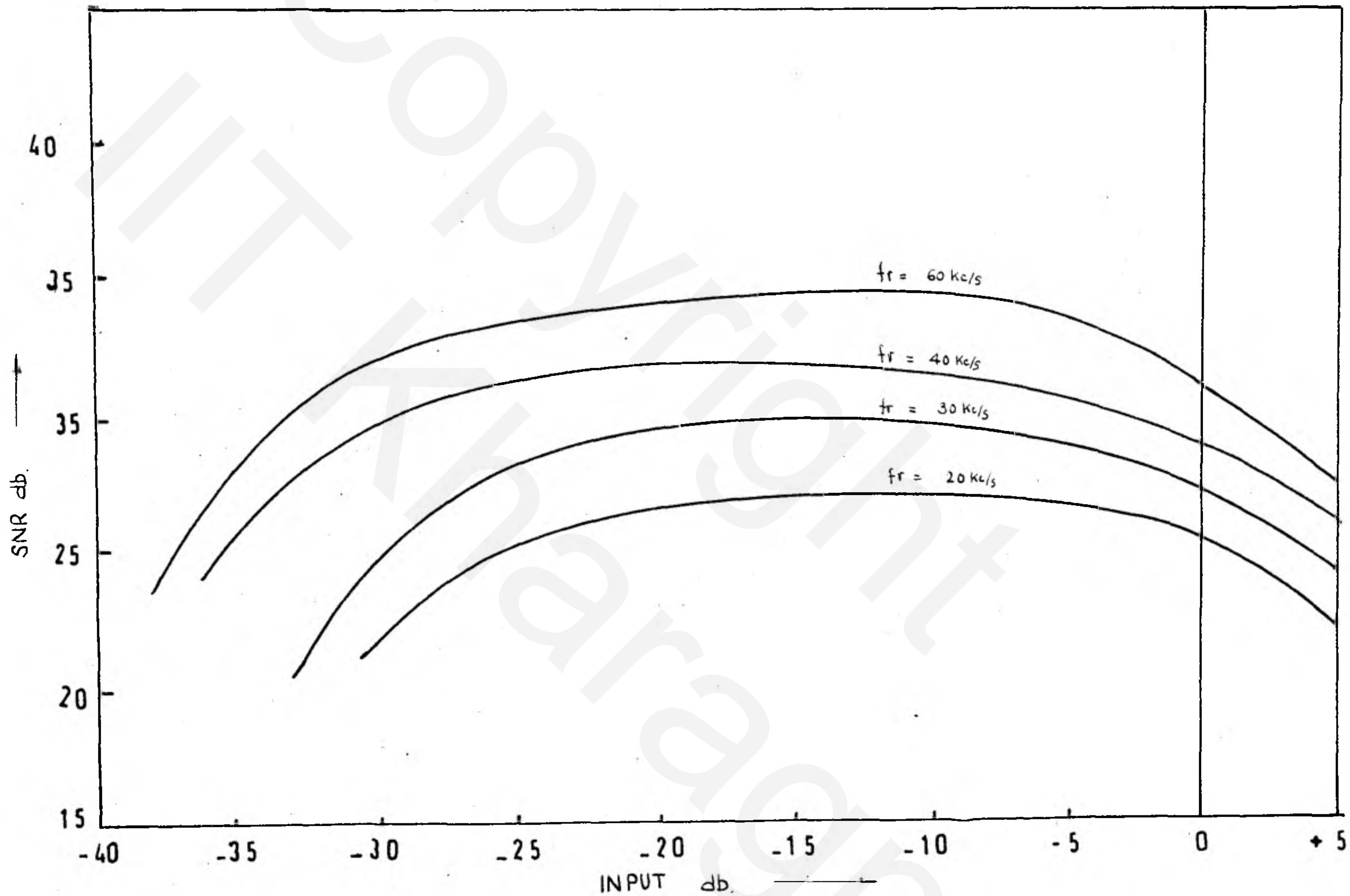


TRIANGULAR INPUT (TOP) AND REPRODUCED SIGNAL (BOTTOM)

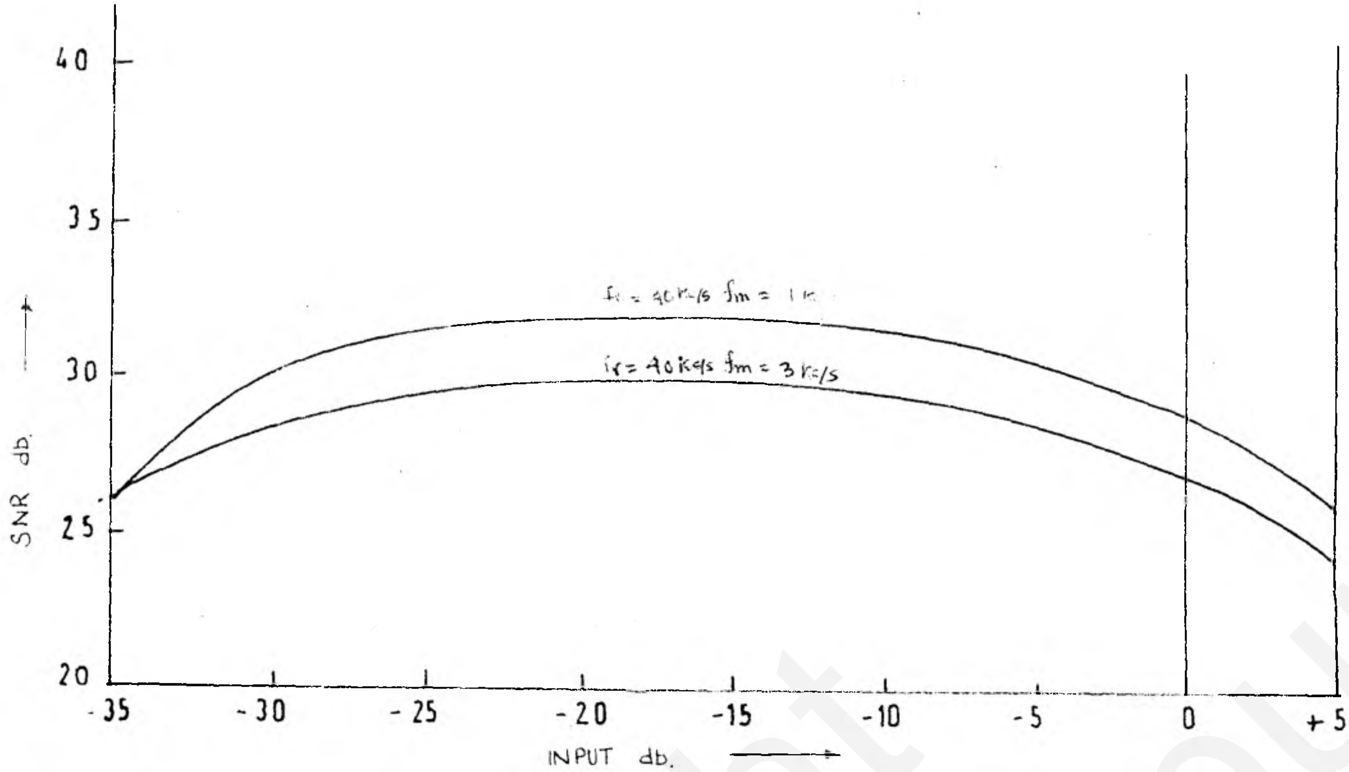


VARIATION OF SNR WITH INPUT LEVEL FOR THE LINEAR COMPANDOR

FIG. NO. 3.17



VARIATION OF SNR WITH INPUT LEVEL FOR THE PARTIAL COMPANDOR; FIG. NO. 3'18
f_m = 1 Kc/s



VARIATION OF SNR WITH INPUT LEVEL FOR THE PARTIAL COMPANDOR, FOR DIFFERENT f_m

FIG. NO. 3·19

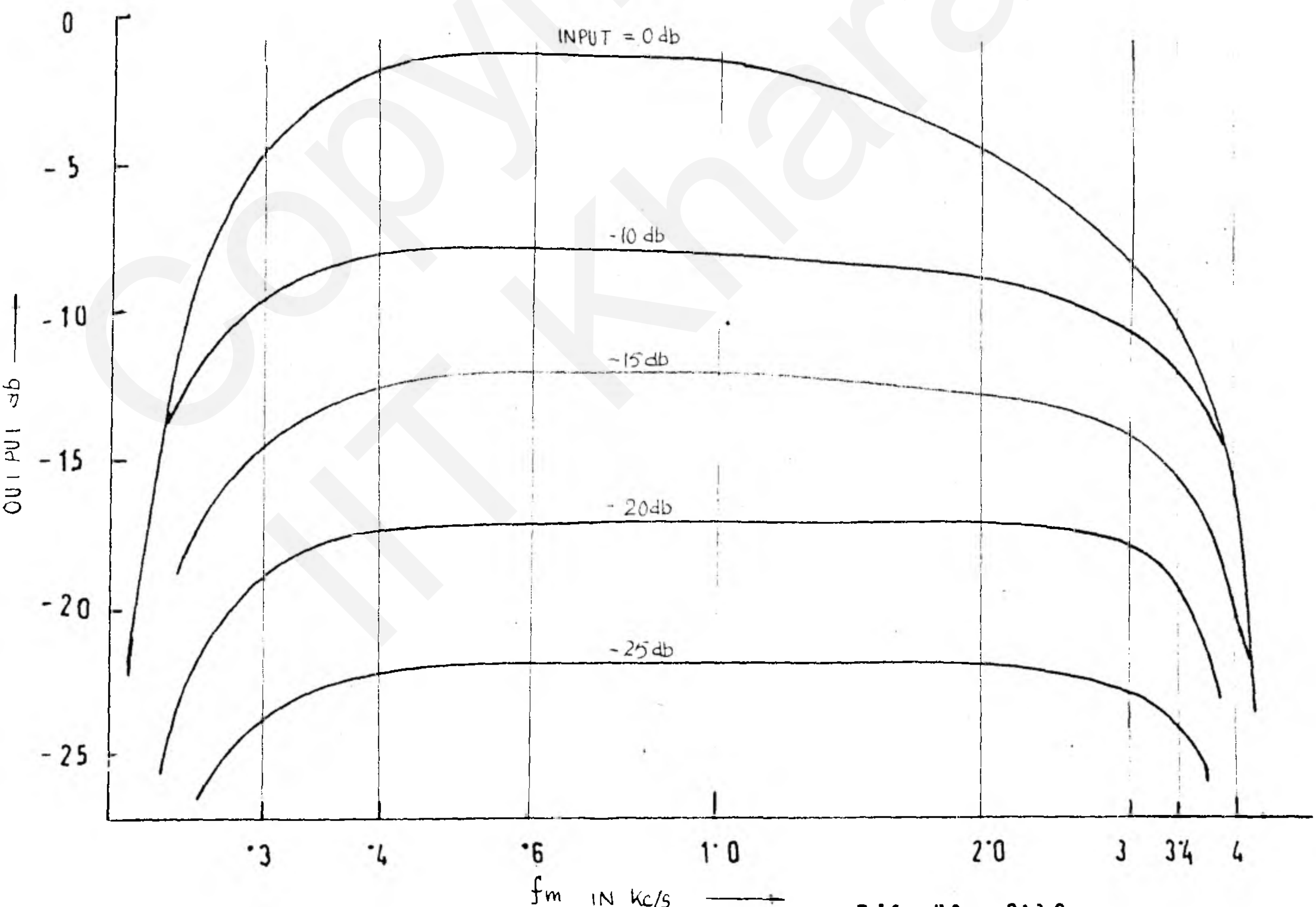


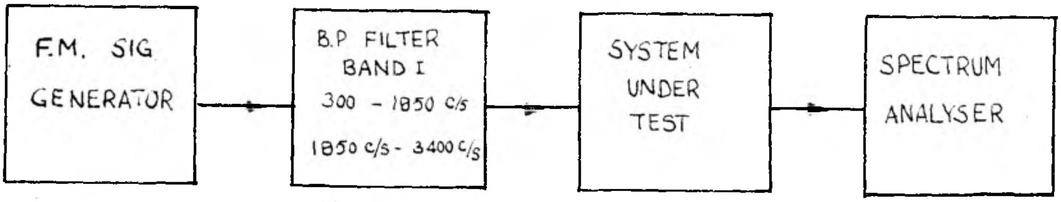
FIG. NO. 3·20

VARIATION OF OUTPUT WITH f_m ; f_r = 40 kc/s shows the output variation with frequency, for different signal input levels. The output at higher frequencies falls because of the same saturation effect, discussed earlier. Since the range of 0 - 20 db is compressed to 0 - 5, which is also the overload range (as seen in Fig.3.13), the overload effect spreads upto -20 db input level. The practical results obtained for the linear and the partial compandor agree fairly well with the theoretically calculated SNR and the dynamic range given in Fig.2.23.

(b) FM Signal Test.

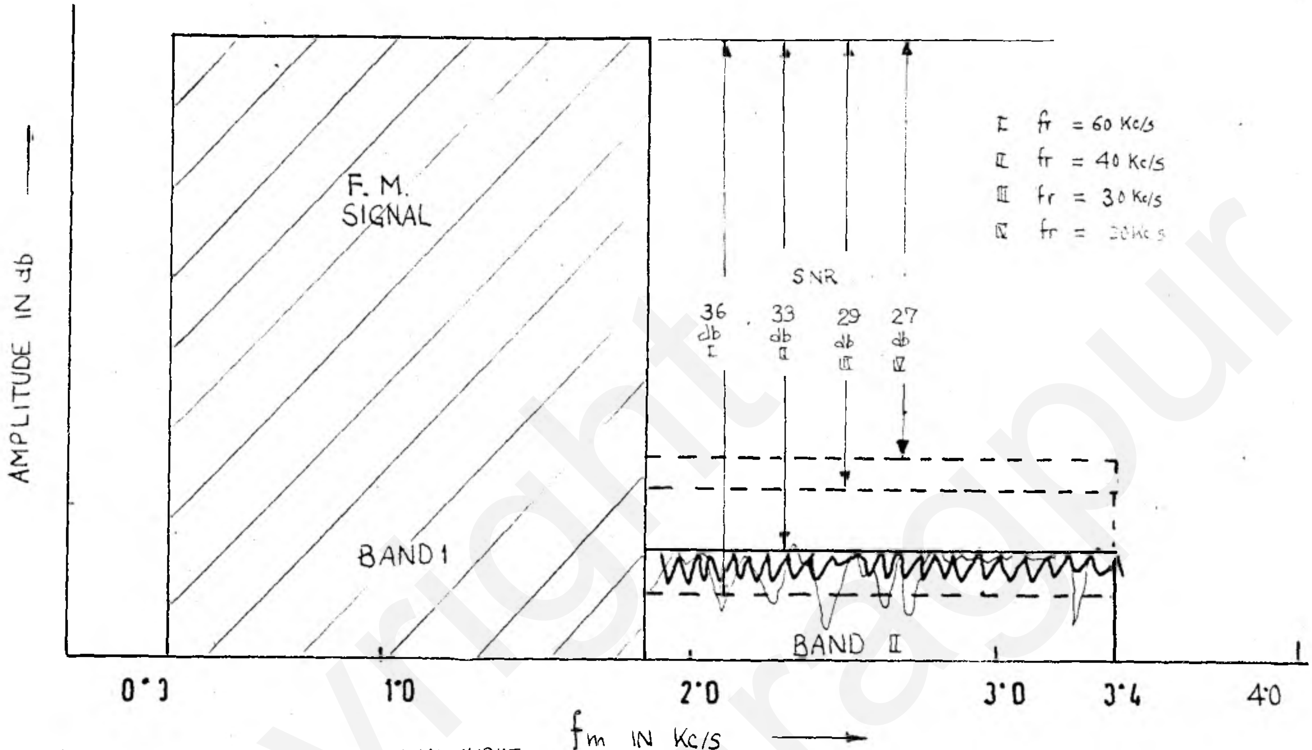
For the processing of FM signals, the input-output linearity is not of any importance, as there is no amplitude variation. The performance of the ternary SQ-PCM system (with feedback) has been tested with FM signals, and the measurement technique is indicated in Fig.3.21. The output of the FM generator is passed through a bandpass filter, whose passband can be made either from 300 c/s to 1850 c/s, called Band I, or from 1850 c/s to 3400 c/s, called Band II. The signal is processed by the system and the output of the receiver is monitored with the Spectrum Analyser (mentioned earlier). If the signal is in Band I, the noise is observed in the Band II, or vice versa.

The SNR for the FM signal input to the system is shown in Fig.3.22, where the signal has been shown to be in Band I and the noise observed in Band II. The average readings of the SNR, for the signal in Band I and in Band II, at PRF's of 60, 40, 30 and 20 Kc/s, are 36, 33, 29 and 26 db, respectively. A narrow band



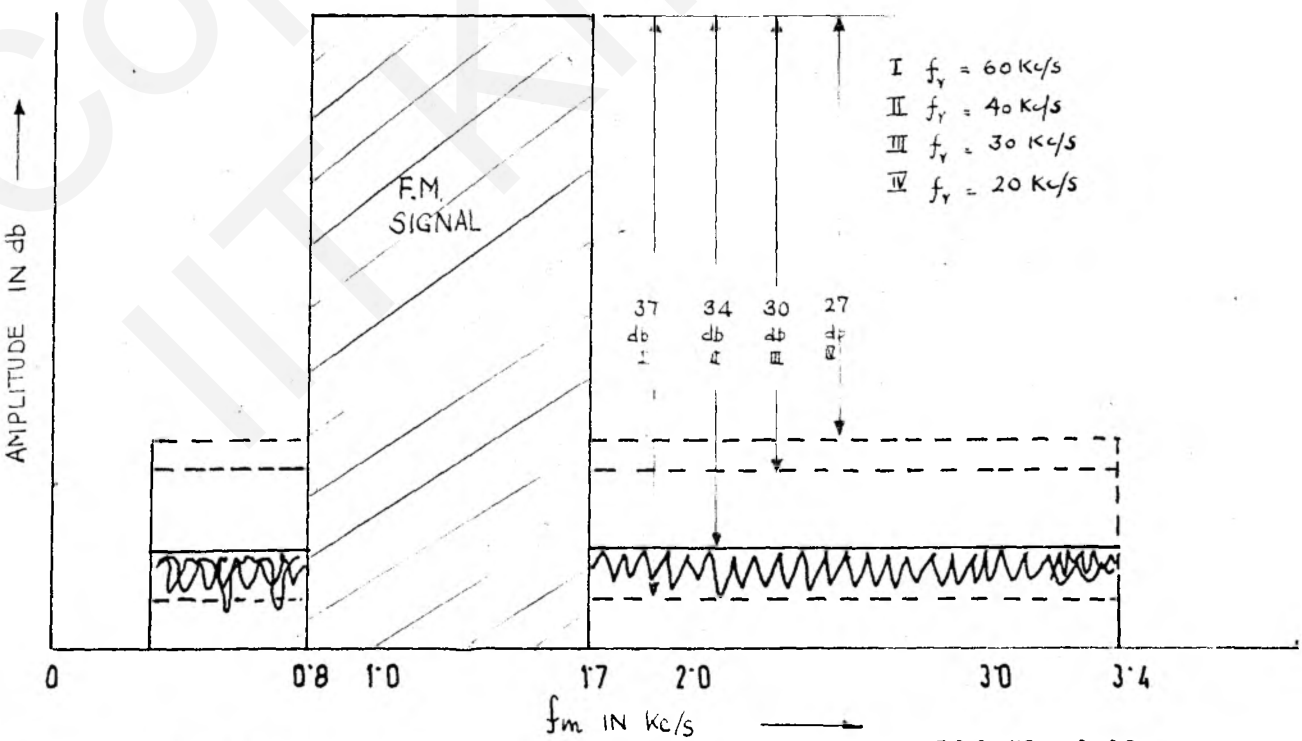
AN EXPERIMENTAL SET-UP FOR MEASUREMENT WITH FM SIGNALS

FIG. NO. 3·21



SNR FOR 1/2 BAND FM SIGNAL INPUT

FIG. NO. 3·22



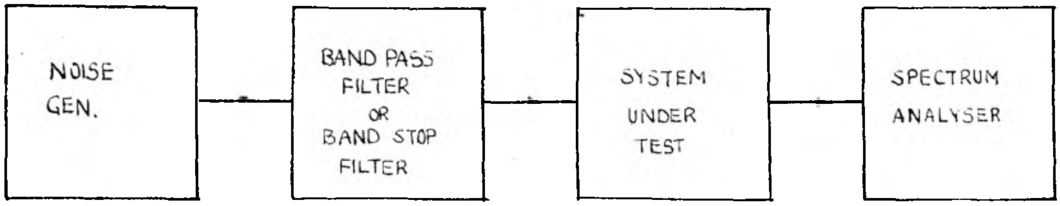
SNR FOR NARROW BAND FM SIGNAL INPUT

FIG. NO. 3·23

FM has also been used as a test signal. The signal occupies a band of 800 - 1700 c/s and the SNR at the different PRF's of 60, 40, 30 and 20 Kc/s is 37, 34, 30 and 27 db, respectively, as shown in Fig.3.23. A photograph of the input and the reproduced signal, when the input is an FM signal (at a PRF of 40 Kc/s), is given in Fig.3.24, and it is seen that the signal has been reproduced fairly accurately.

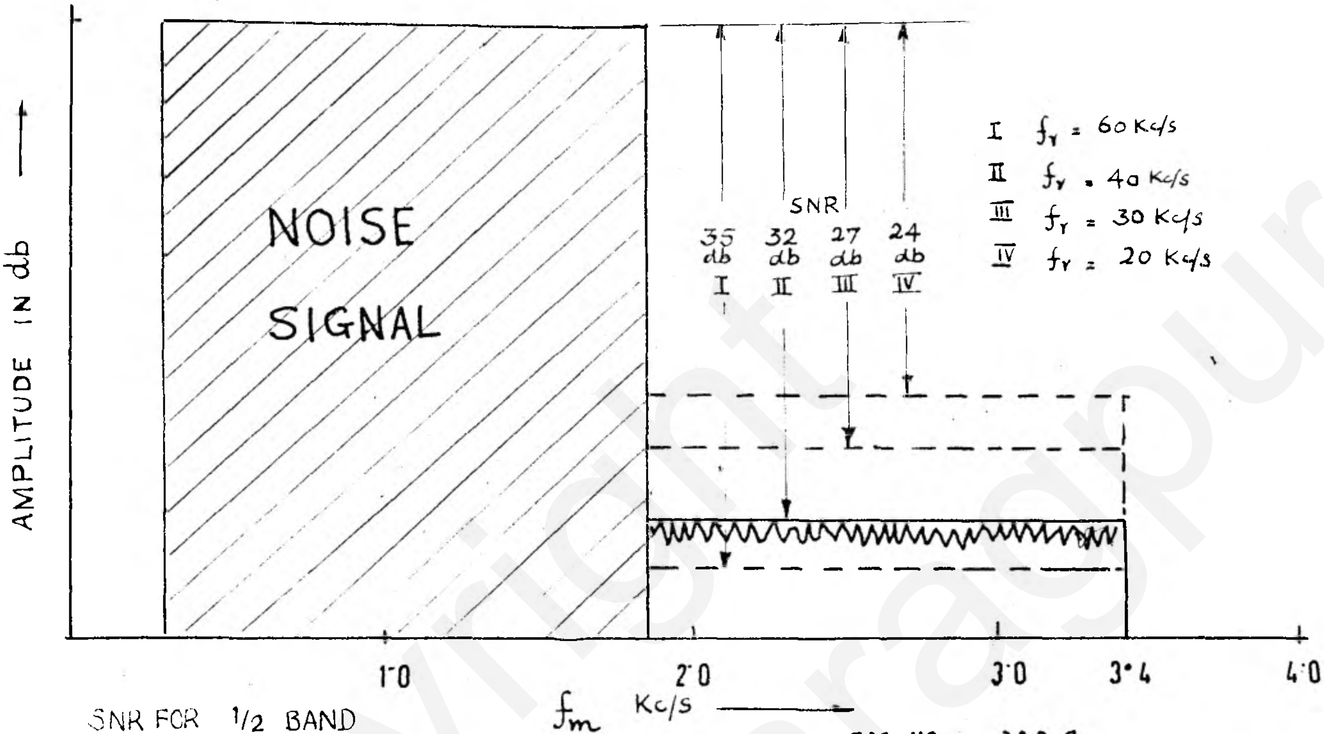
(c) Noise Signal Test.

White noise is the most general signal among the complex signals which can be used for testing the performance of any system. The white noise has a continuous distribution of frequency components, and is, theoretically, capable of assuming infinitely great values of instantaneous voltage at infrequent instants of time. Bennett[17] has remarked that it is an experimentally observed fact that, white noise has never been observed to exceed appreciably a voltage four times its rms value. Hence the rms value of the input noise signal is taken to be one fourth or 12 db below the overload voltage for the sinusoidal signal. The technique of measurement is similar that given for the FM signals, and is shown in Fig.3.25. Two kinds of measurements have been done. In one method, a bandpass filter is used so that the total audio band is divided into two, Bands I and II, as before. The quantizing noise has been assumed to be of the white noise type (refer Section 2.4), having a uniform power spectral density, and as the Bands I and II are equal, the total quantizing noise is double of the noise observed in Band I or II.



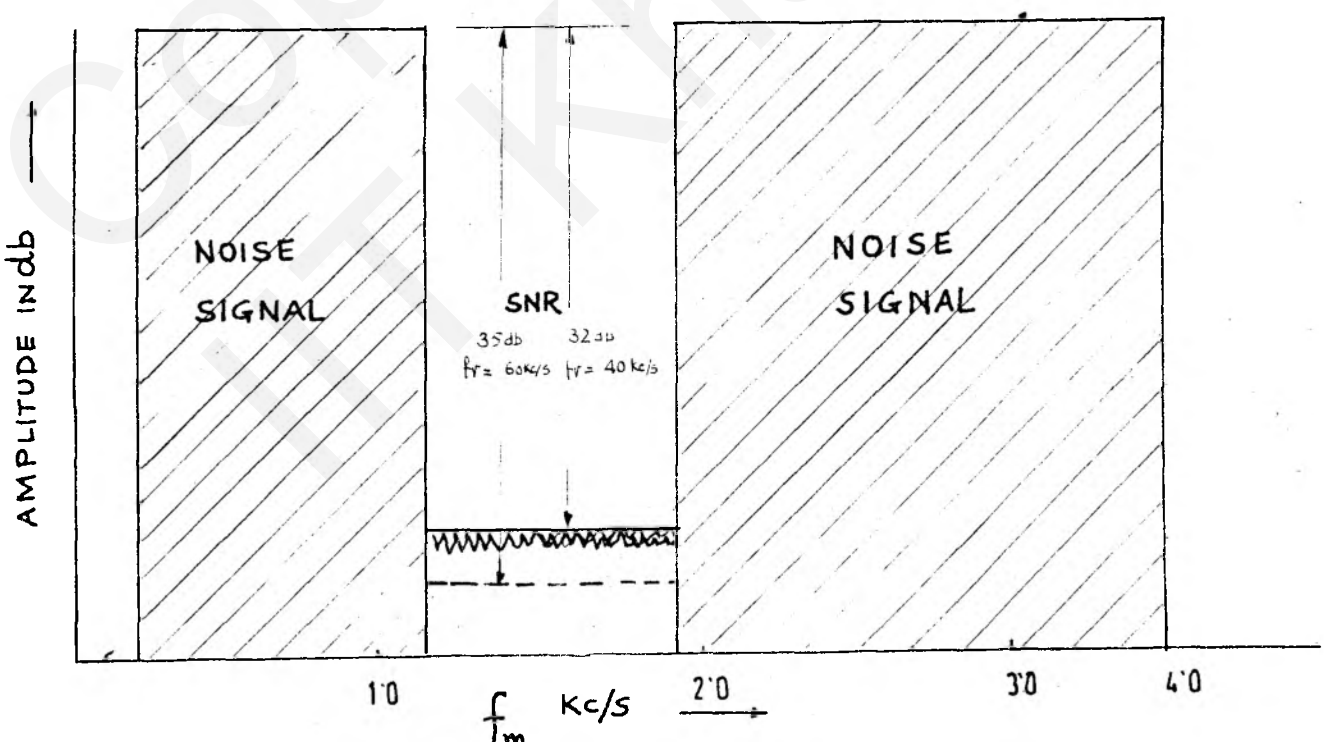
AN EXPERIMENTAL SET-UP FOR MEASUREMENTS WITH NOISE SIGNALS INPUT

FIG. NO. 3.25



SNR FOR 1/2 BAND NOISE SIGNALS INPUT

FIG. NO. 3.26



EQUALISED SNR FOR NOISE SIGNALS INPUT USING A BAND-STOP FILTER

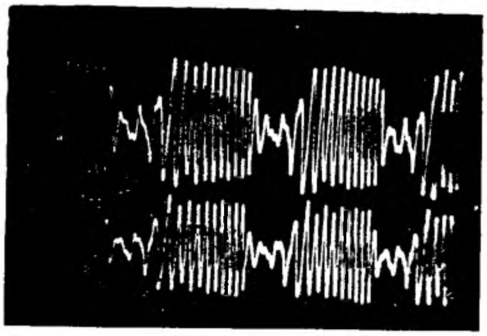
FIG. NO. 3.27

The input signal is also a noise signal, and therefore, the noise signal power in any one of the Bands is half of the total signal power in the audio band. The signal to noise ratio can therefore be directly read out from the Spectrum Analyser by observing the noise signal level in one of the bands and the quantizing noise level in the other band. A result of such a measurement is shown in Fig.3.26, where it is seen that the SNR is 35 db, 32 db, 27 db and 24 db for the PRF's of 60, 40, 30 and 20 Kc/s, respectively.

Another method of measurement consists in using a band stop filter after the noise generator such that, there is no noise signal input in the band of, say, 1100 - 1900 c/s. The output of the system, with this type of signal input, will have some quantizing noise in the stop band also. The readings are equalised for the bandwidths, and the results of SNR are given in Fig.3.27 for the PRF's of 60 and 40 Kc/s, and are 35 db and 32 db, respectively. The use of the compandor for the noise signals input shows that a deterioration in SNR of about 2 db occurs as compared to the SNR in the system without the compandor. This deterioration is explained as the direct result of the extra overloading (at high frequencies) trouble encountered when the compandor has been used.

Fig.3.28(a) shows a photograph of the noise input signal of half bandwidth, Band I, and Fig. 3.28(b) shows a photograph of the output of the system, where it is seen that there is some quantizing noise in Band II. SNR from the photograph is of the order of 32 db for a PRF of 40 Kc/s.

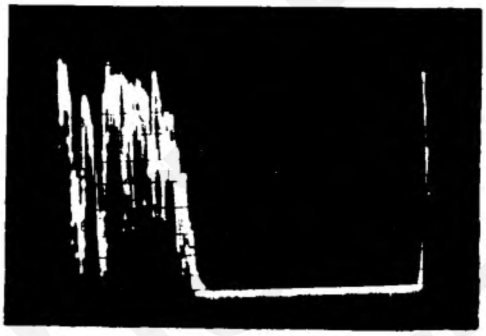
FIG 3.24



FM TEST SIGNAL (TOP) AND REPRODUCED SIGNAL (BOTTOM)

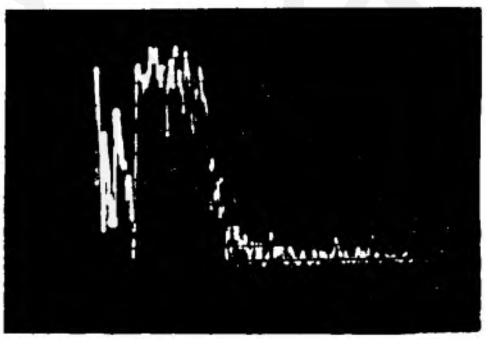
FIG 3.28

(a)

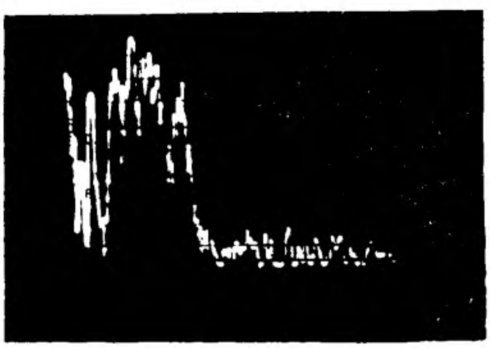


HALF BAND NOISE TEST SIGLAL

(b)



RECEIVER OUTPUT SHOWING QUANTIZING NOISE AT PRF 40 Kc/s



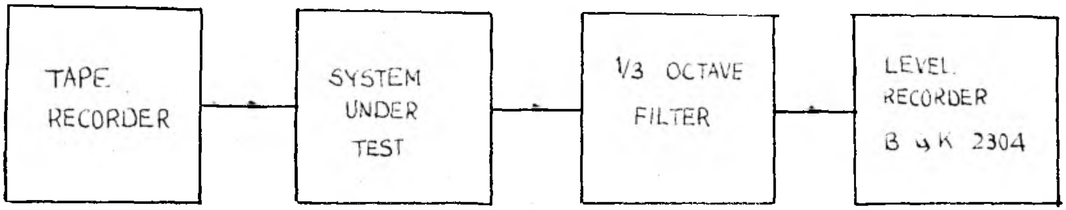
SAME AS (b) AT PRF 30 Kc/s.

(d) Speech Test.

Davenport[26] and others have shown that the probability distribution of the amplitudes of a speech signal is almost Gaussian with a crest factor of about 3.5, a value very close to that of white noise. The noise test in the previous section, therefore, gives a fairly good idea of the performance of the system for speech signals. A point of essential difference between the noise signal and the speech signal is that the former has a flat frequency spectrum, whereas the frequency spectrum of the latter is far from flat. Also, for the convenience of communication, the total range of level variation of speech signals can be substantially reduced without any noticeable deterioration in intelligibility and quality, and this artifice is good enough for commercial purposes.

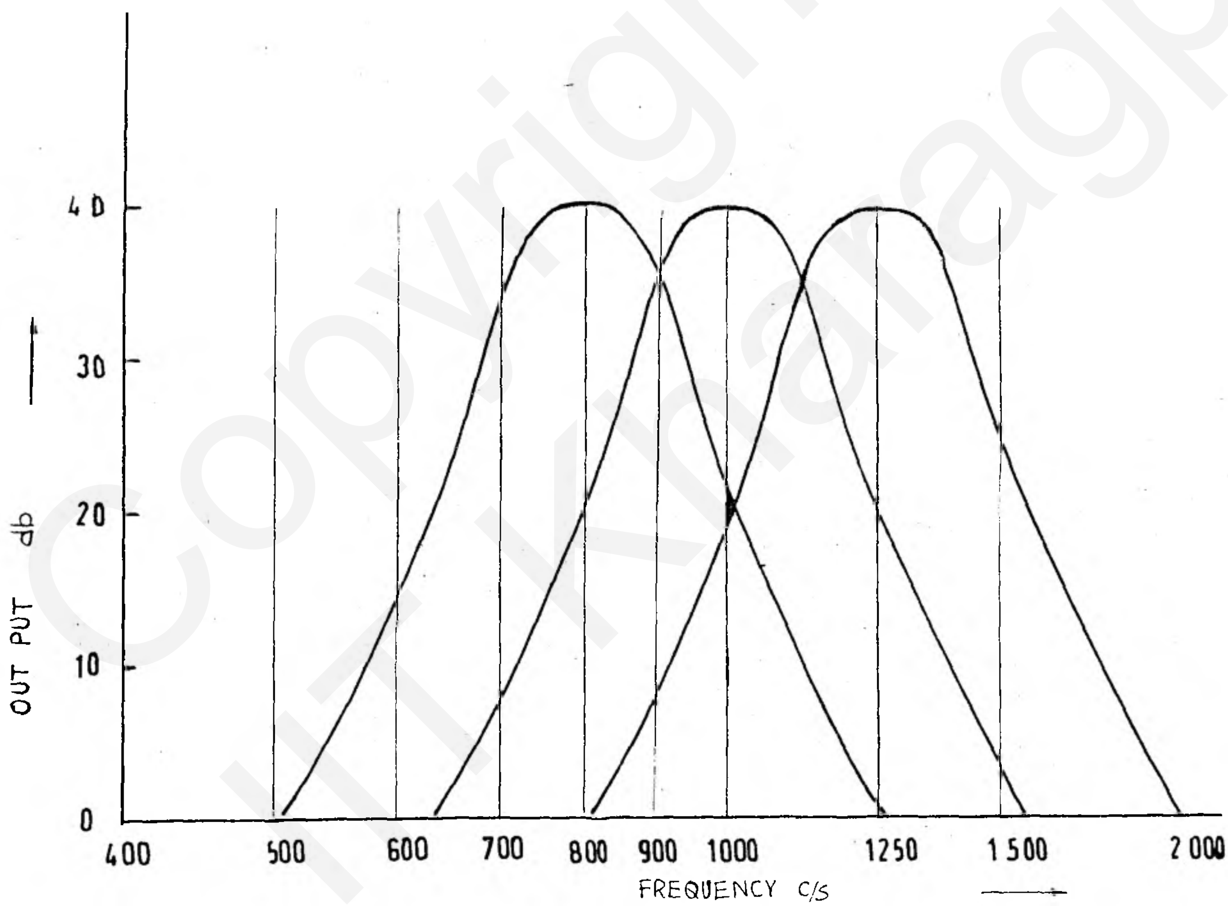
Listening tests on the reproduced speech signals for continuous speech passages gave excellent results at 40 and 60 Kc/s PRF. It is found that for a PRF of 40 Kc/s and above, there is no appreciable difference between the original and the reproduced signal. At 20 Kc/s PRF, however, some background crackling noise appears which is tolerable, and the intelligibility of speech is quite good. However, quantitative analysis of processed speech is generally done by speech spectrographs. In the absence of a speech spectrograph, a amplitude-time-frequency characteristics of the processed speech has been obtained and compared with the original in the following way.

The original speech, and the reproduced speech, are recorded on a Level Recorder by using a technique shown in Fig.3.29. A sen-



AN EXPERIMENTAL SET-UP FOR MEASUREMENTS WITH SPEECH SIGNALS

FIG. NO. 3.29



FREQUENCY Vs OUTPUT CHARACTERSTICS OF 1/3 OCTAVE FILTER

FIG. NO. 3.30

tence is recorded on the Tape Recorder and then played back through a narrow band 1/3 octave filter to be recorded on the Level Recorder. The centre frequency of this 1/3 octave filter is variable over the audio band in discrete steps, and can be set at 315 c/s, 400 c/s, 500 c/s, 630 c/s, 80 c/s ...... 3150 c/s, 4000 c/s etc. The frequency vs. output characteristics of this 1/3 octave filter is shown in Fig.3.30, for three consecutive centre frequencies of 800 c/s, 1000 c/s and 1250 c/s. The speech energy passing through the passband of this filter set at any centre frequency, therefore, is recorded on the Level Recorder. The Level Recorder used for the experimental purpose here is Bruel and Kjaer, Type 2304. Many recordings have been taken, but the records of the centre frequencies of 500 c/s, 1000 c/s and 3150 c/s only have been given below, as they are the representative recordings of the lower, centre and higher ends of the audio band.

The original recording of the chosen sentence, "Pandit Jawaharlal Nehru died on 27th May, 1964, Bharat Ki Jai", is therefore rerecorded on the Level Recorder with the 1/3 octave filter having the three centre frequencies of 500 c/s, 1000 c/s and 3150 c/s. The same taped sentence is now processed by the system at PRF of 40 and 20 Kc/s, and the output is again recorded for the same three centre frequencies. Next the processed signal, with the compandor in the system, is similarly recorded.

Fig.3.31 shows, for the centre frequency of 500 c/s, the four recordings (i) original, (ii) reproduced signal without compandor at 40 Kc/s PRF, (iii) reproduced signal without compandor at 20 Kc/s PRF and (iv) reproduced signal with compandor

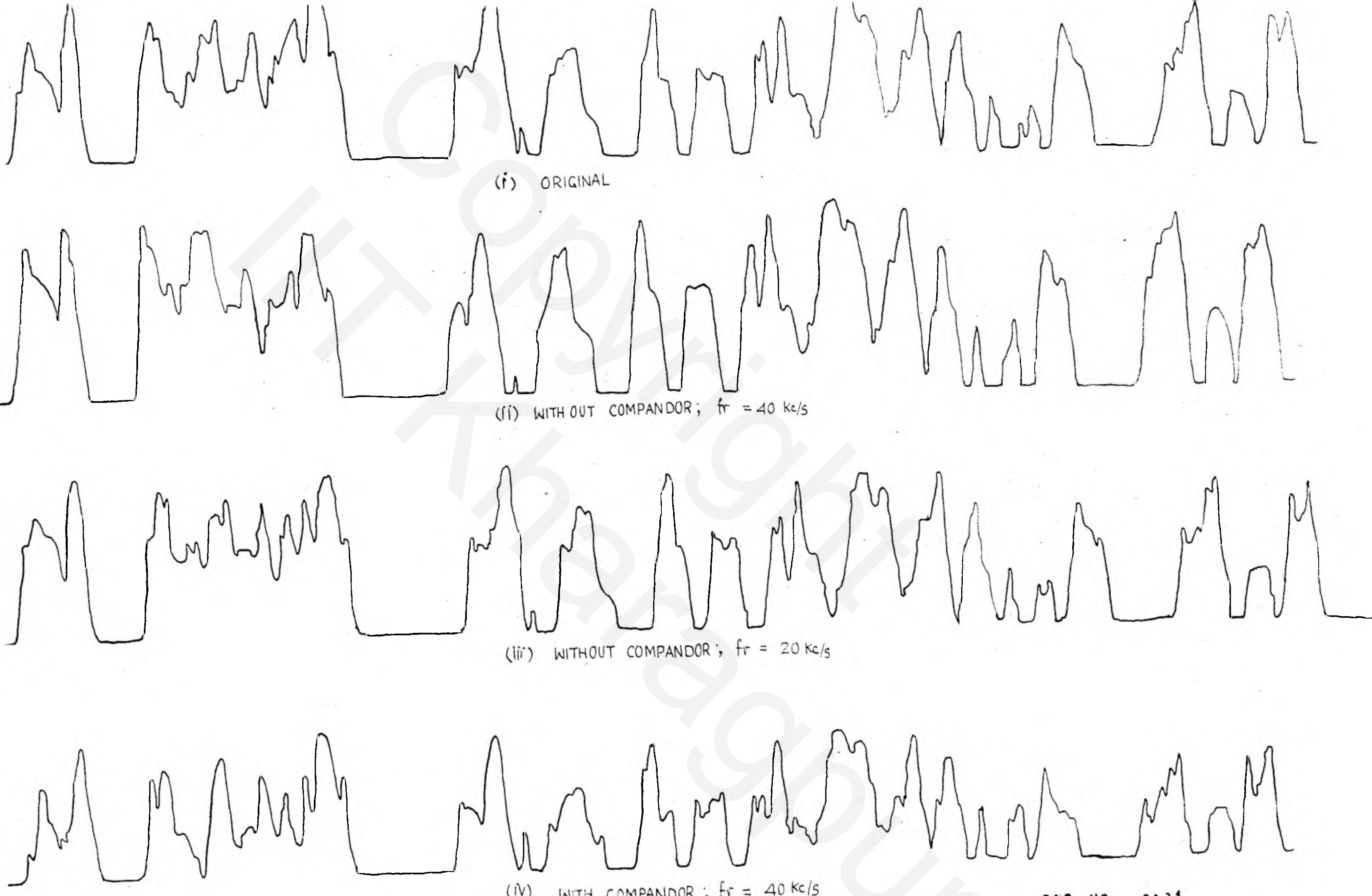


FIG. NO. 3·31

AMPLITUDE - TIME - FREQUENCY RECORDINGS OF THE 'SENTENCE' ON THE LEVEL RECORDER; CENTRE FREQUENCY 500 c/s

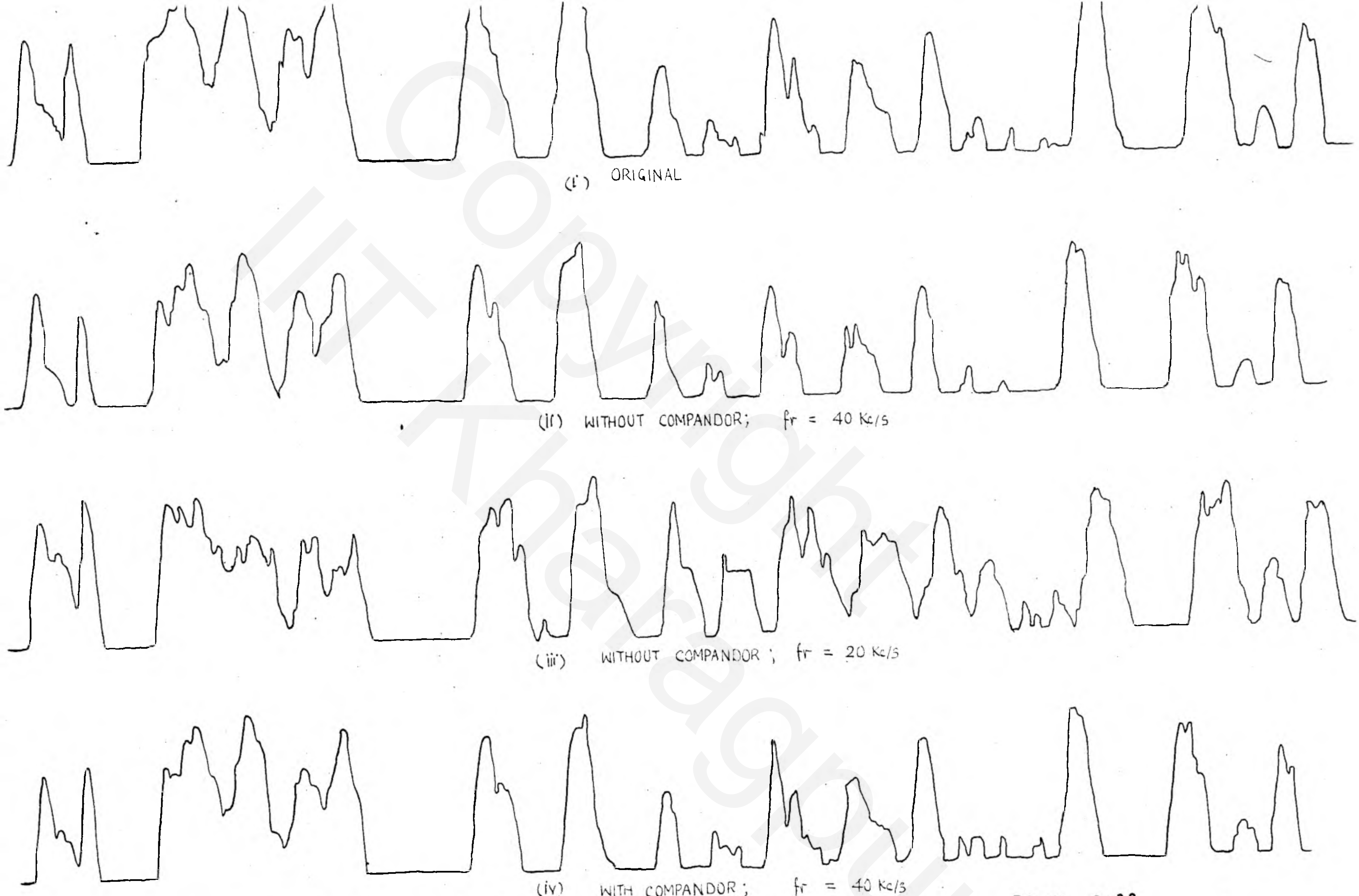


FIG. NO. 3-32

AMPLITUDE -TIME-FREQUENCY RECORDINGS OF THE 'SENTANCE' ; CENTRE FREQUENCY 1000 c/s

AMPLITUDE-TIME-FREQUENCY RECORDING OF THE 'SENTENCE', CENTRE FREQUENCY 3150 c/s

FIG. NO. 3·33

included (at 40 Kc/s PRF), for a comparison. Fig.3.32 shows the four recording for a centre frequency of 1000 c/s and Fig.3.33 shows the same four recordings for a centre frequency of 3150 c/s. It is seen that at 20 Kc/s PRF, the 3150 c/s band of the signal frequency is not well reproduced, but the reproduction at 1000 c/s band is tolerable. The processing through the compandor is seen to be quite satisfactory, and the quality of the speech, on listening, is good.

3.1.2. STABILITY.

The stability of the Ternary SQ-PCM system with feedback will depend upon the proper operation of the constituent circuits. Disregarding any major breakdown when the system does not operate at all, the deterioration of the system performance with drift, either in the power supply, or in the reference voltage, is of particular interest.

The three important quantities, a variation of which is normally expected, are the reference bias voltage of the Schmitt trigger, the bias voltage of the bisymmetrical sampling circuit, and the ratio of the amplitudes of the positive and negative pulses at the transmitter output. As the Schmitt trigger is the quantizer, and the selection of the triggering level, which decides the error pulses to be ignored, depends on the bias voltages, a stability measurement will require the determination of the deterioration in performance with the variation of this bias. As the bias voltage is changed in the positive direction, the triggering level of the Schmitt trigger becomes higher and higher, so that fewer error pulses are selected, and ultimately, a condition is reached when that particular trigger does not produce any pulses. The circuit then degenerates into a Unidirectional Binary circuit and is similar to the SQ-PCM system discussed in section 2.3. The SNR at the output of the system, for a variation of the bias of the positive Schmitt trigger, is shown in Fig.3.34. It is seen that the initial change of bias from the optimum brings down the SNR from 40 db to 38 db, but it remains above 37 db for a change of about 16 volts in the bias. A further increase of the bias voltage, however, results in fewer triggers and the SNR deteriorates to about 32 db, till, at about 120 V bias, the circuit becomes a Binary SQ-PCM system. If the bias is kept at 120 V, and the input level is varied, the SNR with input level variation is as shown in Fig.3.35. At 0 db input, the feedback circuit is not functioning properly. With a further lowering of the input, the feedback voltage is enough to match the input and a SNR curve exactly similar to the Binary SQ-PCM system is obtained.

If the bias voltage, instead of increasing, is decreased below the optimum, the Schmitt trigger first of all triggers on very small noise amplitudes, and a further decrease of the voltage makes the triggers wide and continuous, and thus, the Schmitt trigger loses its properties. A decrease of bias of about 2-3 volts is tolerable and the circuit operates satisfactorily. Thus it is seen that the system with feedback is specially very stable

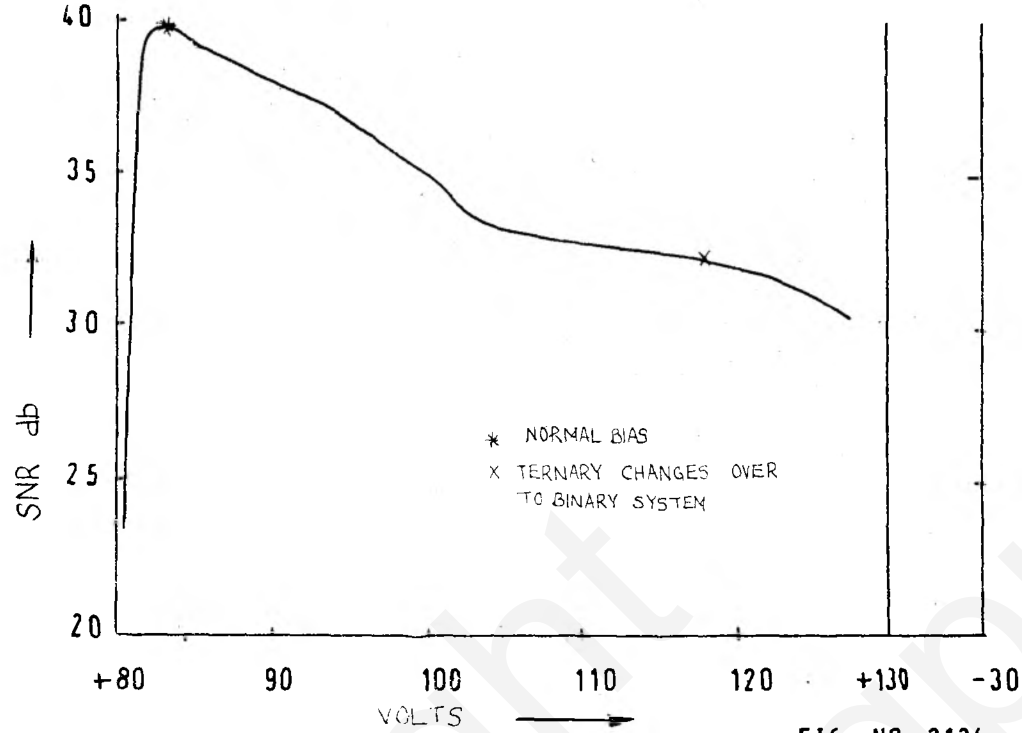
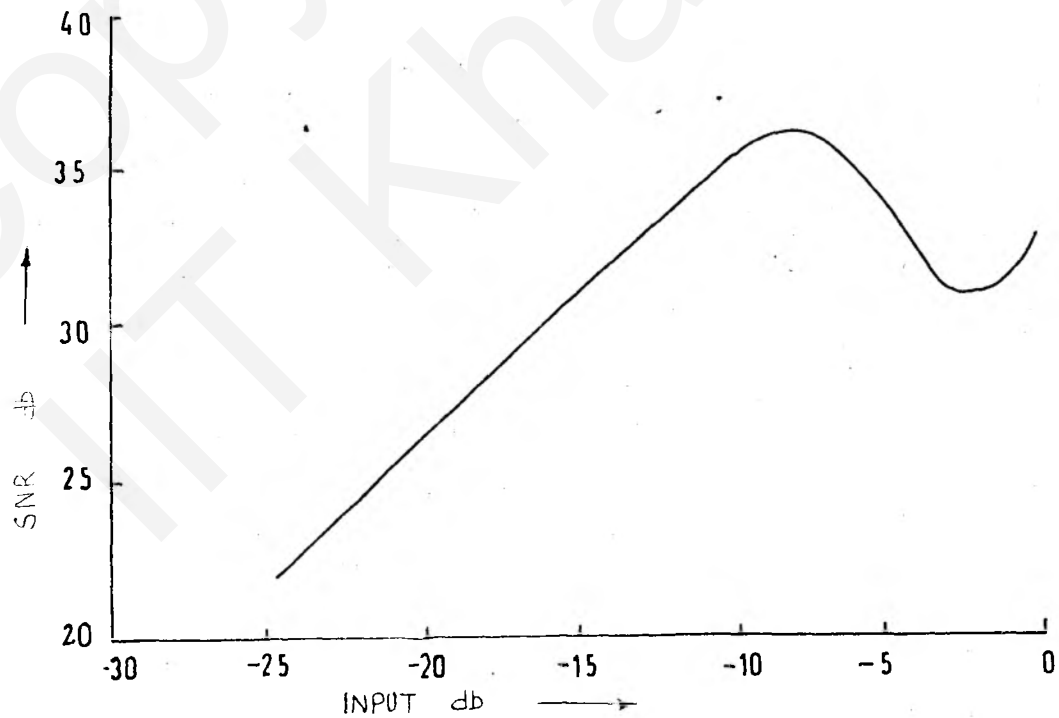


FIG. NO. 3·34

VARIATION OF SNR WITH THE BIAS OF THE SCHMITT TRIGGER



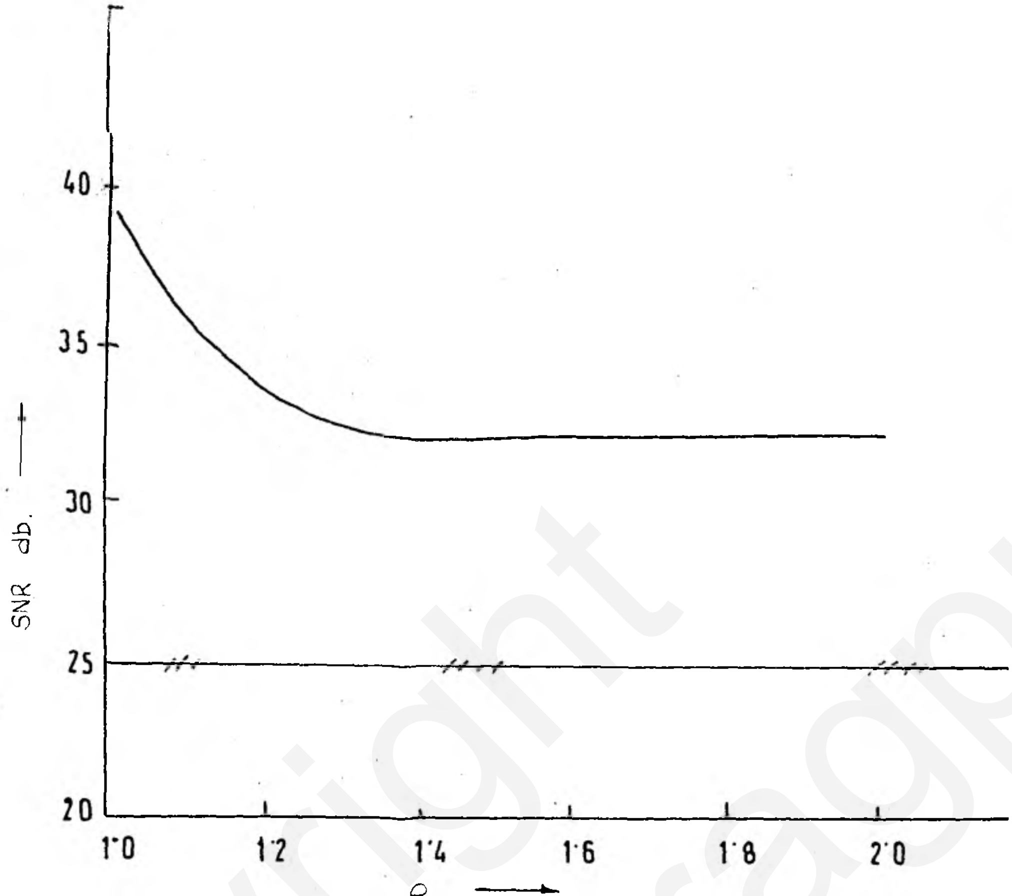
VARIATION OF SNR WITH INPUT LEVEL; BIAS SET AT 120 V

FIG. NO. 3·35

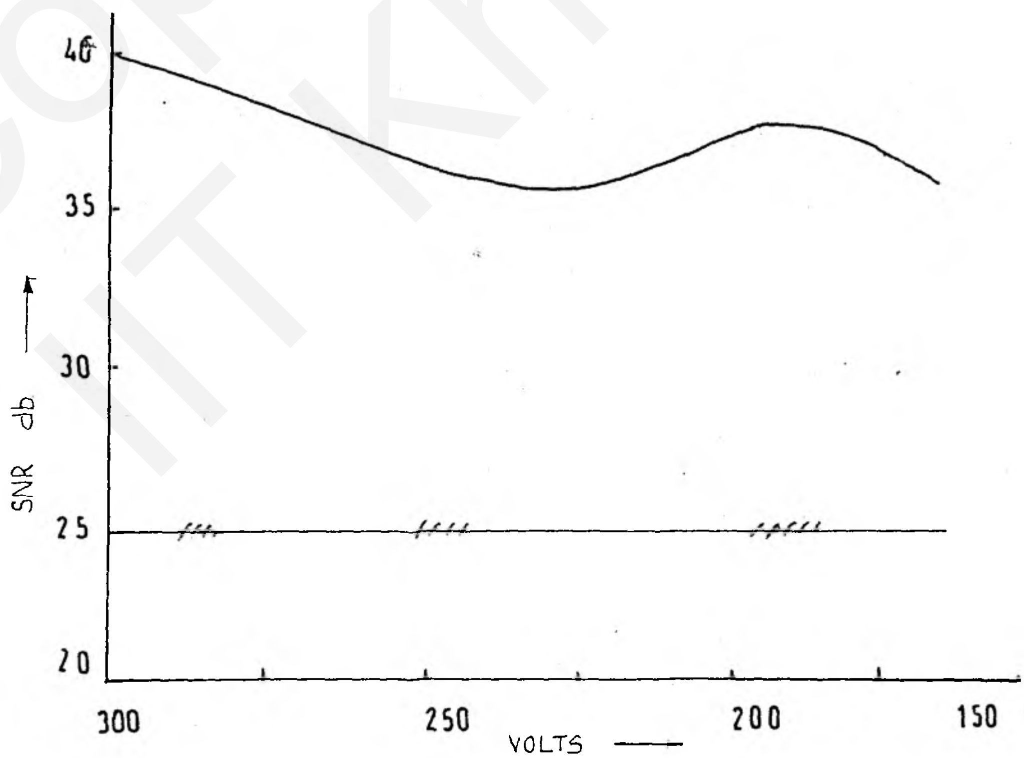
against the reference voltage variations, and the SNR is within acceptable limits for a large amount of variations of the bias.

The results given above are for the Schmitt trigger working on the positive error samples. The modified Schmitt trigger of Fig.3.3 which is used for the quantization of the negative error samples, is more sensitive to voltage variations as compared to the positive Schmitt trigger. However, a voltage variation of 1% in the reference voltages results in an SNR impairment of less than 1 db. All the measurements on the system's performance have been done with the modified Schmitt trigger, but a greater stability, than the figure given above, can be readily obtained by inverting the negative error samples, and then using the more rugged positive Schmitt trigger for quantizing. The positive pulse output of this trigger, however, has to be inverted again to get the correct +1, 0, -1 pulses at the output of the transmitter.

The next important parameter which can change is the size of the positive and negative pulses. If the pulses are equal in height, the SNR is optimum. But it may happen that due to some unbalance in circuitry, the amplitude of the positive pulses increases and becomes more than the negative pulses. The effect of variation of ρ (defined as the ratio of positive pulse amplitude to the negative pulse amplitude at the output of the transmitter) with SNR is shown in Fig.3.36. It is seen that the increase of ρ by 10% deteriorates the SNR by about 4 db, and



VARIATION OF SNR WITH ρ FIG. NO. 3·36



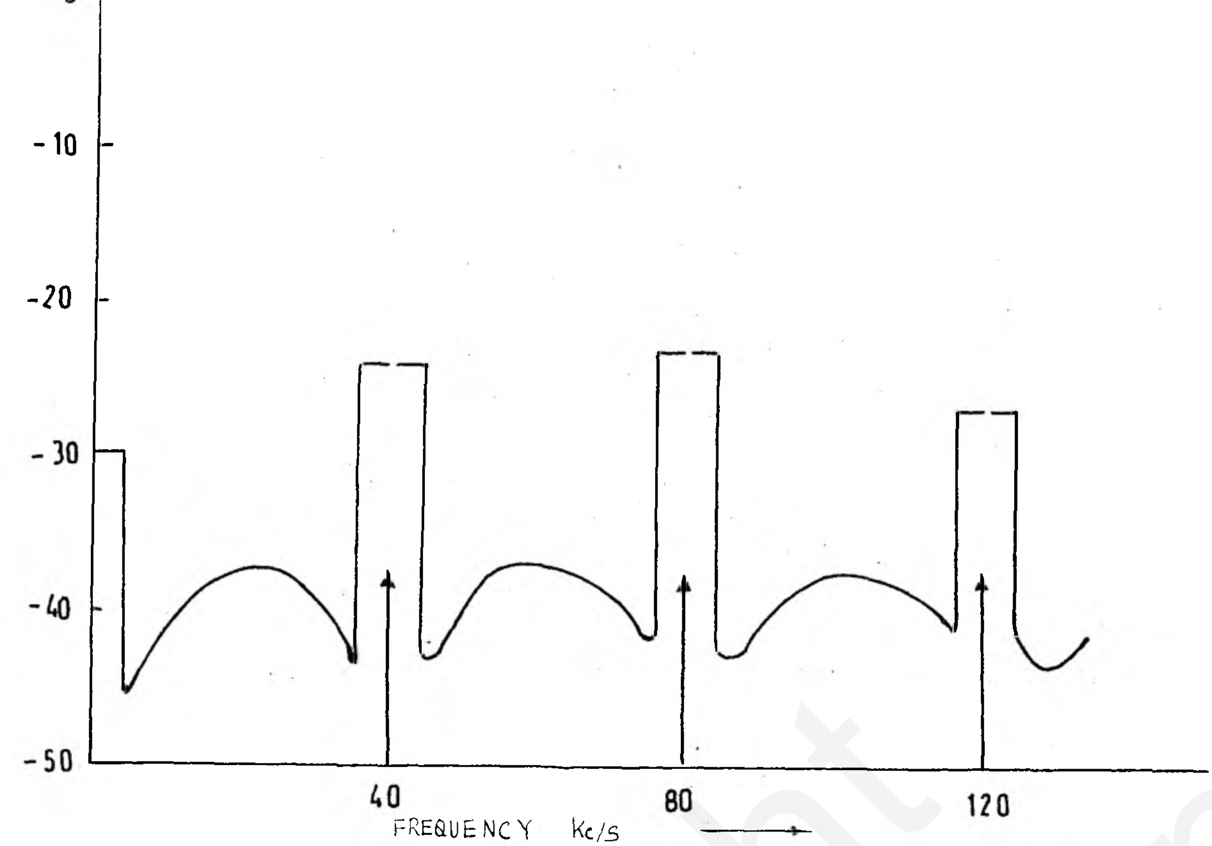
VARIATION OF SNR WITH THE BIAS OF SAMPLING CIRCUIT FIG.NO. 3·37 an increase of 30% deteriorates the SNR by about 8 db. Any further increase does not have much effect, and the worst SNR is 32 db, even for an increase in ρ by 100%, as the system again becomes equal to Binary SQ-PCM.

The effect of the variation of the bias in the Bisymmetrical sampling circuit is shown in Fig.3.37. It is seen that, a change of 50 V in the bias produces a change of about 4 db in the SNR, and the circuit is fairly stable against any such variations. In fact, it is observed that, even without the negative supply, the circuit operates satisfactorily, except for a 'mushy' triggering in the negative triggers which gives some extra noise i.e. the SNR is about 36 db and the circuit is slightly oscillatory at lower input levels. Thus, it may be concluded that the Ternary SQ-PCM system with feedback is quite rugged and an acceptable operation is possible even when the reference voltages etc. are drifting. Normally, the reference voltage is taken from a well regulated power supply, and such major changes in its voltages which will effect the circuit operation will be rare.

3.1.3. SPECTRUM OF QUANTIZING-NOISE.

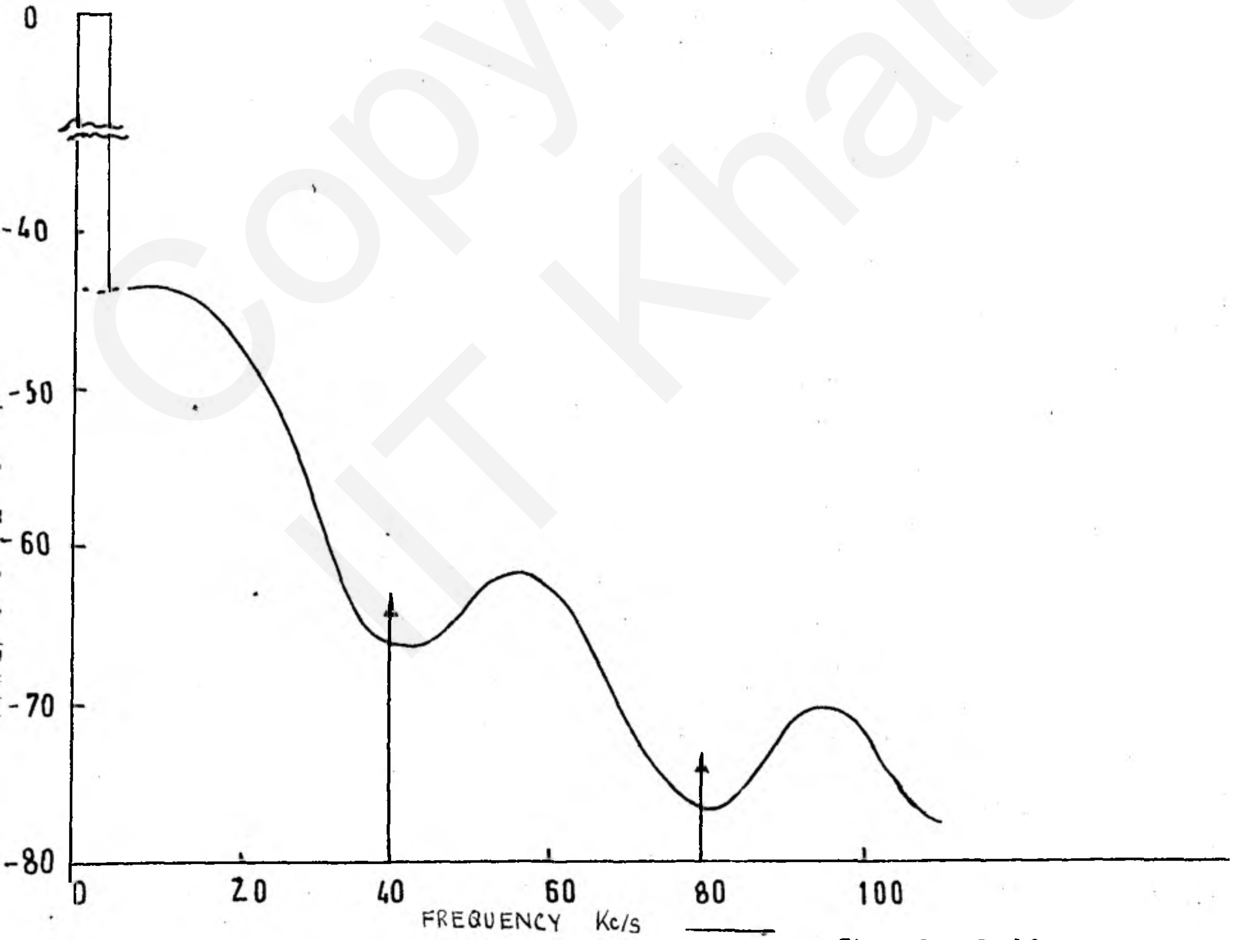
The quantizing-noise characteristics of the SQ-PCM system has been discussed in Section 2.4. The spectrum of the output pulses of the transmitter and the receiver output (before filter) has been experimentally measured to coroborate the assumptions made there. The measurement of the spectrum has been made with a tunable filter extending over the range of 0 - 600 Kc/s. The bandwidth of the filter is 1.5 Kc/s at any setting of the centre frequency and the output of the filter is metered with an rms meter. The input signal to the system is taken from a Multifrequency generator giving nine discrete frequencies in the band of 300 c/s to 3.4 Kc/s. Fig.3.38 shows the spectrum of the pulses at the output of the transmitter.

The spectrum consists of a continuous spectrum (as given in eq. 5.30), and a line spectrum, at the pulse repetition frequency. It is seen that the audio power in the transmitted signal is -30 db, the reference 0 db is the AF output of the receiver. The sidebands' power, upper and lower, around the fundamental and 2nd harmonic of the PRF is about -23 db. The carrier is suppressed by 16 db as compared to the sidebands, and the continuous noise spectrum shows dips around the harmonics of the PRF, as was discussed in Section 2.4. After integration, the noise spectrum is modified as shown in Fig.3.39, and it is seen that the shape of the spectrum matches the theoretically assumed noise spectrum in the Section 2.4. The signal power in the audio band is equal to the reference 0 db and the continuous noise spectrum outside the AF band is 44 db lower. The fundamental of the PRF gets reduced to -64 db and the harmonic of the PRF are reduced still further. Therefore, it is seen that with the integrator, the energy in the sidebands around the harmonics of the PRF (Fig.3.38) help to build up the signal in the audio band. This is a very important property of the Ternary pulses and it leads to the conclusion that for the video transmission of these



SPECTRUM OF THE TRANSMITTER OUTPUT PULSES
fr = 40 Kc/s

FIG. NO. 3·38



SPECTRUM OF THE
TRANSMITTED SIGNAL AFTER INTEGRATION fr 40 Kc/s

FIG. NO. 3·39

pulses through cables etc. the d.c. and the low frequencies need not be transmitted very efficiently[27]. This will lead to considerable saving in the cost of transformers etc., because in the ternary system the low frequency suppression is allowable without disturbing the main spectrum. In fact, it is known that in AQ-PCM, where 0/1 pulses are normally generated, the pulse train is converted into a Pseudo ternary pulse train before being transmitted on the cables, as the spectrum of the 0/1 pulse train contains discrete lines at low frequencies, and therefore, will suffer most from the low frequency suppression which is inevitable in cable transmission. The timing pulses in the Ternary SQ-PCM can be extracted by first converting the ternary pulses into binary pulses by rectification in the receiver, and then using the larger amplitude of the line spectrum of the Binary pulses (Section 4.2.2) for the purpose.

3.1.4. SUMMARY OF THE RESULTS.

The performance of the Ternary SQ-PCM system (with feedback) has been given for four different types of input signals such as Sinusoidal, FM, Noise, and Speech, and for different pulse repetition frequencies of 60, 40, 30 and 20 Kc/s. For a sinusoidal frequency of 1 Kc/s, the best SNR of 44, 41, 37 and 33 db is obtained for the four different PRFs, respectively. The SNR falls linearly with input below the overload level and falls sharply for the inputs above this. The reason is that the quantizing-noise is random, with a uniform power spectral density in the audio band, and the SNR falls linearly as the signal decreases. Above the overload level, there is a clipping leading to large amounts of harmonic distortion and a decrease in SNR. The dynamic range of input, with an SNR of above 25 db, is 31 db, 29 db, 22 db, and 16 db for the PRFs of 60, 40, 30 and 20 Kc/s respectively. For a sinusoidal frequency at the higher end of the audio band, the overload level of the input is about 3 db below the overload level at 1 Kc/s. The effect produced is also reflected in the decrease of the SNR and the increase of frequency distortion for higher frequency signals at 0 db input level. Hence, the SNR and the output fall by about 6 db and 3 db respectively, at 40 Kc/s PRF. However, at lower input levels, the SNR and the output are constant for a variation of the signal frequency. The input-output relation is linear below the overload. Similar results are obtained for the other PRFs, and the improvement in the best SNR is about 8 db/octave change of PRF, against an expected value of 9 db/octave.

An instantaneous compandor improves the dynamic range of the input considerably. Two types of compandors have been experimented with; a linear compandor, where the output expansion matches the compression at the input, and a partial compandor, where the expansion is less than that required for a match. The dynamic range of 41 db and 37 db is obtained at a PRF of 60 and 40 Kc/s, and the SNR is above 31 and 27.5 db, respectively. The SNR is above 24 db and 20 db for a range of input of 32 db and 27 db at a PRF of 30 and 20 Kc/s, respectively. The compandor advantage at a PRF of 40 Kc/s is 22 db i.e. the SNR at an input of -40 db is

25 db with the compandor included, whereas without the compandor the SNR would have fallen to 3 db. The partial compandor, on the other hand, gives a dynamic range of 42, 40, 34, and 26 db, for a SNR of above 25 db at 60, 40, 30 and 20 Kc/s PRF, respectively. The best SNR is 34, 32, 29 and 27 db, for the four PRF's, respectively. Since the overload level of the higher-end input frequency is about 3 db less than the centre frequency, a further frequency distortion occurs in the output of the system with compandor. The output at 3.3 Kc/s falls by about 9 db as compared to the 1 Kc/s signal at 0 db input level, and the output becomes constant with frequency only at -15 db input level.

The FM signals are reproduced with good fidelity, and the SNR obtained for the half band FM signals is 36, 33, 29 and 27 db, at 60, 40, 30 and 20 Kc/s PRF, respectively. A narrow band FM signal has the SNR of 37, 34, 30 and 27 db, at the four PRF's, respectively.

The results of the noise signals input are quite satisfactory, and these results give a correct idea of the performance of the systems as the noise signals are the most general complex signals. The SNR for the noise signal has been measured by two techniques, one using the 1/2 band audio noise signal, and other using a band stop filter in the full audio band noise signal. The results of SNR are 35, 32, 27 and 24 db, at the PRF's of 60, 40, 30 and 20 Kc/s, respectively.

Listening tests with speech signals as input gave a very satisfactory performance at 60 and 40 Kc/s PRF's, while the results at 20 Kc/s are also satisfactory except for a slight background noise. A record of reproduced speech (one sentence) through 1/3 octave filters at 40 Kc/s PRF, with and without compandor, and at 20 Kc/s PRF without compandor has been given, together with the record of the original speech signal for comparison. The comparison for the 40 Kc/s PRF shows a good reproduction, while the higher frequencies are badly reproduced at 20 Kc/s PRF.

The stability of the system has been measured against drift and change of voltages in the circuit. It has been shown that a bias voltage variation of one of the Schmitt triggers produces a change in the number of triggers, and a large change of bias voltage will not produce any positive (or negative) trigger. The circuit then degenerates into the Binary SQ-PCM system, but continues working. Similarly, the bias voltage of the sampling circuit and ratio of the positive and negative pulses at the transmitter output have been changed. It has been shown that even for appreciable change in these quantities the SNR degenerates but remains at about 32 db.

The spectrum of the quantizing noise observed experimentally agrees fairly well with the one predicted theoretically. In the spectrum of the transmitter pulses, the energy at low frequency (audio band) is lower, compared to the sideband energy distributed around the PRF and the harmonics of the PRF. With the integration at the receiver, the energy from the sidebands around the PRF and its harmonics builds up the signal to the same reference value. The output of the system is suitable for transmission through cables without any preconversions etc., as required in the binary pulses.

3.2.0. TERNARY SQ-PCM SYSTEM WITHOUT FEEDBACK.

If the feedback loop in the previous system described in Section 3.1 is not used, and a differentiating circuit is put before the sampling circuit, the system will become a Ternary SQ-PCM without feedback. The theoretical justification for including a differentiating circuit has been given in Section 2.2. A block diagram of the Ternary SQ-PCM system without feedback is given in Fig.3.40. The input signal is amplified, differentiated and fed into the Bisymmetrical sampling circuit through an amplifier. The symmetrical positive and negative pulses are fed to the sampling circuit from the Pulse Generator. The sampled signals are separated into positive and negative samples and fed to the two Schmitt triggers whose bias is adjustable. The positive samples above a particular positive threshold give a +1 pulse, the negative samples below a particular negative threshold give a -1 pulse, and all sample values between the thresholds give 'no' pulse. The output of the Schmitt triggers is differentiated, amplified (to remove any width variations), and combined in an adder circuit to form the output of the transmitter.

The receiver consists of a cathode follower followed by, either an ordinary integrator with negative feedback, or a double integrator including an active integrator with a positive feedback, and a filter. The ordinary integrator with a negative feedback

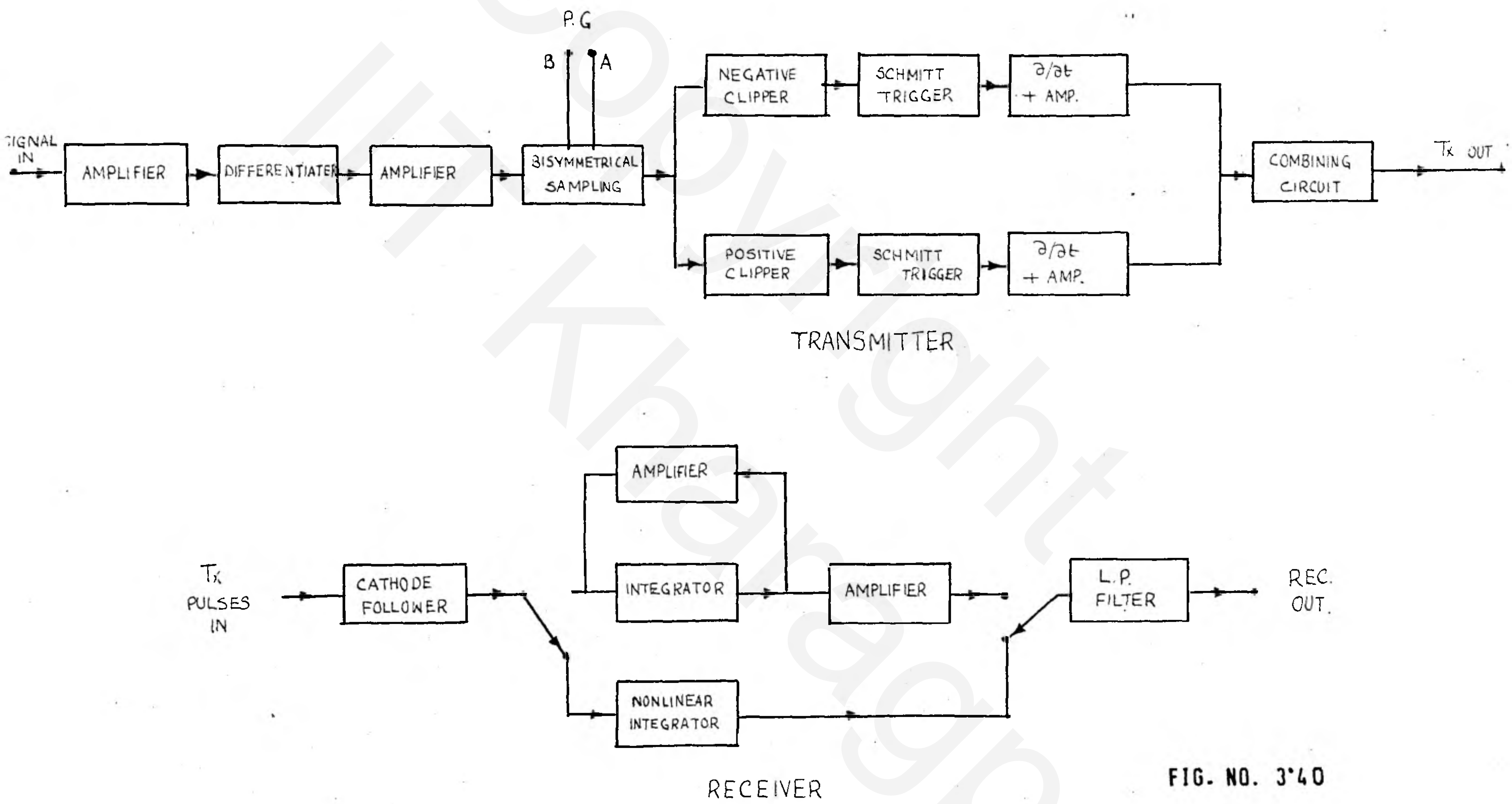


FIG. NO. 3'40

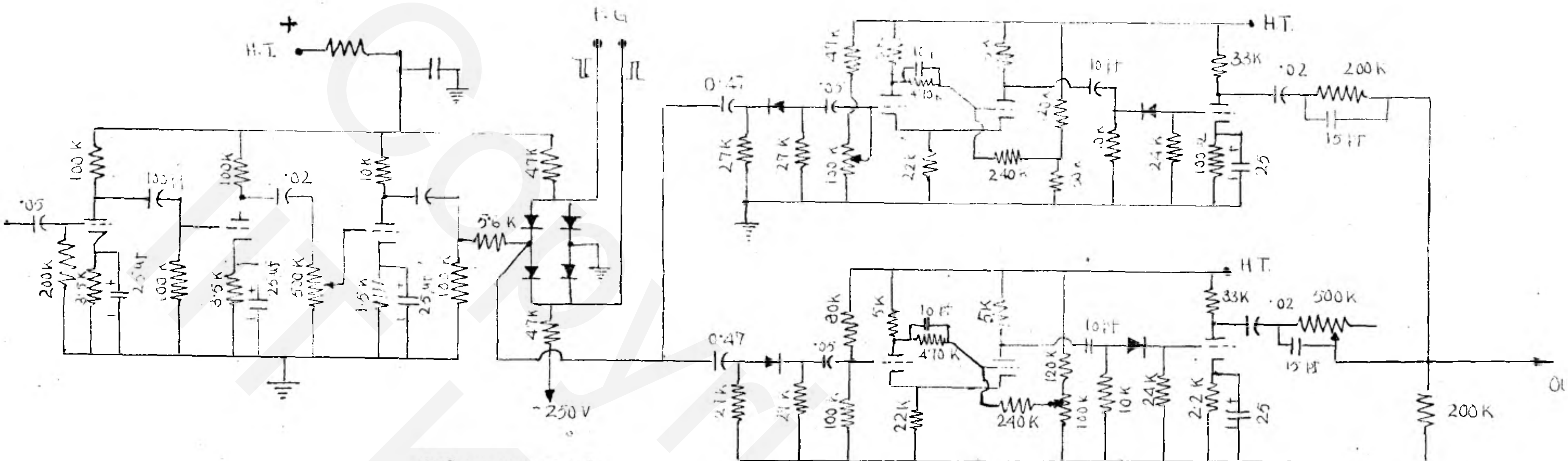
BLOCK DIAGRAM OF THE TERNARY SQ-PCM SYSTEM WITHOUT FEEDBACK loop has been described in Section 3.1 (Fig.3.2). The active integrator with a positive feedback is the M.I.T. integrator followed by an equaliser to correct the frequency response. The integrator, together with the saturating amplifier, forms a circuit which is called here the nonlinear integrator. A complete circuit diagram of the system is given in Fig.3.41. The test signals for estimating the performance of the system are the same as described in Section 3.1.

3.2.1. SYSTEM CHARACTERISTICS.

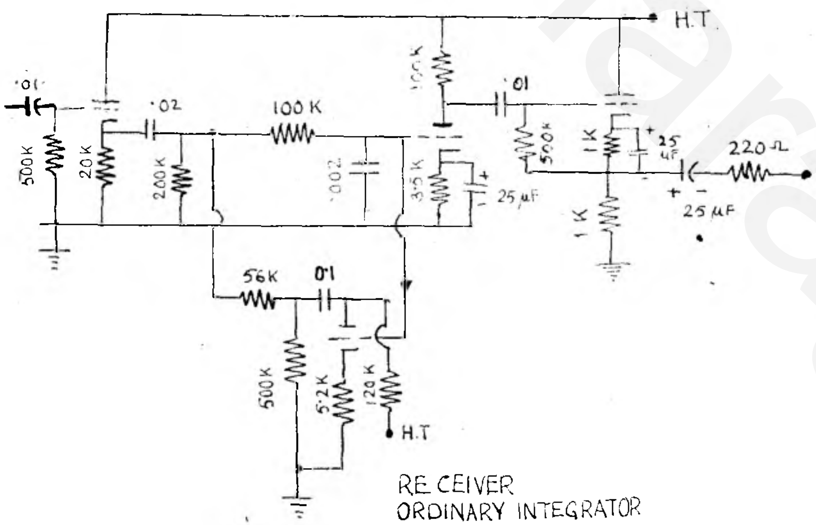
(a) Sinusoidal Signal Test.

The method of measurement of SNR, output, frequency distortion etc. is the same as given in Fig.3.7. The pulse repetition frequencies between 20 Kc/s to 60 Kc/s have been used for estimating the performance of the system. The results given here are for the PRF of 40 Kc/s. The results for the other PRF's have the same proportional relation as given in Section 3.1 for the system with feedback. The use of the active integrator gives some instantaneous compression due to the nonlinear amplitude characteristics, and decreases the 3rd harmonic considerably.

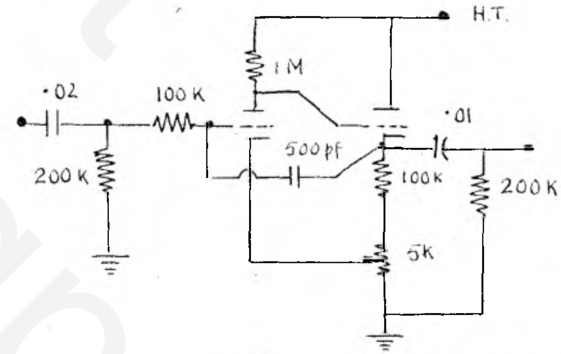
The variation of SNR with input level is shown in Fig.3.42, where the dotted curve is for the ordinary integrator, and the solid curve is for the active integrator. For the ordinary integrator, it is seen that the SNR shows a peak at a certain input level. At higher input levels, there are either a positive or a negative group of pulses, with a very small gap of 'no' pulses between them.



TRANSMITTER



RECEIVER
ORDINARY INTEGRATOR



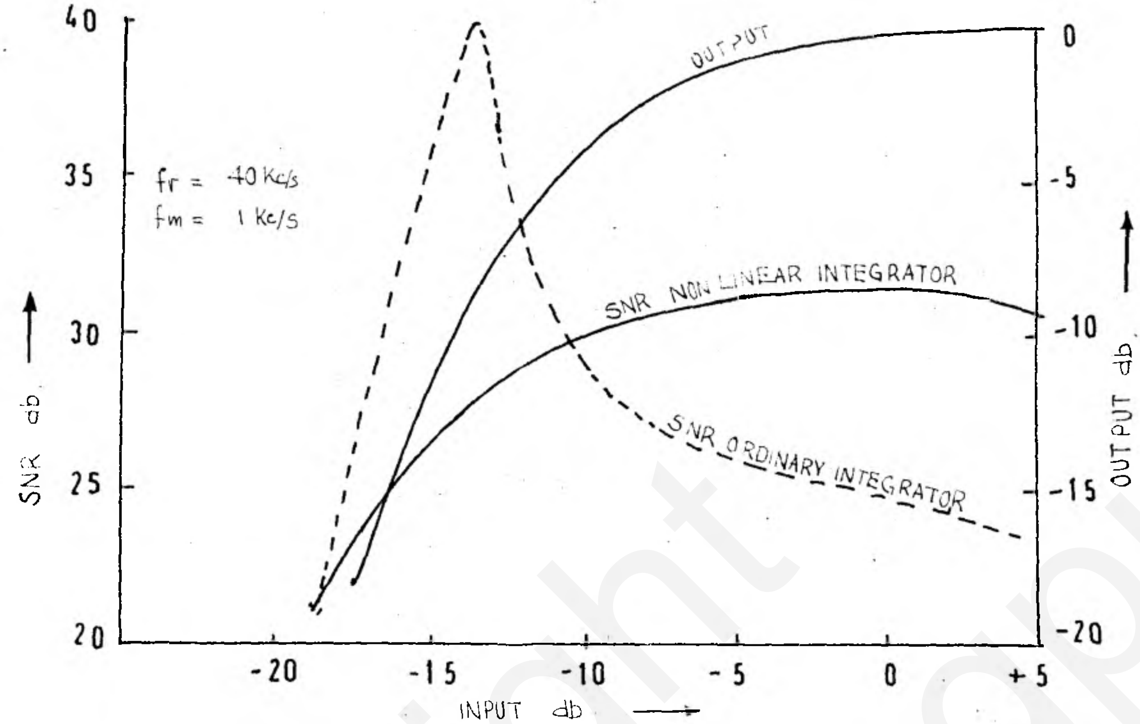
M.I.T. INTEGRATOR

FIG. NO. 3·41

COMPLETE CIRCUIT DIAGRAM OF THE TERNARY SQ-PCM WITHOUT FEEDBACK

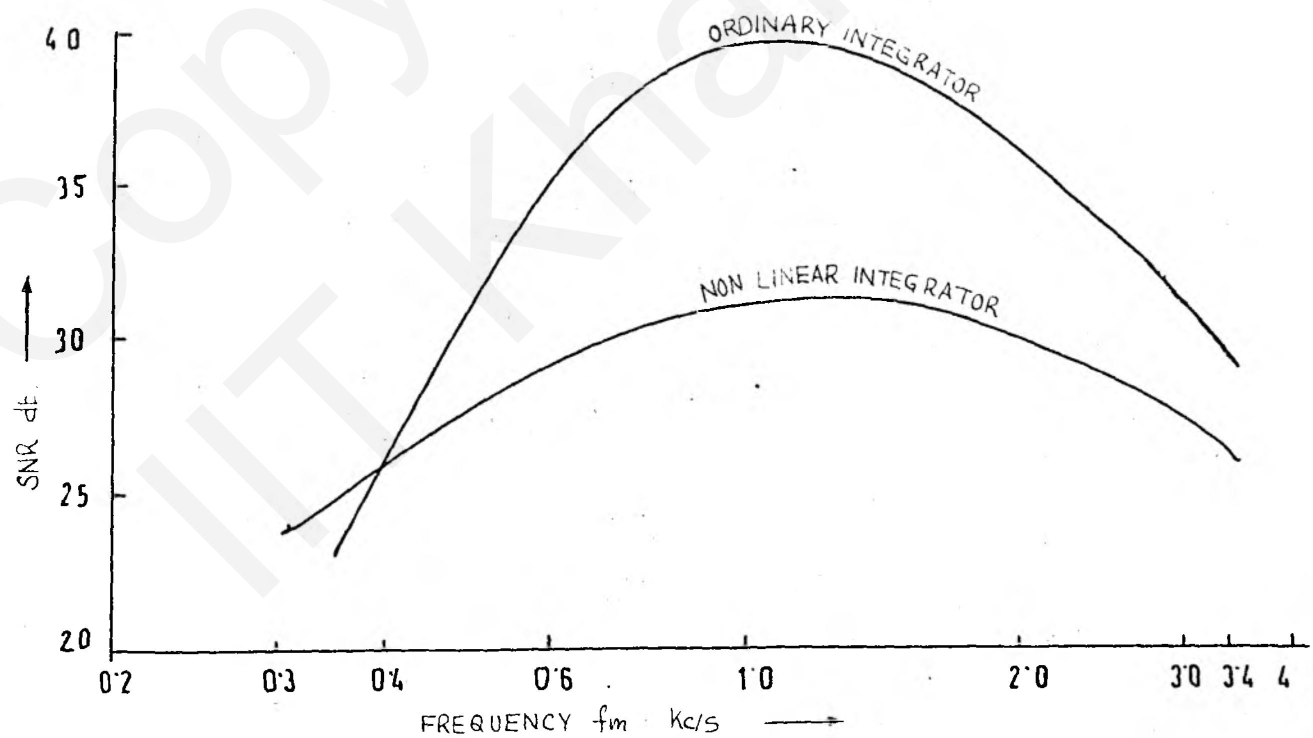
As the input is decreased, the threshold level (which is kept constant) of the quantizers rises in comparison with the rms value of the signal samples. The result is that, fewer +1, -1 pulses occur, and the number of 'no' pulses increases. The low SNR is mainly due to the harmonics produced because of the saturation, and as the input is decreased, more gaps appear between the pulses resulting in a decrease of the harmonics. Hence, an optimum input is reached when the harmonics are minimum and the SNR is maximum. The SNR falls rapidly for a further decrease in the input, because the number of triggers becomes less and less very quickly, till there is no output. For a signal of 1 Kc/s the SNR is about 25 - 28 db for an input range of 0 to -10 db, mainly consisting of 3rd harmonic. The SNR increases to 40 db at an input level of -14 db, to fall again to 20 db for an input of -18.5 db. The use of the nonlinear active integrator improves the SNR (for the 0 to -10 db input) to 30 - 32 db, but at the cost of the 40 db SNR obtained previously at -14 db input level. The SNR with the input level variation is now almost flat within 25 - 32 db for an input range of 21 db.

The input-output voltage relation is not linear, as shown in the Fig.3.42. A 10 db decrease in the input, from 0 db reference, results in only a 3.5 db decrease in the output. This is again because of the same saturation effect as encountered in the SNR characteristics. Next 5 db decrease results in a 7.5 db decrease in the output, and any further decrease in input results in a very steep fall in the output, e.g. a decrease of 5 db gives an output change of 26 db because the +1 and -1 pulses are very few. The



VARIATION OF SNR AND OUTPUT WITH THE INPUT LEVEL

FIG. NO. 3·42



VARIATION OF SNR WITH fm ; fr = 40 Kc/s

FIG. NO. 3·43

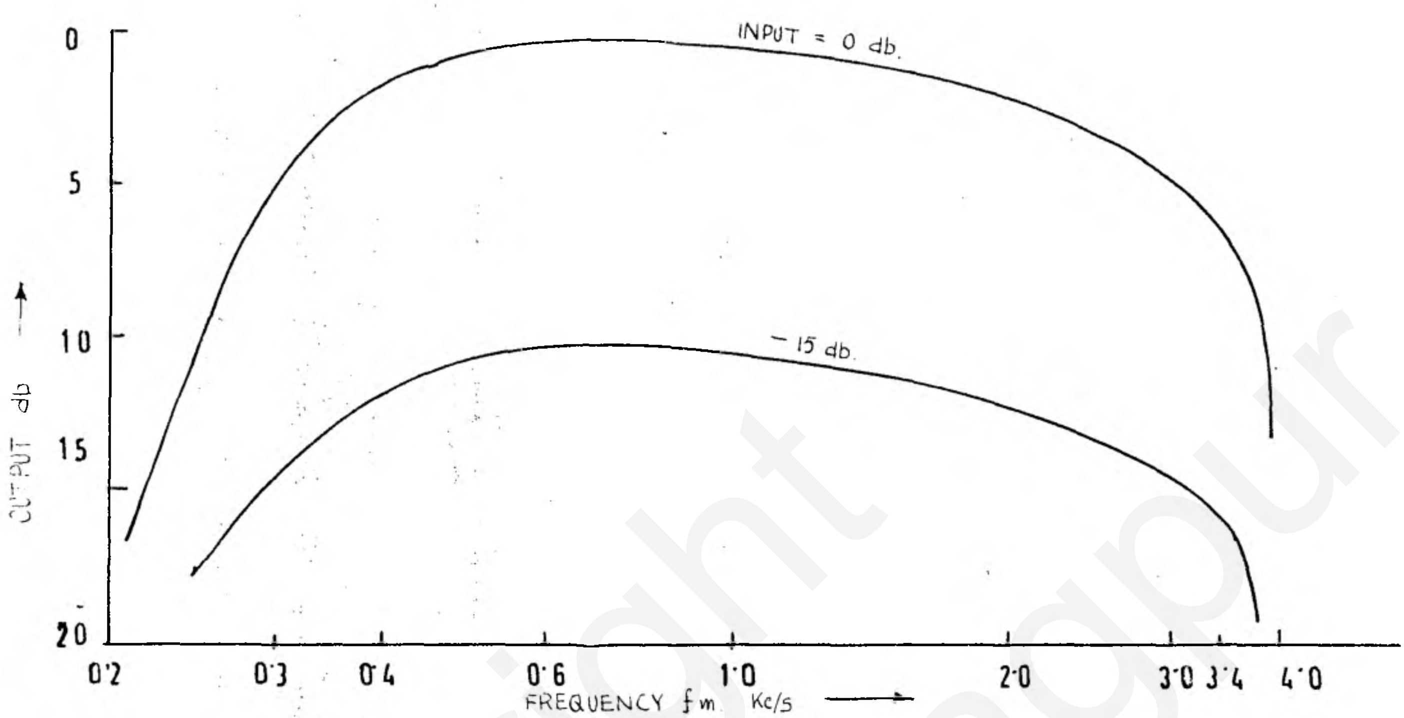
variation of the optimum SNR for the different signal frequencies is shown in Fig.3.43. It is seen that the SNR at 400 c/s falls by about 12 - 14 db as compared to the SNR at 1 Kc/s, and at 3 Kc/s it falls by about 10 db. However, with the nonlinear integrator, the variation of the SNR with frequency is much less, and the SNR falls by about 6 db at the two ends of the frequency band.

The variation of the output with the variation of signal frequency for two input levels is shown in Fig.3.44. The output at 3 Kc/s falls by about 5 db as compared to the centre frequency. It is shown in the Fig.3.42 that the output is saturated, but for an overall decrease of 15 db, the output changes by 10 db at the centre frequency as is seen in Fig.3.44. The variation of optimum SNR with PRF is shown in Fig.3.45 for both the integrators. The SNR varies by 7.5 db/octave change of PRF and agrees fairly with the theoretical increase of 9 db/octave.

A photograph of the output transmitter pulses for a sinusoidal input is shown in Fig.3.46(a), and a photograph of the output of the receiver showing the negative and the positive steps is given in Fig.3.46(b).

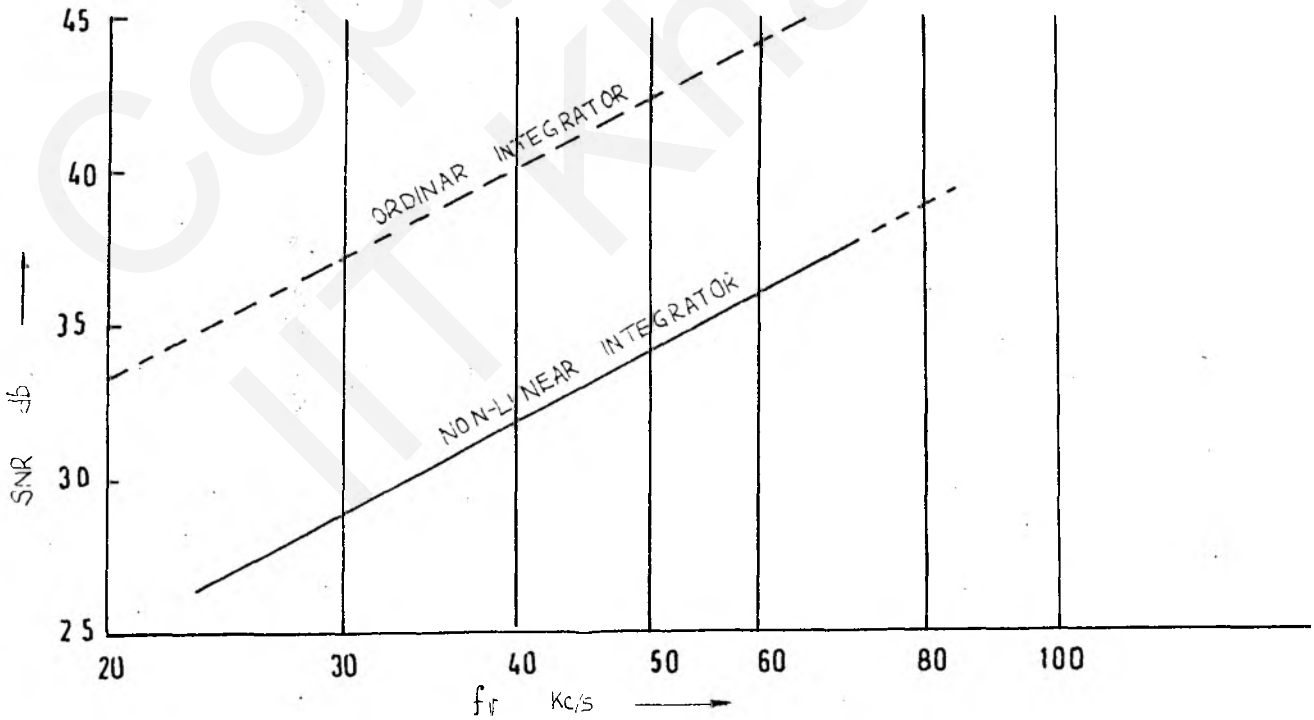
(b) <u>FM Signals</u>.

Since the frequency modulated signals have practically no amplitude variation, the input-output amplitude linearity of the system is of no importance. The Ternary SQ-PCM system without feedback will be quite suitable for such applications. It is also seen that the SNR at -14 db input level is 40 db for a sinusoidal frequency of 1 Kc/s, and if the amplitude of the FM signal



VARIATION OF THE OUTPUT LEVEL WITH fm FOR DIFFERENT INPUT LEVELS; fr = 40 kc/s

FIG. NO. 3·44



VARIATION OF SNR WITH PRF

FIG. NO. 3·45

is adjusted to be in this range, the reproduction of the signals will be pretty good. The non-linear integrator, however, levels off this peaky SNR to a lower but flat SNR value, and will give poorer results for the FM signals.

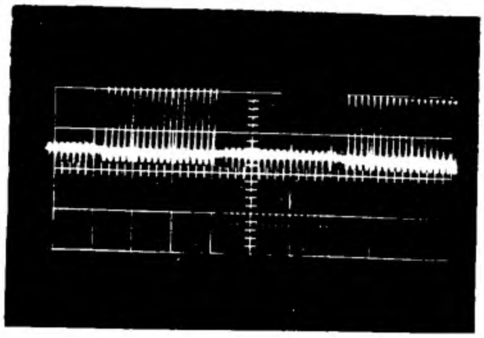
The measuring technique of the performance of the system for the wideband and the narrow band FM signals is the same as given in Fig.3.21. The average SNR of the two measurements, one, when the signal is in Band I and the noise is measured in Band II, and the second, when the signal is in Band II and the noise is measured in Band I, are shown in Fig.3.47. The optimum SNR at PRF's of 60, 40, and 30 Kc/s, is 32, 30 and 26 db, respectively, when the ordinary integrator is used, and is 29, 26 and 21 db respectively, when the non-linear integrator is used.

The system without feedback is also useful for narrow band FM signals. The FM signal is confined to a band of 800 - 1700 c/s as shown in Fig.3.48 and the average SNR is 34 db, 32 db and 28 db, for the three PRF's of 60, 40 and 30 Kc/s, respectively, with the ordinary integrator in the circuit. At a PRF of 40 Kc/s, with the nonlinear integrator, the SNR for a narrowband FM signal is 30 db.

Another technique of measuring the SNR for the narrowband FM signals input has been to examine the output of the system through a narrowband 1/3 octave filter described in Section 3.1.1(d), and the characteristics of which are shown in Fig.3.30. The voltage output of each of these bands is measured by an rms meter with the original signal, and then the reproduced signal, as its input.

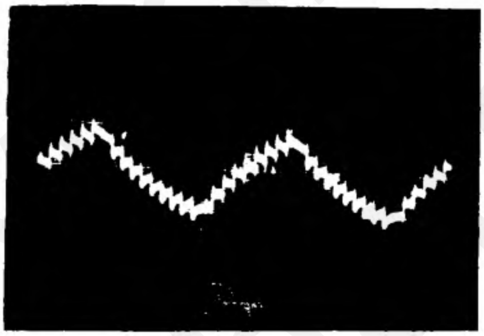
FIG 3.46

(a)



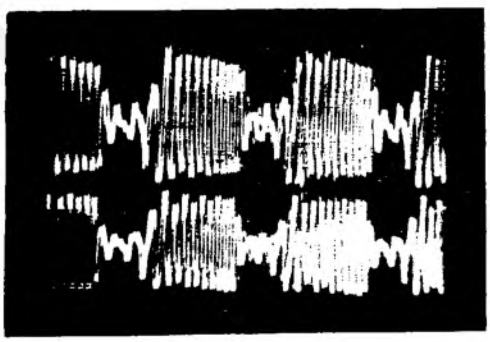
TRANSMITTER PULSES OF TERNARY SQ-PCM WITHOUT FEEDBACK

(b)

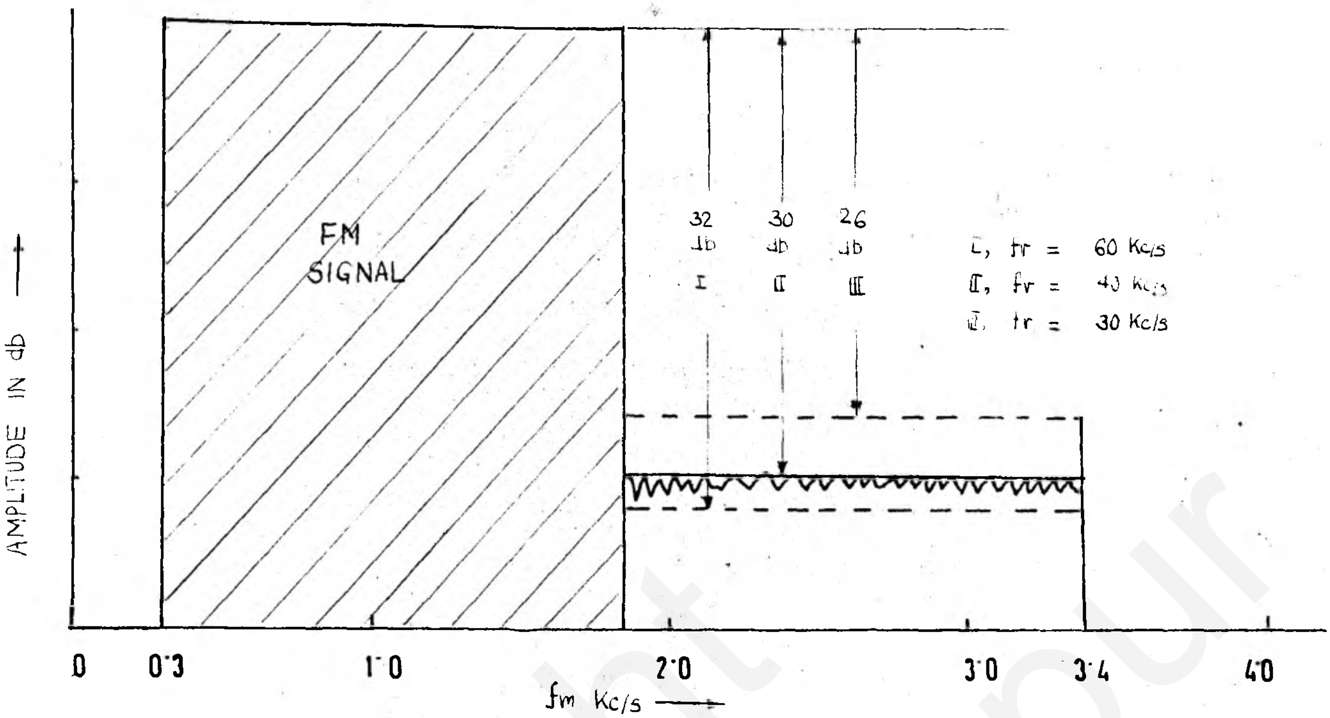


RECONSTRUCTED SIGNAL AT THE RECEIVER

FIG 3.51



FM TEST SIGNAL (TOP)
REPRODUCED SIGNAL (BOTTOM)



SNR FOR 1/2 BAND FM SIGNAL WITH ORDINARY INTEGRATOR IN THE RECEIVER

FIG. NO. 3·47

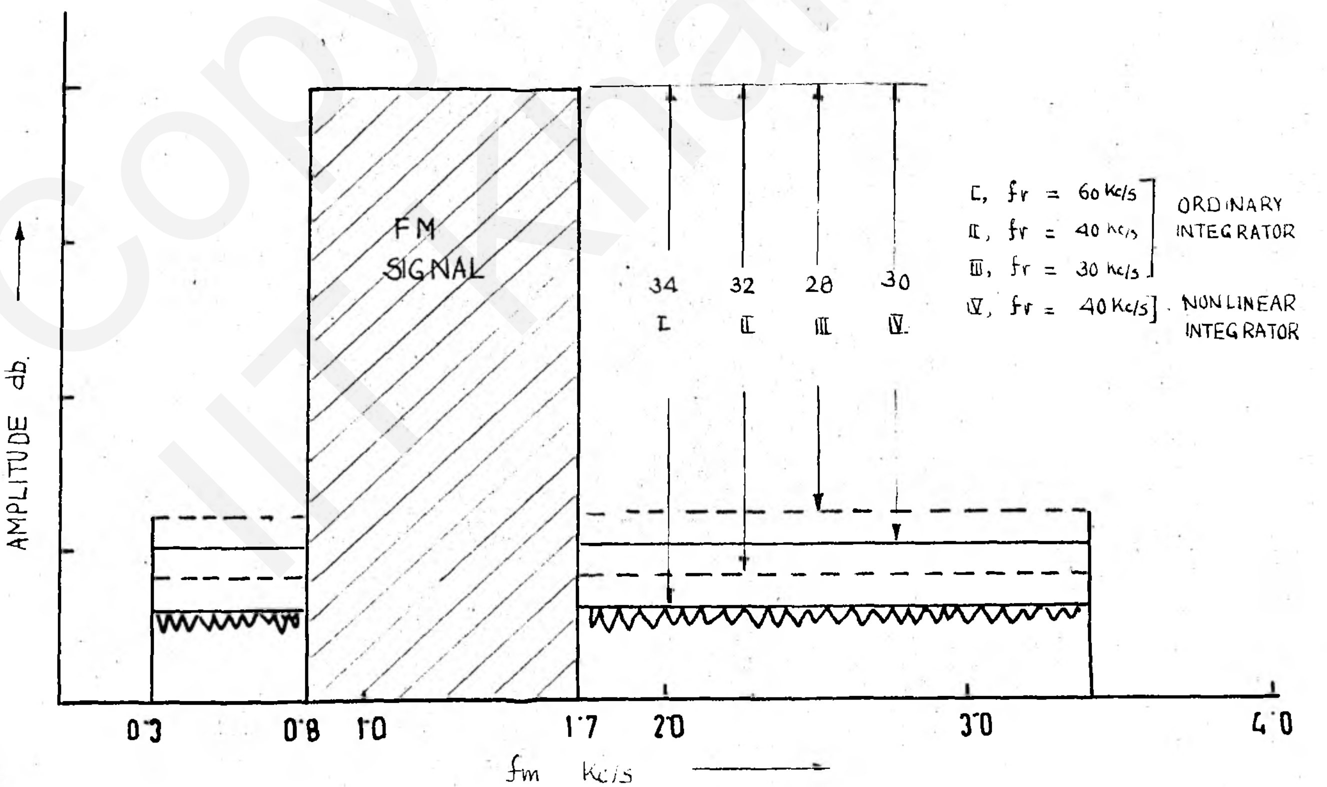


FIG. NO. 3·48

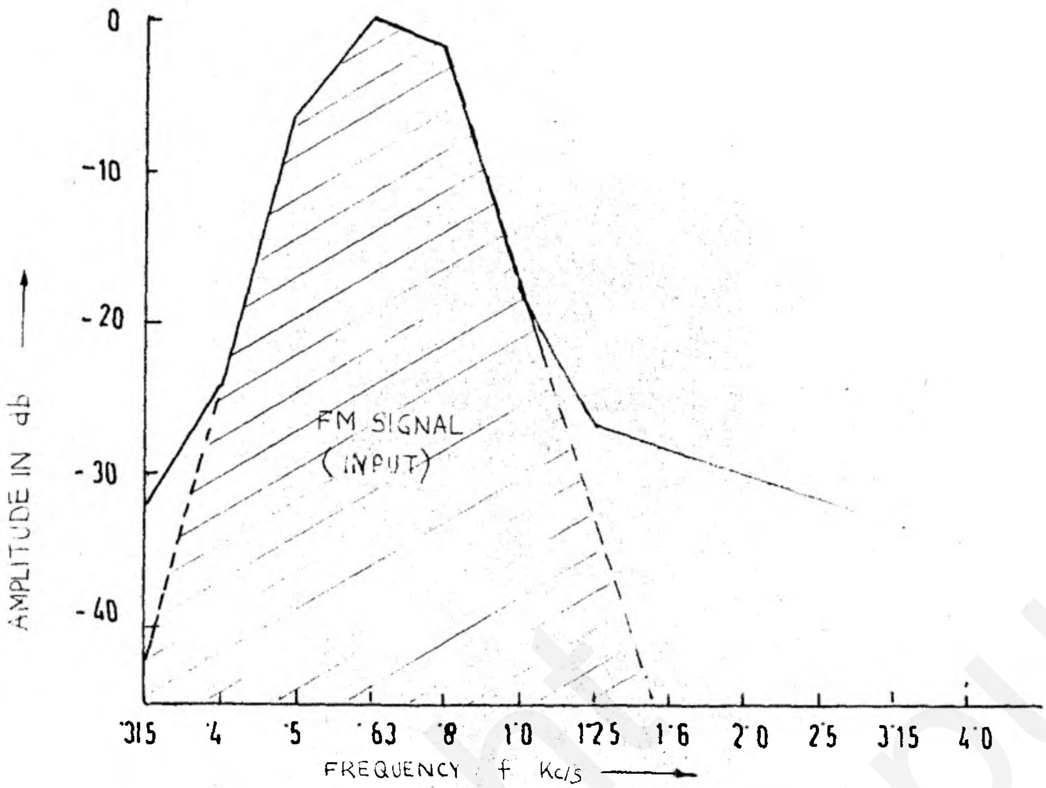
SNR FOR NARROWBAND FM SIGNAL

A plot of two such measurements are shown in Fig.3.49 and 3.50. The input FM signal is in the lower end of the audio band in Fig.3.49, where the shaded area represents the original signal, and the solid line curve represents the output of the system at each centre frequency of the 1/3 octave filter. Fig.3.50 shows a similar plot when the input signal is at the higher end of the audio band. Both these experiments have been done at a PRF of 40 Kc/s with the nonlinear integrator in the receiver circuit. It is seen from these figures that the noise is about 30 db below the signal, a result which agrees with the measured values given in Fig.3.48.

A photograph of the FM signals reproduced by the system is shown in Fig.3.51, where the input has been adjusted for the best SNR reproduction at the PRF of 40 Kc/s. The top curve shows the original input and the bottom curve is the reproduced signal by the system. Fig.3.52(a) shows the photograph of another FM signal input, and Fig.3.52(b) shows a photograph of the output of the system when the input level is not so carefully adjusted. The photograph clearly shows the compression of the amplitudes at the output, as is expected if the input level is between 0 - 10 db (refer Fig.3.42).

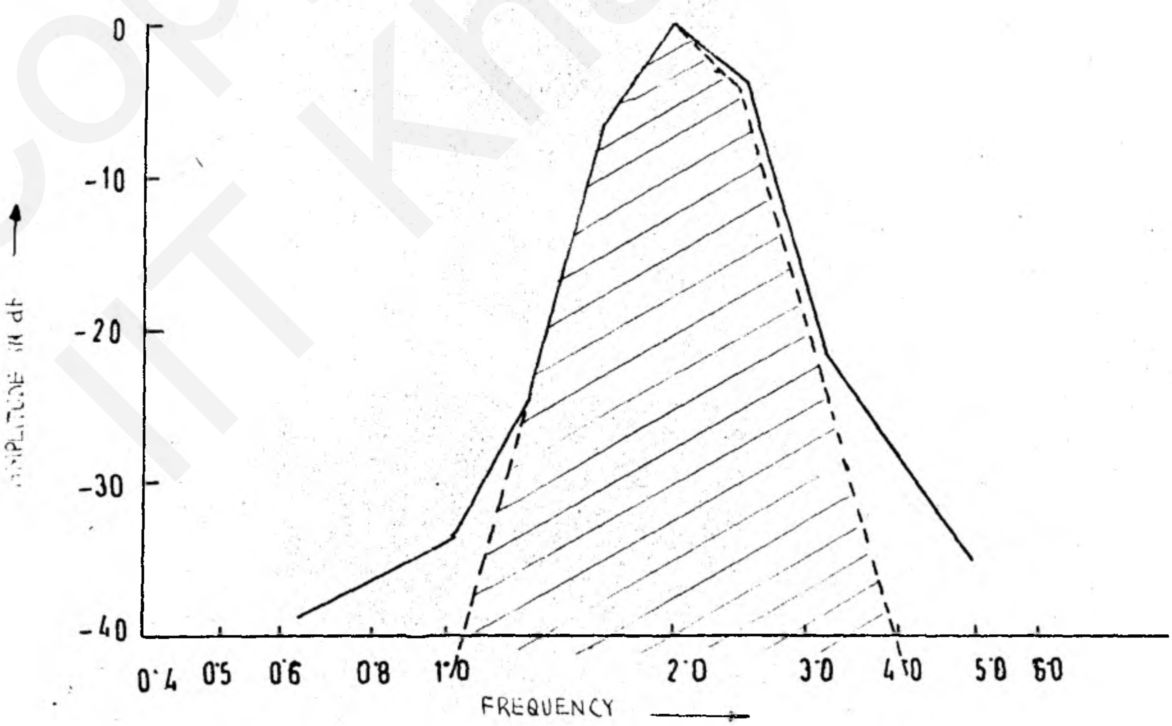
(c) Noise Signal Test.

The performance of the system for the noise signal input has been evaluated by the methods similar to that given in Section 3.1 (Fig.3.25). The SNR at 40 Kc/s PRF for the half band noise



NARROWBAND FM SIGNALS, AND NOISE MEASURED WITH THE 1/3 OCTAVE FILTER; fr = 40 Kc/s; NON LINEAR INTEGRATOR IS USED IN THE RECEIVER; fm AT THE LOWER END OF THE A.F. BAND

FIG. NO. 3·49

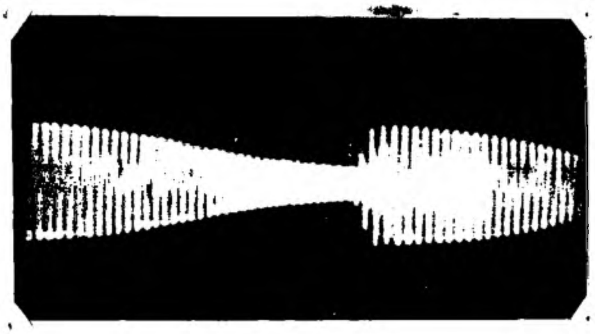


NARROW BAND FM SIGNALS, AND NOISE MEASURED WITH 1/3 OCTAVE FILTER; fr = 40 kc/s, NON LINEAR INTEGRATOR USED IN THE RECEIVER; fm AT THE HIGHER END OF THE AF BAND

FIG NO. 3·50

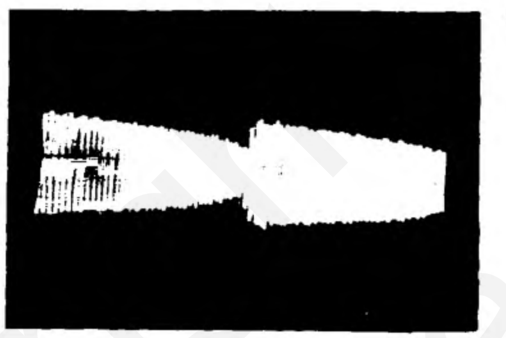
FIG 3.52

(a)



ANOTHER FM TEST SIGNAL

(b)



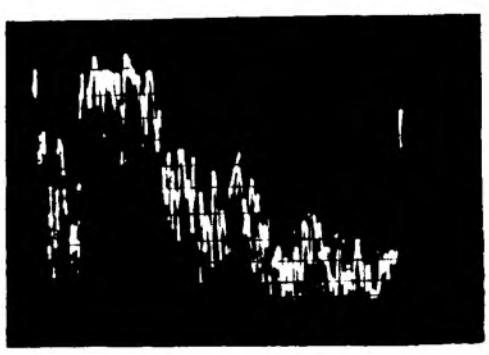
REPRODUCED SIGNAL WITH INCORRECT ADJUSTMENT OF INPUT

FIG 3.55

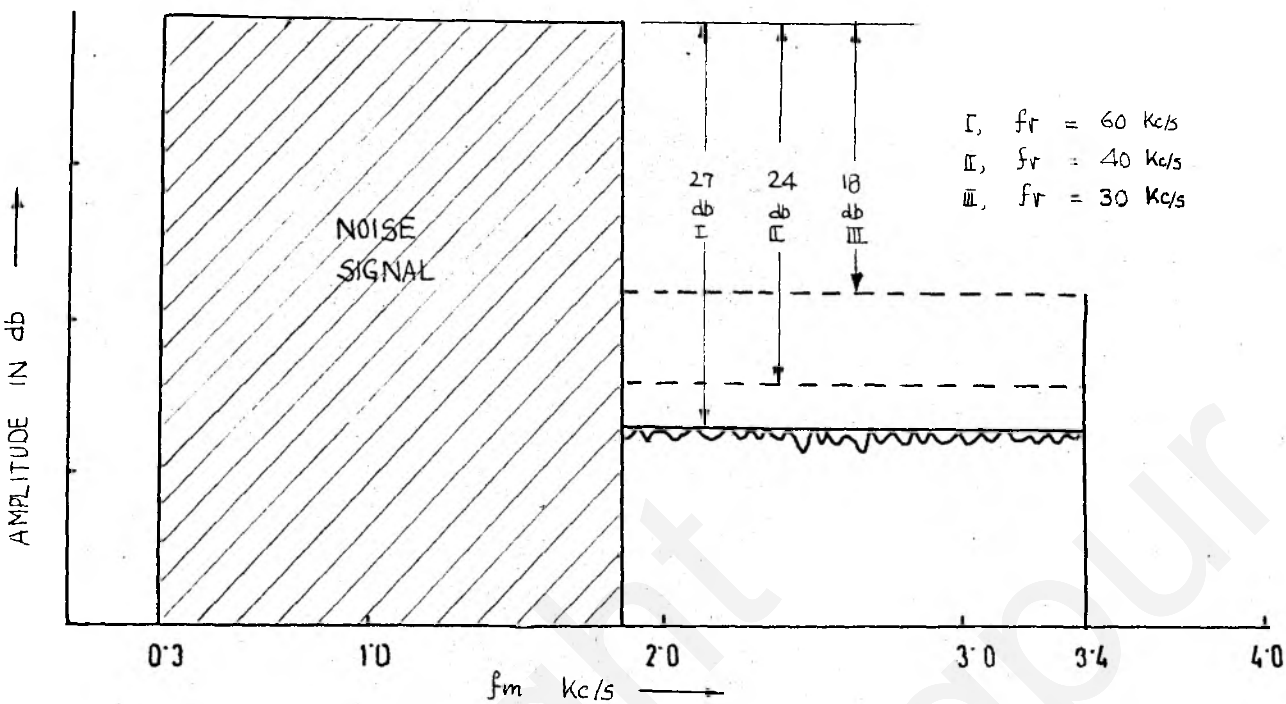
(a)

HALF BAND NOISE TEST SIGNAL

(b)

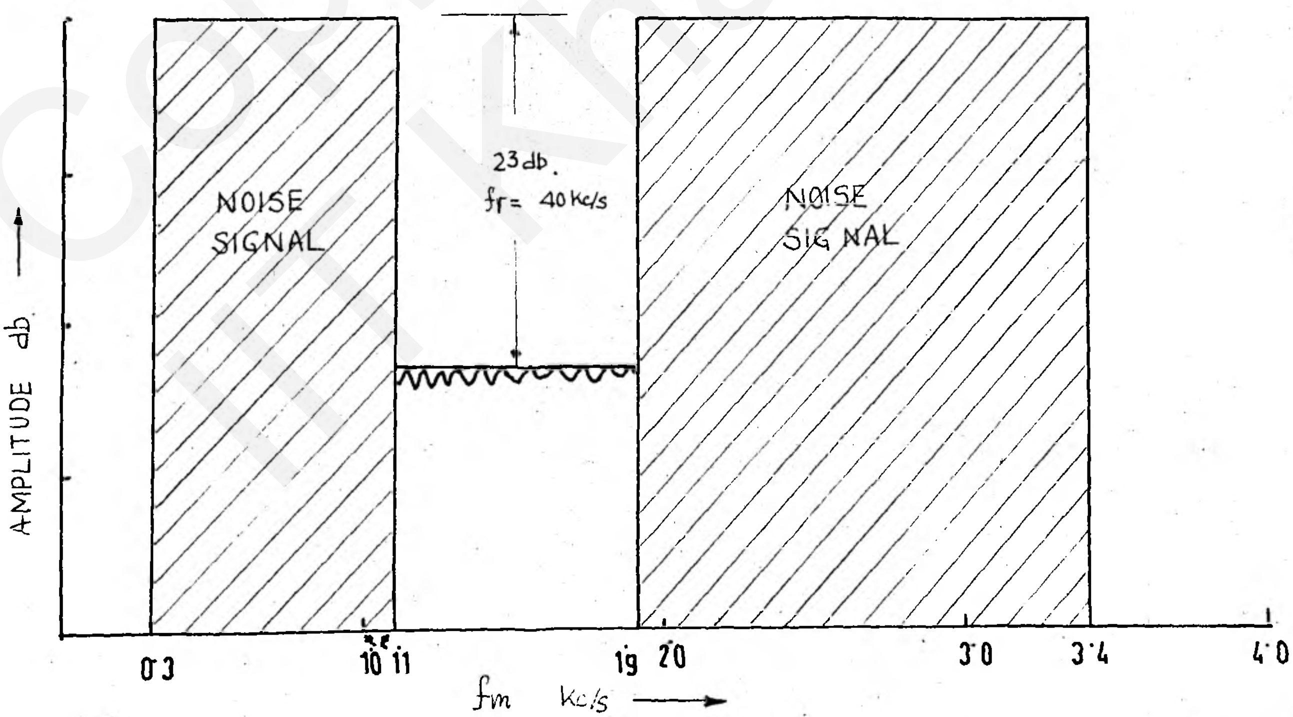


RECEIVER OUTPUT SHOWING QUANTIZING NOISE



SNR FOR 1/2 BAND NOISE SIGNALS WITH ORDINARY INTEGRATOR IN THE RECEIVER

FIG. NO. 3·53



EQUALISED SNR FOR NOISE SIGNAL INPUT USING A BAND STOP FILTER

FIG NO. 3·54

signal is 24 db as shown in Fig.3.53, and the other readings at 60 and 30 Kc/s PRF are 27 db and about 18 db respectively. In the other method of measurement, a band stop filter of about 800 c/s band is used, and the equalised SNR (equalised to the bandwidth) is 23 db at 40 Kc/s PRF as shown in Fig.3.54. This agrees fairly well the value obtained in the previous measurement. A photograph of the noise signal in Band I (Fig.3.55 (a)), and the quantizing noise observed in Band II at a 40 Kc/s PRF, is shown in Fig.3.55(b). It is seen that the SNR is 24 db.

(d) Speech Signal Test.

Speech signals test, similar to the one given in Section 3.1, are also made. Listening tests showed that the connected speech is perfectly intelligible and the quality of reproduction is satisfactory at the PRFs of 60 and 40 Kc/s. The lowest PRF which can give some intelligible speech is 20 Kc/s. Here, some of the words are missed and there is a persistent crackling sound in the background which is sometimes annoying. The quality of transmission is of the order wire grade (Grade III).

A record of the same sentence, as in Section 3.1, is made through the 1/3 octave band filter described earlier. Fig.3.56 shows a record of (i) the original sentence, together with (ii) reproduced output at 40 Kc/s, and (iii) at 20 Kc/s PRF, at the centre frequency of 500 c/s (of the 1/3 octave filter). Similarly Figs.3.57 and 3.58 show the same three records at the centre frequencies of 1000 c/s and 3150 c/s respectively. It is seen that the reproduction at 20 Kc/s is unsatisfactory for all the centre frequencies in the audio band.

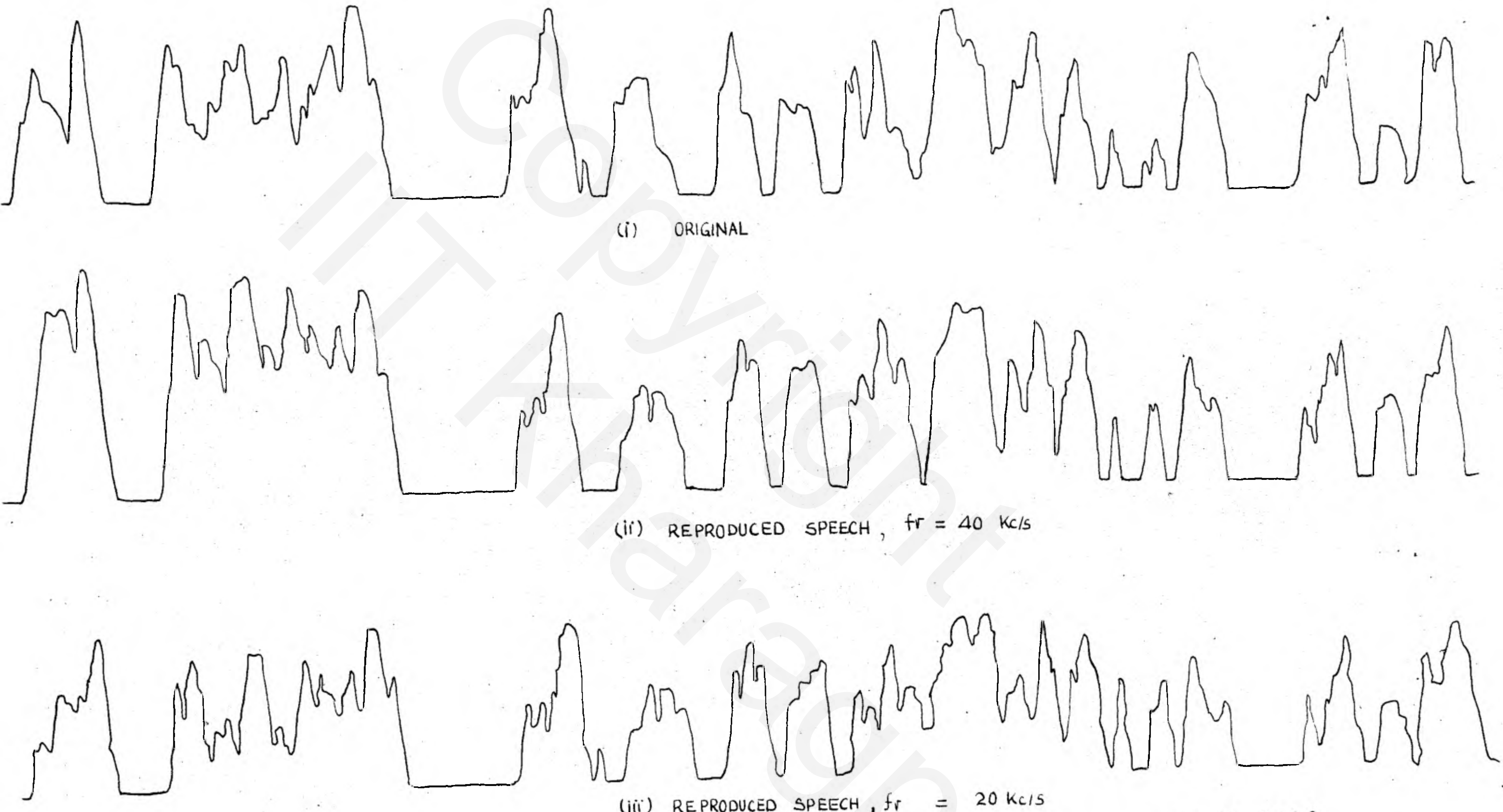


FIG. NO. 3·56

AMPLITUDE - TIME - FREQUENCY RECORDING OF THE 'SENTENCE'; CENTRE FREQUENCY 500 c/s

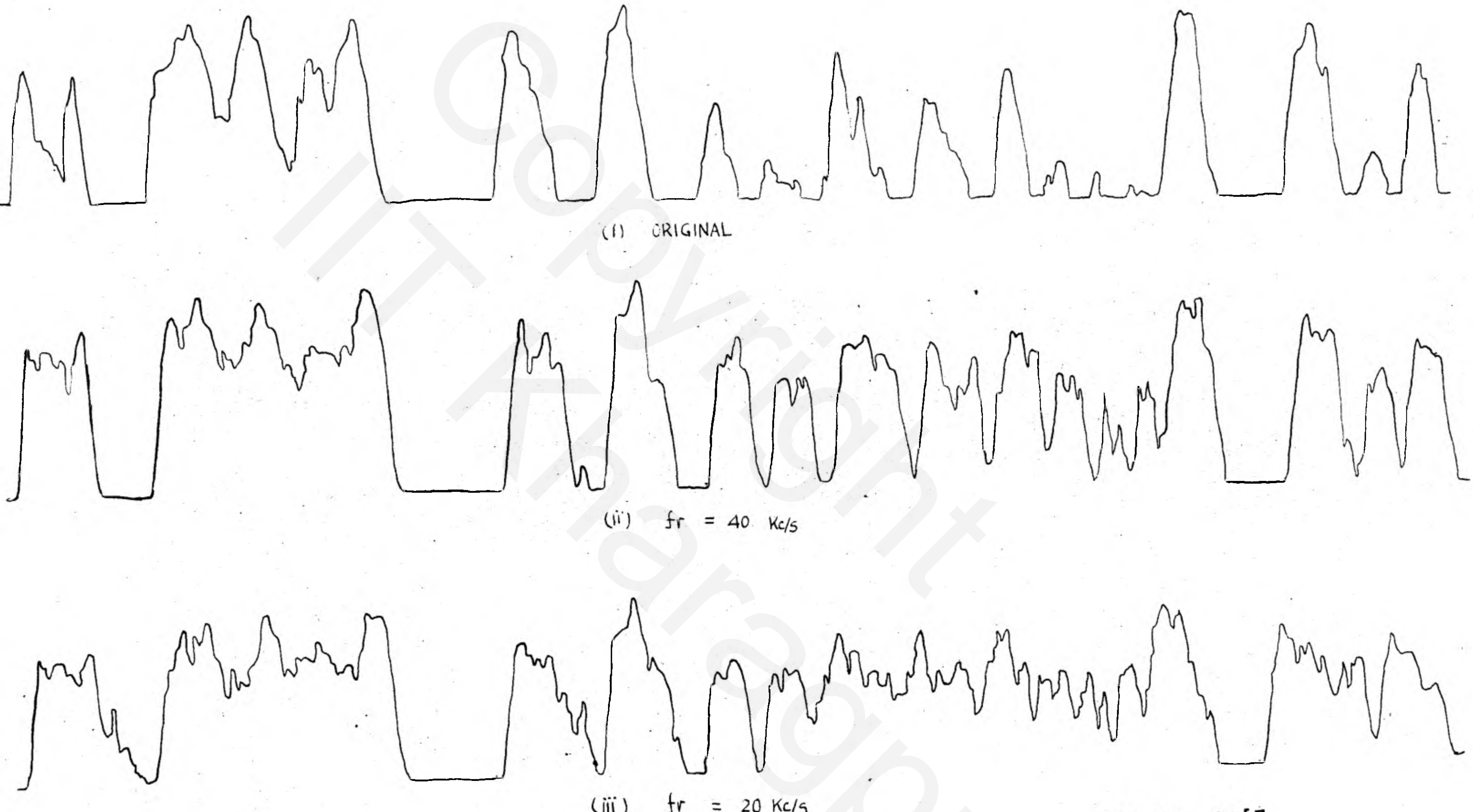
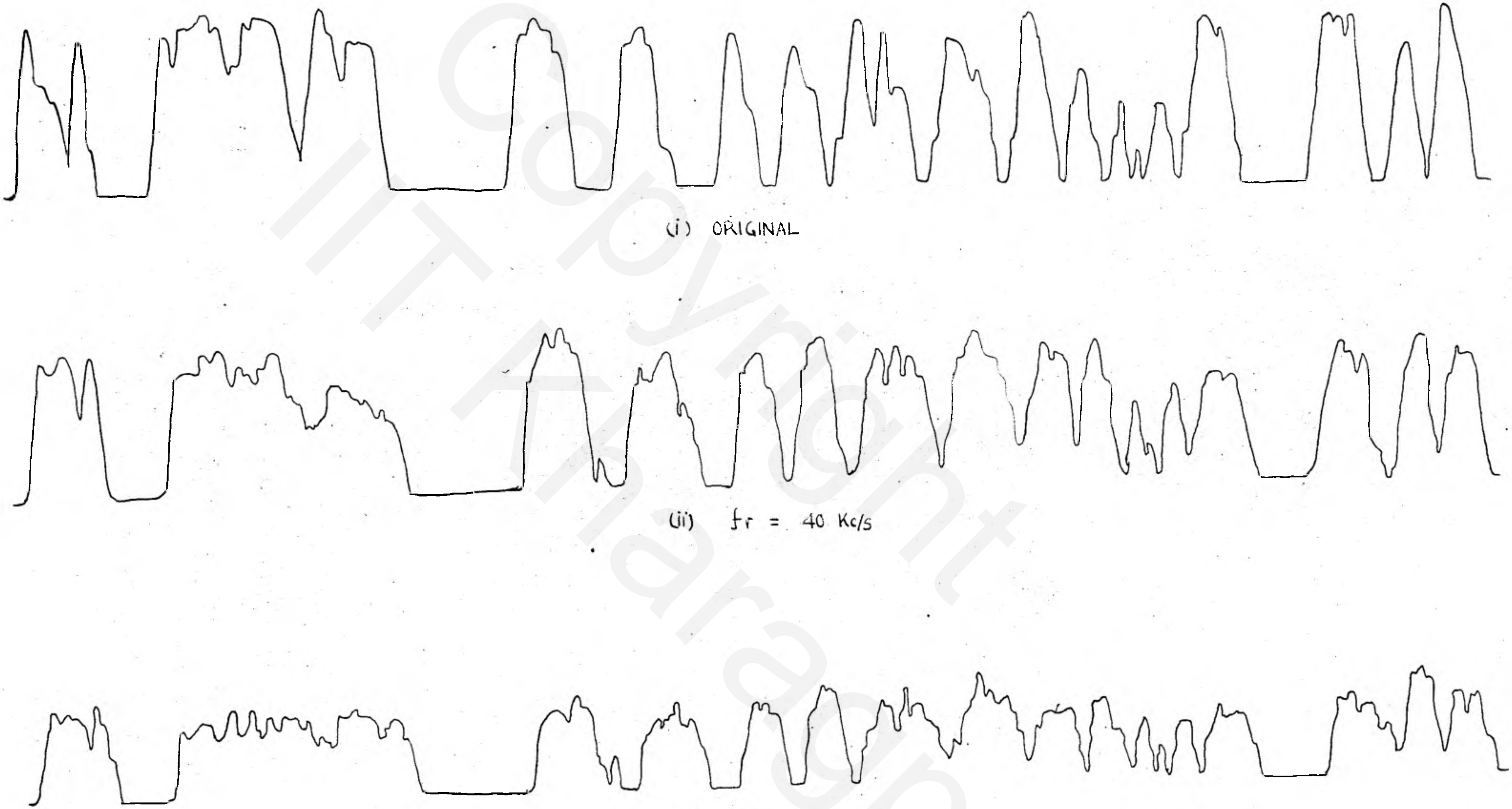


FIG. NO. 3·57

AMPLITUDE- TIME- FREQUENCY RECORDINGS OF THE 'SENTENCE'; CENTRE FREQUENCY 1000 c/s.



(iii) fr = 20 Kc/s

FIG. NO. 3·58

AMPLITUDE - TIME - FREQUENCY RECORDINGS OF THE 'SENTENCE'; CENTRE FREQUENCY 3150 C/S

3.2.2. SUMMARY OF THE RESULTS.

The Ternary SQ-PCM system without feedback gives an SNR of about 32 db for sinusoidal frequency of 1 Kc/s at a PRF of 40 Kc/s. The receiver uses a nonlinear integrator, which suppresses the strong 3rd harmonic present at the output when an ordinary integrator is used. The ordinary integrator, however, gives a peaky SNR of 40 db at an input level of -14 db at the same PRF. The SNR, for inputs below this input level, falls off very rapidly. The system suffers from the one major defect of nonlinear relation between the input and the output voltage of the system. The optimum SNR falls by about 10 db at 3 Kc/s, and by about 13 db at 400 c/s as compared to the SNR at 1 Kc/s. The frequency distortion in terms of a 5 db decrease in the output at 3 Kc/s as compared to the 1 Kc/s signal, is similar to the system with feedback and is not very objectionable. The variation of SNR with PRF is such that the best SNR at 60, 30 and 20 Kc/s PRF's are 44, 37 and 33 db, respectively, for an ordinary integrator, and 36, 29, 25 db, respectively, for the nonlinear integrator.

Since the system without feedback shows no particular linear relation between the input and the output voltage, it is suitable for signals having no, or little, amplitude variation. FM signals, therefore, are reproduced excellently when the input is adjusted to coincide with the input level for the best SNR. A half band wide FM signal has an SNR of 32, 30 and 26 db, for the PRF's of 60, 40 and 30 Kc/s, respectively, for the ordinary integrator, and 29, 26 and 21 db, respectively, for the nonlinear integrator. The system is also suitable for narrow band signals. The narrow band FM gives an SNR of 34, 32, 28 db, for the PRFs of 60, 40 and 30 Kc/s, respectively.

The noise signal performance of the system is just about acceptable. The SNR for the half band filter method, as well as for the band stop filter method, is 24 db at 40 Kc/s PRF. The results for 60 Kc/s and 30 Kc/s PRFs are 27 db and 18 db, respectively.

The listening tests on speech signals gave quite a satisfactory performance at 60 and 40 Kc/s PRF. The reproduction at 20 Kc/s is intelligible in parts and some words are missed occasionally, but it has an annoying background noise. A record of the sentence through the three centre frequencies of the 1/3 octave filter also show the same effect.

3.3. DISCUSSION.

The Ternary SQ-PCM system without feedback gives quite a satisfactory performance for some special class of input signals like FM. It, however, has a nonlinear relation between the input and the output voltages resulting in a saturation of the output. Complex signals like noise etc. have amplitude and frequency distortion and have a poorer SNR at the output. Two types of integrator at the receiver have been used, where it is found that a nonlinear integrator suppresses the 3rd harmonic normally produced at the output of an ordinary integrator. The overall SNR become lower with the nonlinear integrator in the receiver, and, therefore, it is not suitable for FM signals. The ordinary integrator, on the other hand, produces a peak in SNR at one input level. The amplitude of the FM signal is adjusted to this input value and the SNR obtained is comparable to that of the system with feedback.

The feedback from the output to the input of the transmitter, through a local receiver, improves the performance to a considerable extent. The input and the output voltage relation is linear and the SNR also falls linearly with a reduction of the input level, as expected (eq.2.30). The practical value of the best SNR comes out to be almost equal to the theoretical result obtained in Section 2.4. Any increase in the input level, above a certain reference input, brings in an overload distortion resulting in production of harmonics and a sharp decrease in the SNR. In effect, above the threshold, all the advantages of the feedback are lost, and the system characteristics become similar to the system without feedback. The overloading occurs at a lower input level for frequencies at the higher end of the signal band, as expected, because (refer to eq.2.30) the value of K being the same, the overloading is inversely proportional to the frequency of the signal. This leads to some frequency distortion, and it is felt that a decrease in the output of the order of 3 to 4 db at the higher frequency is not very serious for most of the applications and no equalising will be necessary.

Companding has been shown to lead to some definite advantages in increasing the dynamic range of the system, and the companding advantage is 22 db at a PRF of 40 Kc/s. The system without feedback, however, has already a saturated output and therefore, no compressor is necessary at the input. An expandor at the output does not produce any expansion because the contour of the output waveform does not match the contour required at the input of the expandor for its proper functioning. The practical results obtained here agree very well with the theoretical results given in Section 2.5.

The system with feedback reproduces the FM signals fairly well and a satisfactory SNR at the output is obtained for a Noise signal input. Listening tests on the system with feedback have shown that the quality of reproduction at 40 Kc/s PRF is very good, but at 20 Kc/s PRF, it is just tolerable. In the system without feedback the performance at 40 Kc/s PRF is satisfactory, but at lower PRFs, the performance is not good, and there is an uncomfortable crackling noise.

The system with feedback is particularly very stable against the variations of bias supply and reference voltage. The advantage of the feedback circuit is, in a sense, remarkable, and the SNR does not drop by more than 10 db for any variation of these parameters. The Ternary system changes over and becomes a Binary system if one of the trigger circuits starts malfunctioning and produces no output

The spectrum of the output pulses of the Ternary SQ-PCM system with feedback shows that, the major amount of audio signal energy is present as the sidebands of the fundamental and the harmonics of the PRF. In transmission through systems which do not pass the d.c. and the low frequencies well (e.g. cables), a suppression of the low frequencies is allowable without impairing the recoverable energy at the receiver. As against this, the Binary systems will require a good low frequency pass characteristic for an efficient transmission and thus add to the cost of the equipment.

Thus, it is seen that out of these two systems developed in the laboratory, the system without feedback is quite suitable for processing of narrow band signals or signals having very small amplitude variations. The speech is processed correctly because the speech signals are capable of being compressed and be still intelligible. The system with feedback, however, gives quite a satisfactory performance for all classes of signal inputs, and the system at 40 Kc/s PRF gives a performance equivalent to a 60 Kc/s Binary AQ-PCM system in almost all respects. The circuitry is quite simple and many channels can be multiplexed with a common instantaneous compandor and a common coder, as will be shown in Section 5.5. It is felt that the overall cost of the complete system will be quite economical and provide a good grade of service. A Binary SQ-PCM system has also been developed in the laboratory here and a comparison of the Ternary SQ-PCM system with the Binary SQ-PCM system will be given in Section 4.3.

CHAPTER IV.

CHARACTERISTICS OF THE BINARY SQ-PCM.

4.0 GENERAL.

The Ternary SQ-PCM system with feedback, as seen in the previous Chapter, gives fairly good results at a lower PRF, and it is felt that a Binary model of the system employing a quantization of the slopes of the waveform into '0' and '1' pulses will lead to a better approximation than that obtained with the Δ-M and Δ-ΣM systems. The use of an optimum threshold in the quantizer leads to a minimisation of the errors in a manner similar to that in the Ternary system (Section 2.4). In Δ-M and Δ-ΣM systems, either a positive or a negative pulse (depending on the polarity of the error waveform at the sampling instants) after integration, tries to match approximately the corresponding signal waveform. In the Binary SQ-PCM system, however, a special pulse shaping network in the feedback circuit matches the negative slopes of the signal by the cumulative negative slopes of the shaped pulses. The quantizer has a threshold of operation, and combined with the feedback network, the approximation here has been shown to give a better SNR, and less frequency distortion in a practical system.

The dynamic range of input in the system has been found to be small, and an instantaneous compandor has been used successfully to increase it. Actually, Zetterberg[11] has shown that "there is little to be gained from the use of nonlinear compandors with Δ-M ." But the experiments in the laboratory show that the use of a compandor in SQ-PCM helps considerably in having a uniform SNR for the practical input ranges. Of course, due to the limited bandwidth, the input-output voltage characteristics do not come out to be so linear as in the Ternary SQ-PCM system.

The test signals for estimating the performance of the Binary SQ-PCM system are the same as given in Section 3.0, i.e. sinusoidal, FM, noise and speech signals.

Section 4.1 deals with the details of the feedback network and the actual circuit diagram of the laboratory model of the Binary SQ-PCM system. Section 4.2.0 gives the performance characteristics of the system together with its stability, Section 4.2.1 gives the quantizing noise spectrum, and Section 4.2.2 summarises the results. In Section 4.3, a discussion and comparison of the performance of the Binary SQ-PCM with that of the Ternary SQ-PCM is given, and the improvement of the Ternary code over the Binary code in a Uni-Digit Approximation shown.

4.1. CIRCUIT DETAILS OF THE BINARY SQ-PCM SYSTEM.

A functional diagram of the Binary SQ-PCM has been given in Fig.2.15 and a complete block schematic diagram of the system is shown in Fig.4.1. The input signal is fed through an amplifier and a cathode follower to the difference circuit, to which is also given the approximated signal obtained through the feedback circuit.

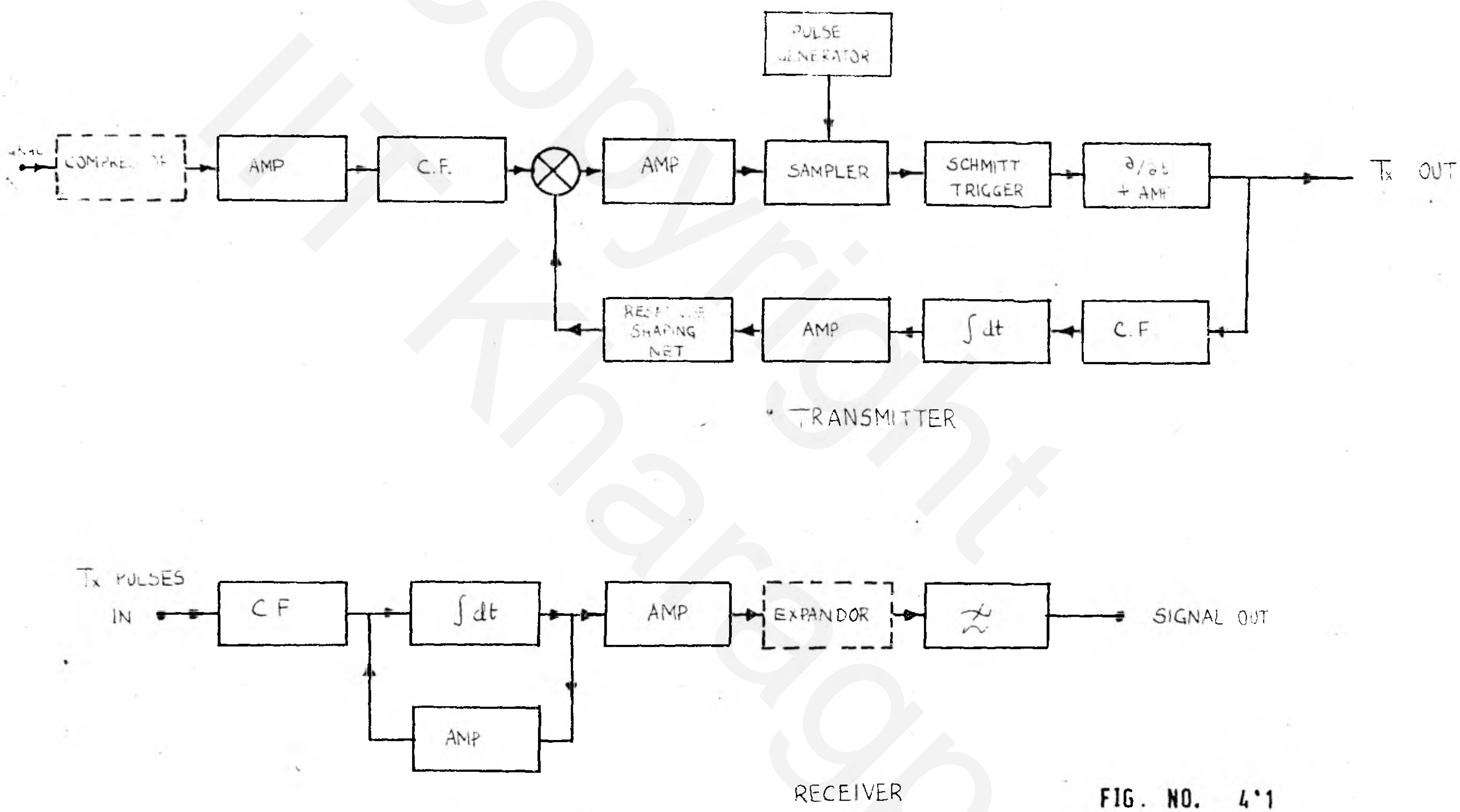


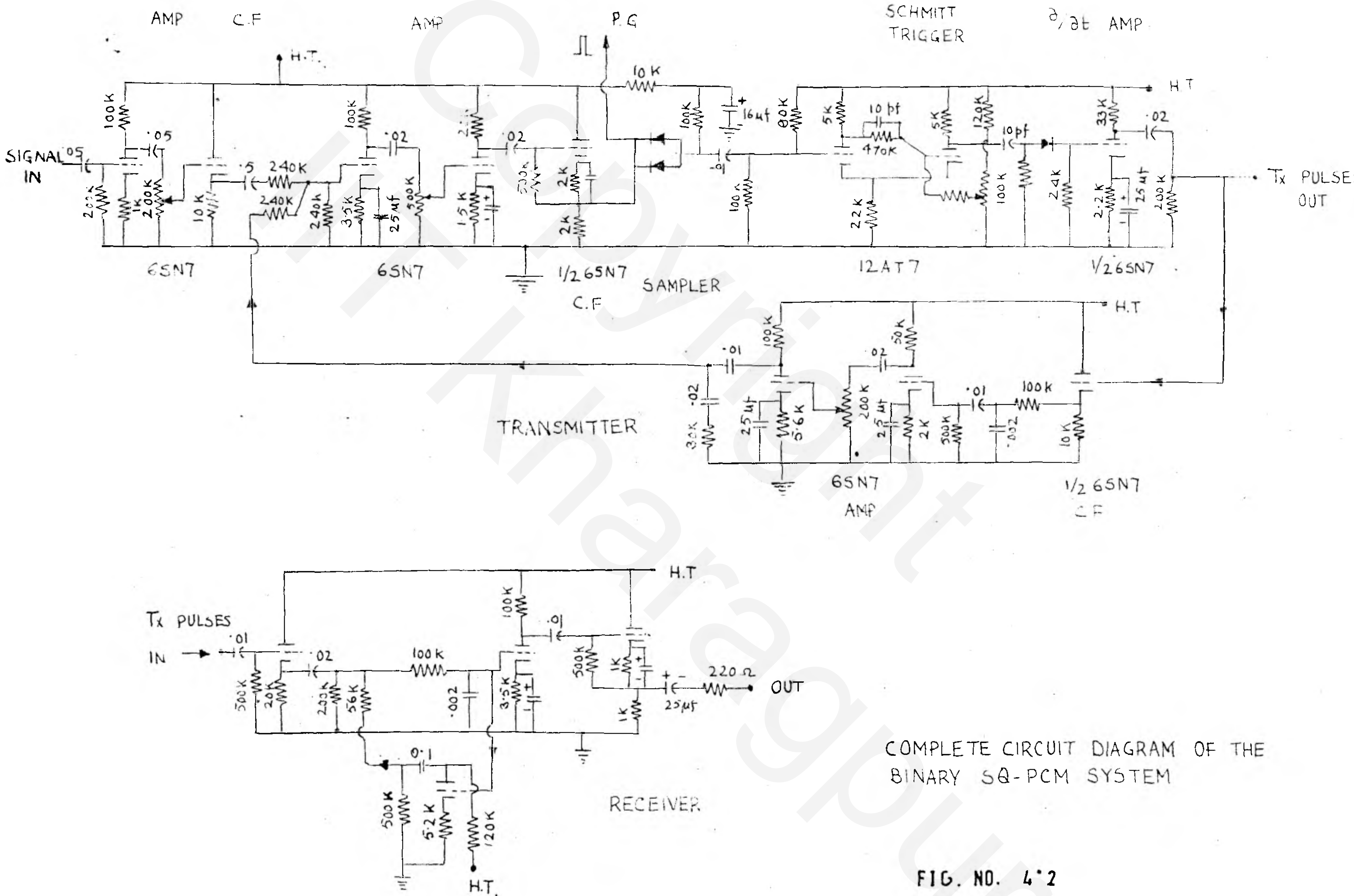
FIG. NO. 4.1

COMPLETE BLOCK SCHEMATIC DIAGRAM OF THE BINARY SQ -PCM SYSTEM

The error signal is amplified and sampled at the desired PRF by the sampling circuit with the Uni-directional pulses from the pulse generator, described in 3.1.0. The error samples are quantized into 1/0 pulses by the Schmitt trigger with a reference bias so adjusted that error amplitudes larger than the threshold are converted into a positive unit pulse, and error amplitudes less than the threshold produce no pulse. The output of the Schmitt trigger, is differentiated and amplified (to remove any width variations) to form the output of the transmitter. A part of the output is taken through a cathode follower, an integrator with a break point at 800 c/s, two stages of amplification and a pulse shaping network to form the feedback circuit. The receiver consists of a cathode follower, followed by a similar integrator and a band pass filter (300 c/s 3.4 Kc/s passband). The receiver circuit is similar to the receiver in the feedback loop, and has a local feedback loop across the integrator, which reduces the hum and noise at lower frequencies, and equalises the frequency response of f(t). The complete circuit diagram of the Binary-SQ-PCM system is given in Fig.4.2. The compressor and expandor of Fig.3.5 are shown dotted in the Fig.4.1, and the circuit performance can be tested with or without them.

The amplitude vs. frequency characteristic of the feedback network is shown in Fig.4.3(a). An approximate transfer function of the network for this frequency response can be derived as,

$$G(s) = 10^4 \frac{s^2}{(s+0.16\times10^4)^2(s+10^4)}$$



COMPLETE CIRCUIT DIAGRAM OF THE BINARY SQ-PCM SYSTEM

FIG. NO. 4·2

and, hence, the impulse response is given as :

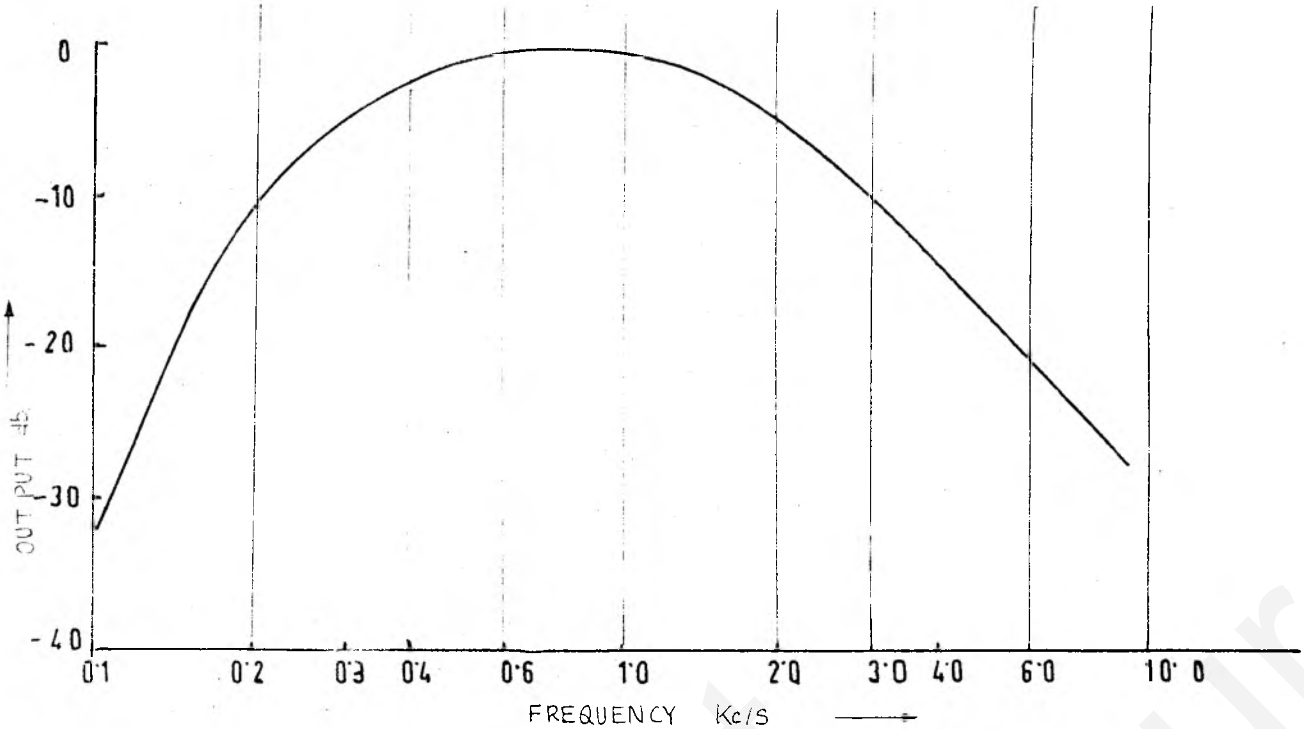
$$f(t/10^4) = 1.42\, e^{-t} + 0.03t\, e^{-0.16t} - 0.42\, e^{-0.16t}$$

The response of the feedback network (having the above impulse response) to unit pulse of 5 μsec. duration is shown in Fig.4.3(b). An actual reconstructed signal for $f_m = 3.30$ Kc/s and PRF = 40 Kc/s, taken from an oscillogram, is shown in Fig.4.3(c). The effect of the optimum threshold of the quantizer and the wider spread in the pulse response is also seen in the figure, and this agrees fairly well with the assumptions made in Section 2.3. As seen in Fig.4.3(c), the signal build-up is such that the peak output flattens after 5 or 6 pulses and the peak-to-peak amplitude of $f(t)$ is approximately equal to $3V_e$, a value expected from eqn. 2.31(a). It is shown there that the peak-to-peak signal $S_v = V_e \cdot f_r/4f_m$ and for a $f_r = 40$ Kc/s and $f_m = 3.3$ Kc/s, the value of S_v is about $3V_e$. To avoid the overloading of the quantizer, the rms value of the input signal, then, has to be less than $0.1\, V_e\, f_r/f_m$. The results have been obtained for PRF's of 30 - 80 Kc/s, but the PRF of 40 - 60 Kc/s has been optimized as this range is equivalent to that of the 5-7 digit AQ-PCM mostly used in practice.

4.2.0 SYSTEM CHARACTERISTICS.

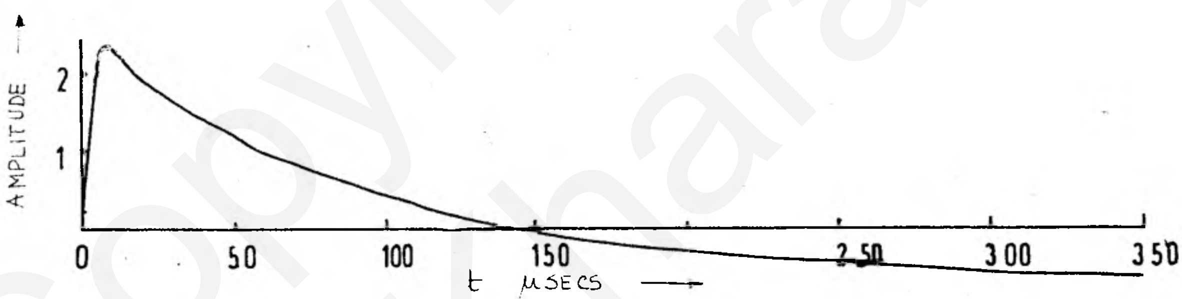
(a) Sinusoidal Signal Test.

The experimental set up for the measurement of the SNR etc., with the sinusoidal signal input, is similar to one given in Fig.3.7. First of all, the compressor and the matching expandor are not used,



AMPLITUDE Vs. FRREQUENCY
CHARACTERSTICS OF THE FEEDBACK NETWORK

FIG. NO. 4·3(a)



RESPONSE OF THE FEEDBACK NETWORK TO
A 5 μSEC. PULSE

FIG. NO. 4·3(b)

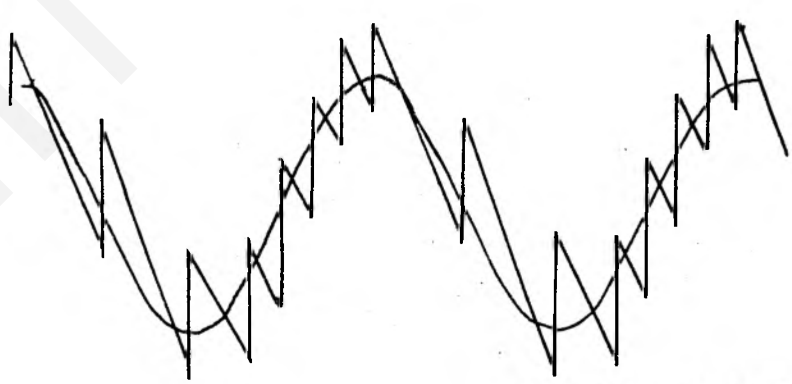
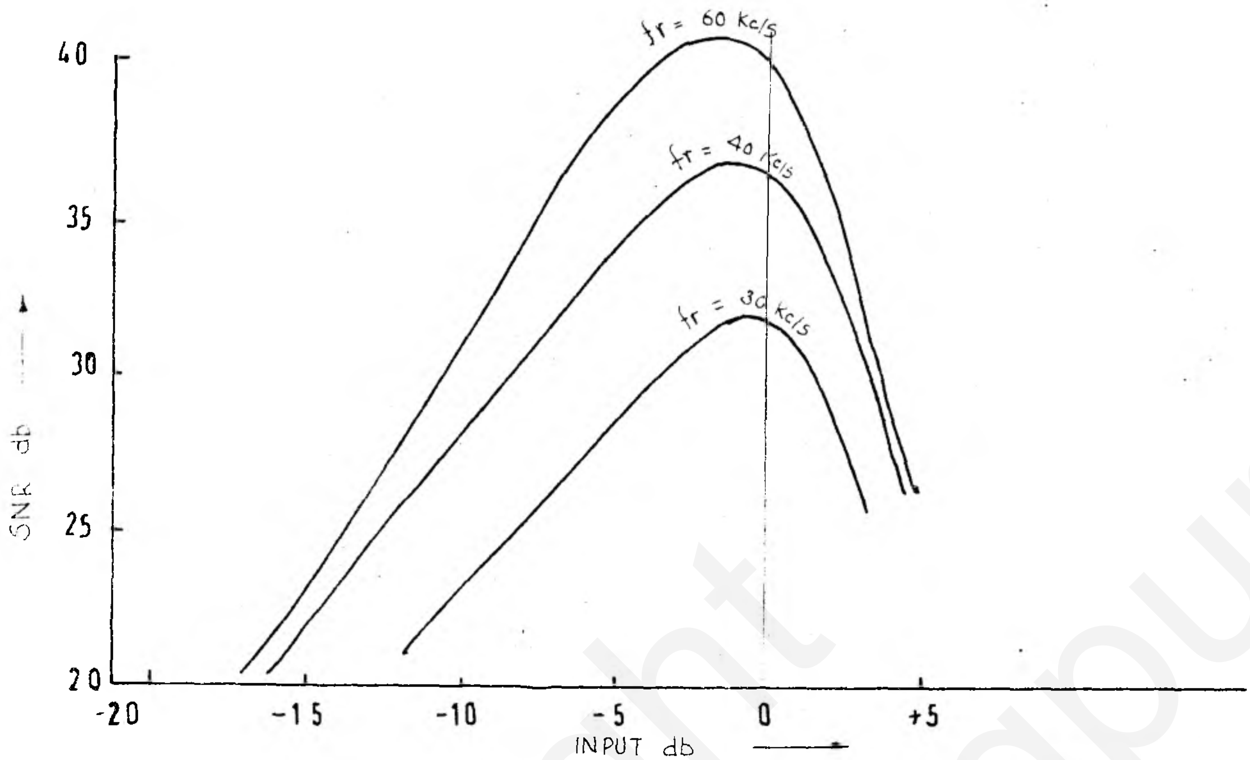


FIG. NO. 4·3(c)

AN ACTUAL RECONSTRUCTED SIGNAL, f_m = 3·30 Kc/s f_r = 40 Kc/s and the system is tested with signals in the band of 300 c/s to 3.4 Kc/s frequency. The variation of SNR with input signal level is shown in Fig.4.4, for three different PRF's of 60, 40 and 30 Kc/s. The signal frequency is 1 Kc/s and the best SNR's are 41, 37 and 32 db, respectively. The SNR is above 25 db for a dynamic range of 20, 17.5, and 12 db for the three PRF's, respectively. The SNR falls sharply above the optimum input level of 0 db because of the overloading effect discussed in Section 2.4. If the input level is increased beyond the reference input, harmonics will be produced due to clipping, as explained in Section 3.1.1. Fig.4.5 shows the experimental values of the amplitudes of the 2nd and 3rd harmonic for such an increase in the input level. A +5 db increase in the signal level results in a -33 db 2nd harmonic and a -35 db 3rd harmonic level below the fundamental. A further increase to 7.5 db input results in a -22 db 3rd harmonic and -32 db 2nd harmonic as compared to the fundamental.

As given in Section 3.1.1, periodic noise in the form of beat notes occur for discrete values of the signal frequencies and, Fig. 4.6 shows the nature of variation of the periodic noise in the SQ-PCM system, where the periodic noise output below the fundamental is plotted for an input signal frequency of 2700 c/s. The frequencies of the periodic components are 650 c/s and 2 Kc/s in this particular measurement. The points shown as circles and crosses are the measured values and the arrows indicate the input levels for which the periodic component is negligibly small; the continuous curves are extrapolated from this.

Fig.4.7 shows the variation of the SNR with input level for the different signal frequencies of 400 c/s and 3 Kc/s. These



VARIATION OF SNR WITH INPUT LEVEL, fm = 1 Kc/s

FIG. NO. 4·4

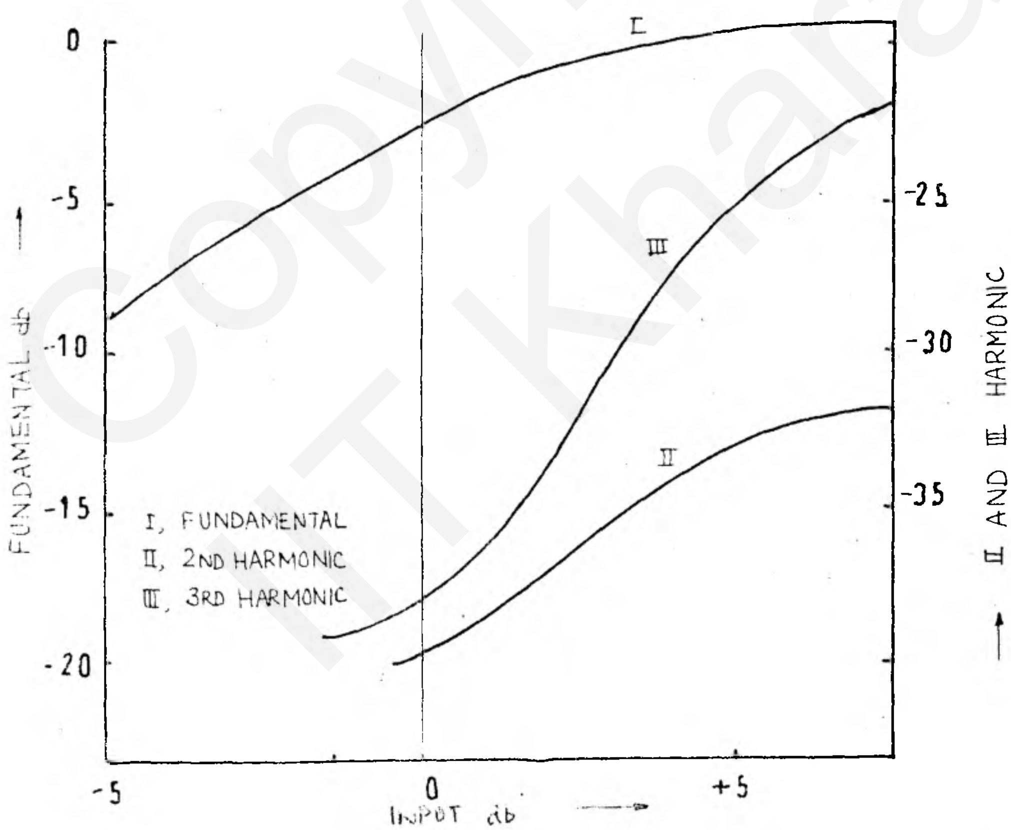


FIG. NO. 4·5

AMPLITUDE OF THE HORMONIC DUE TO OVERLOAD

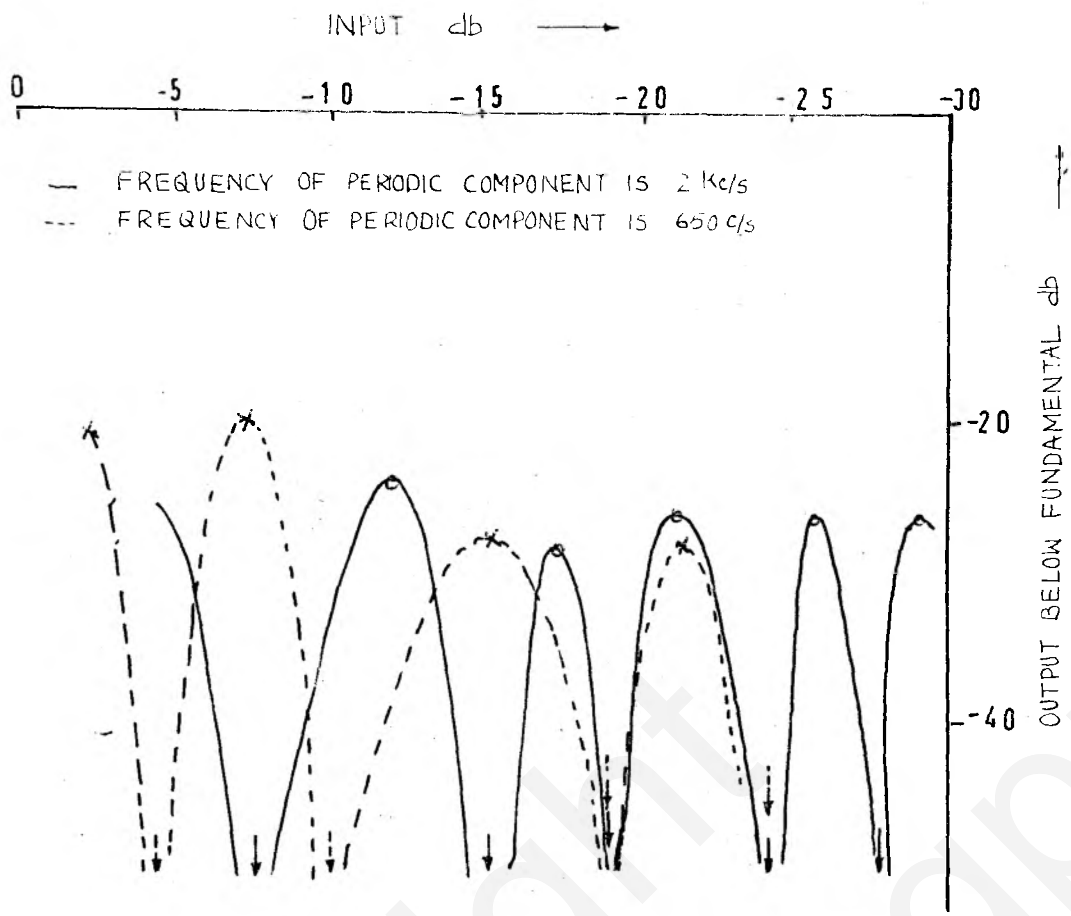
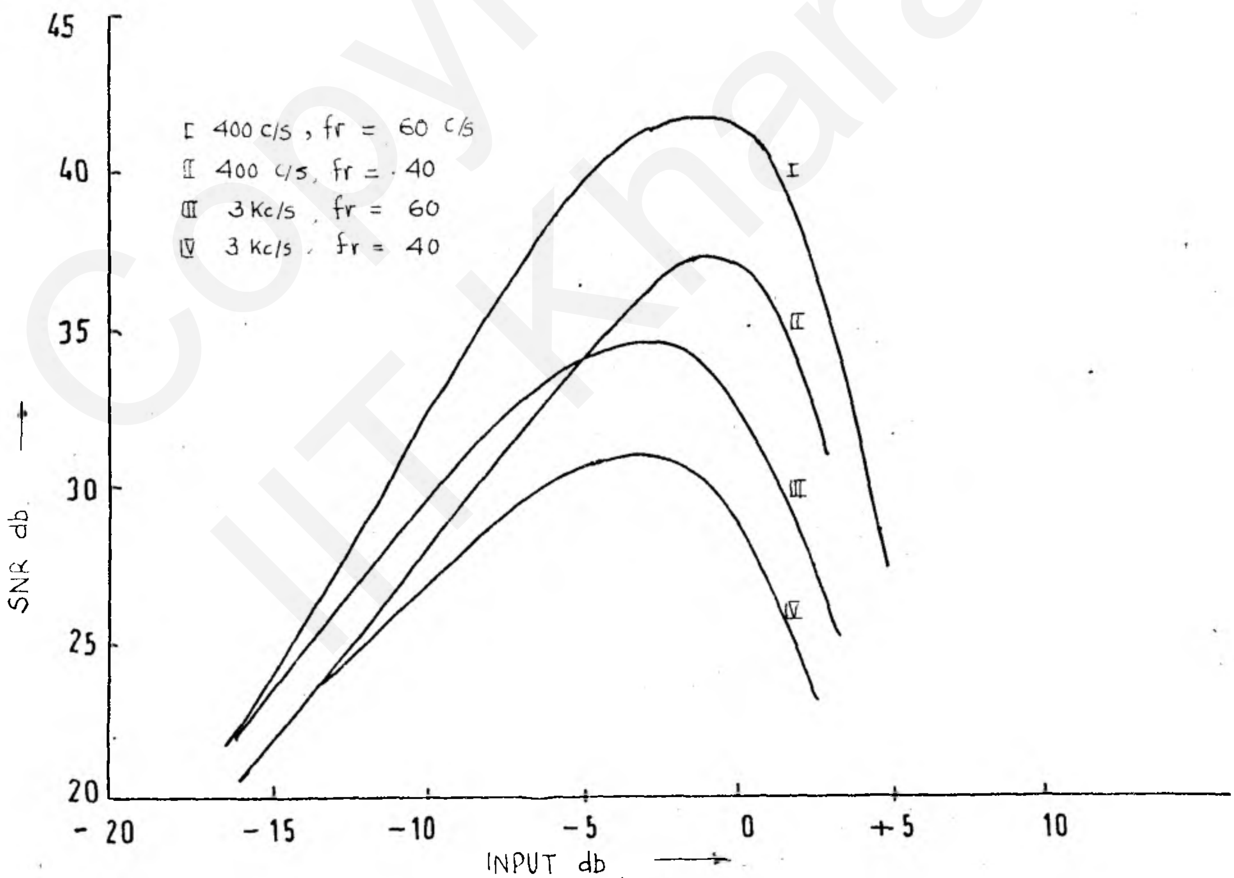


FIG. NO. 4·6

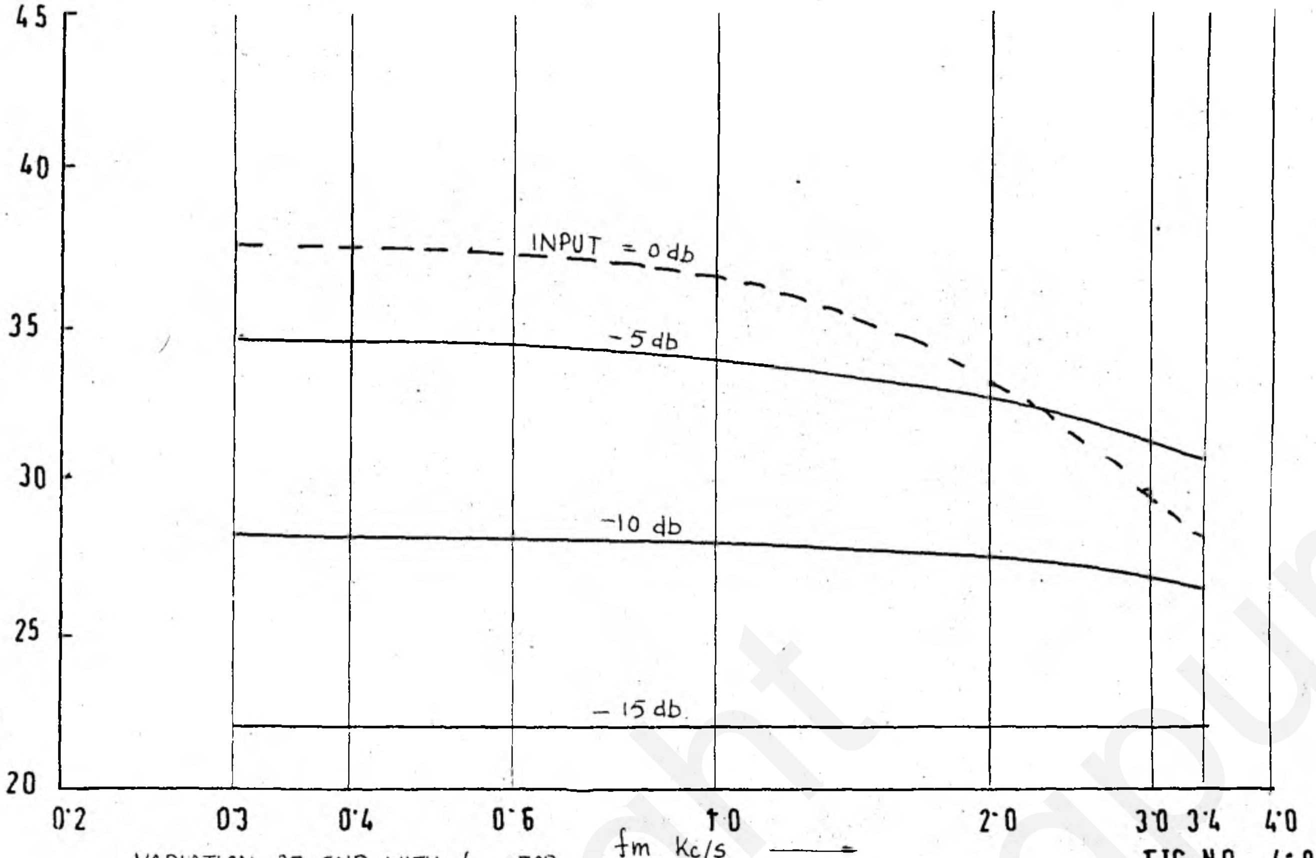


VARIATION OF SNR WITH INPUT LEVEL FOR DIFFERENT fm

FIG. NO. 4·7

curves have been drawn for the two PRF's of 60 and 40 Kc/s, and it is seen that the SNR for the 3 Kc/s signal deteriorates by about 8 db (at 0 db input level), as compared with the SNR for the 1 Kc/s signal. At 400 c/s, the SNR is almost the same or about one db better than that for the 1 Kc/s signal. In Fig.4.8, the variation of SNR with signal frequency is shown for different input levels at a PRF of 40 Kc/s. It is seen that for the -5 db input level, the SNR at 3 Kc/s falls by about 3 db only, as compared to the SNR at 1 Kc/s, and at still lower inputs the SNR is almost constant with frequency. As has been explained for the Ternary system (Section 3.1.1), this is a direct result of the decrease in the output for 3 Kc/s (at the 0 db input level) and, thus, results in a saturation at the higher frequency. The variation of output levels with the input signal frequency for different input levels has been shown in Fig.4.9, where it is seen that, the output at 3 Kc/s falls by about 5 db as compared to the output at 1 Kc/s. The output for frequencies between 400 c/s and 1 Kc/s is almost constant. All the figures (Fig.4.7, 4.8 and 4.9), therefore, show the effect of the overloading in terms of poorer results for the higher frequency signals, at comparatively higher input levels. The input vs. output voltage relation is shown in Fig.4.10 for the 1 Kc/s and the 3 Kc/s signal.

The variation of SNR with PRF is shown in Fig.4.11 for the optimum input. Referring to eq.(2.33), the SNR is dependent upon PRF raised to the power 3/2 i.e. the SNR increases by 9 db for an octave change of PRF. As seen from the figure the slope of the



VARIATION OF SNR WITH fm FOR DIFFERENT INPUT LEVELS, fr = 40 Kc/s

FIG. NO. 4·8

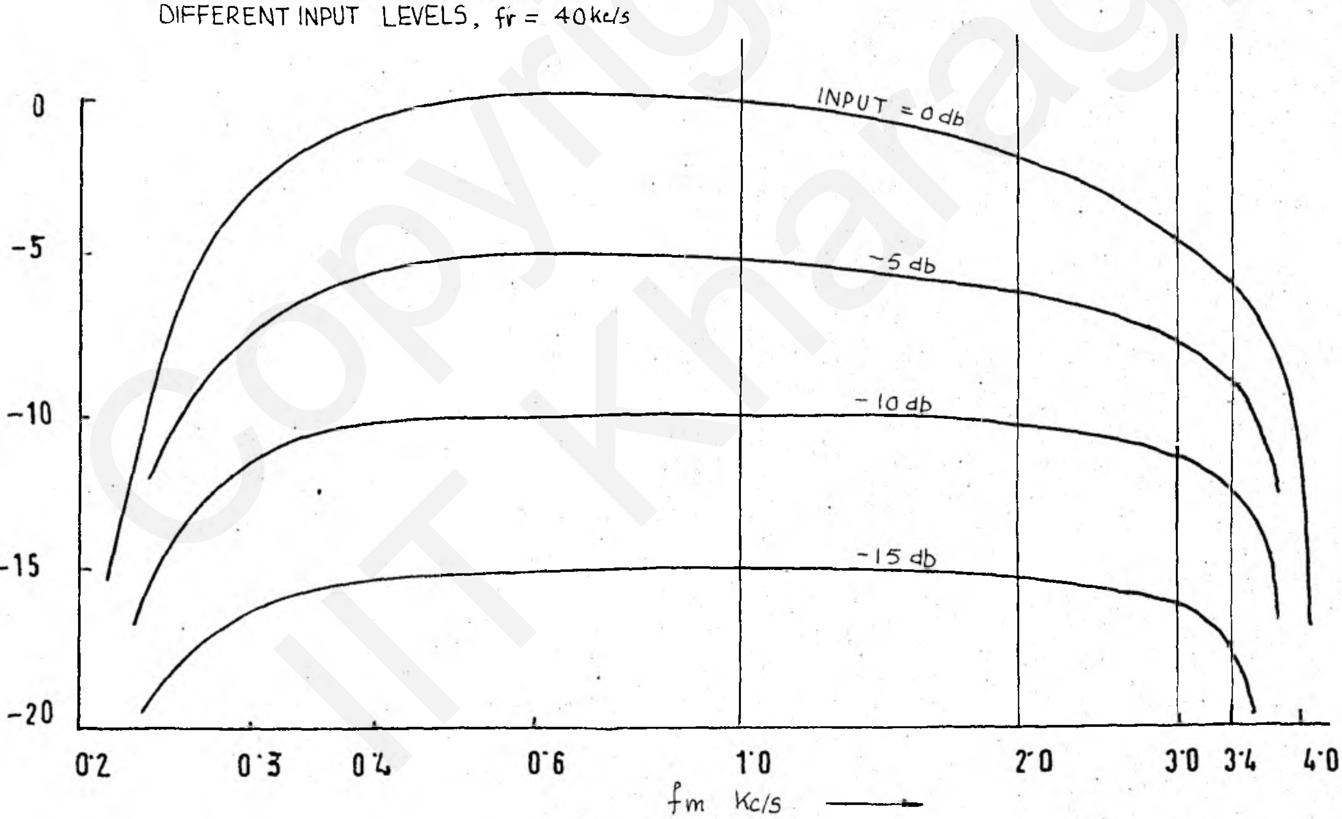
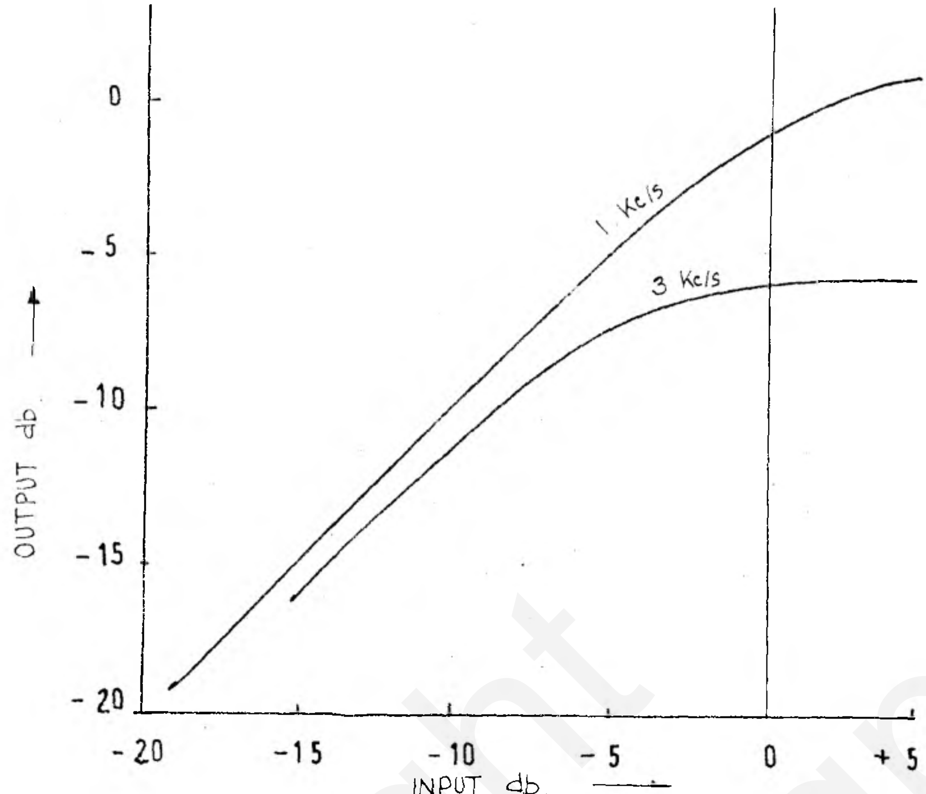


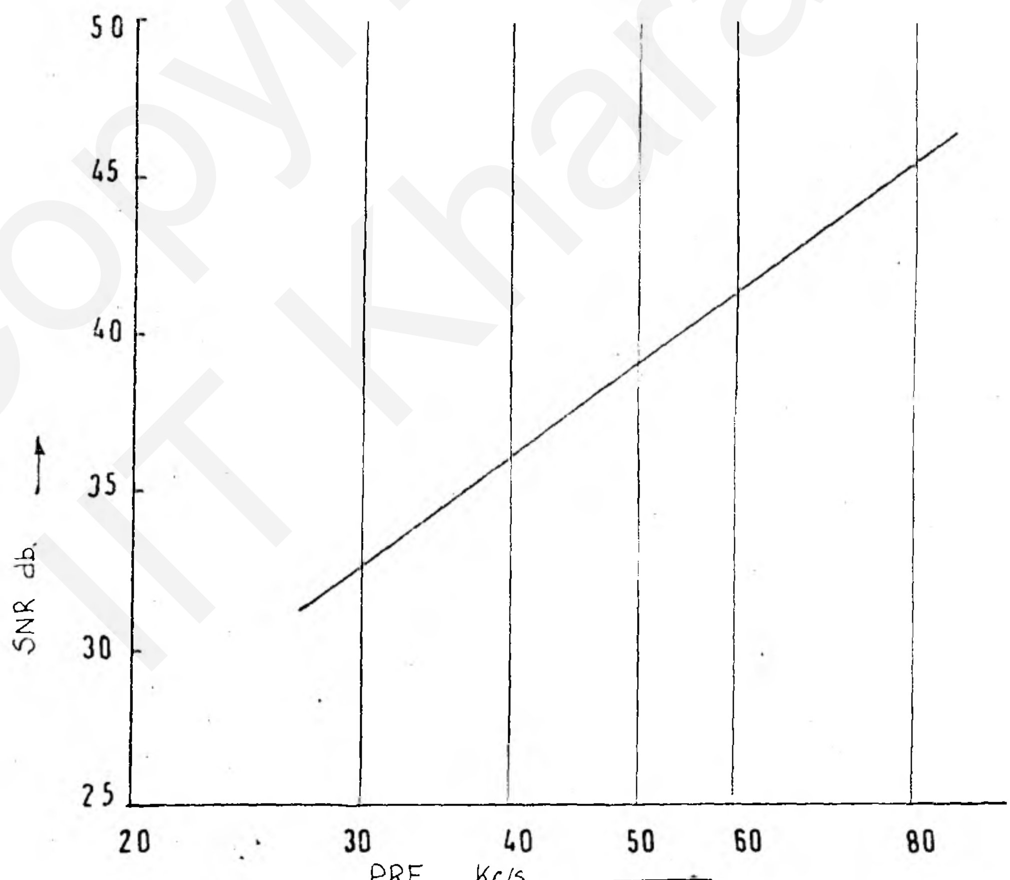
FIG. NO. 4·9

VARIATION OF OUTPUT WITH fm FOR DIFFERENT INPUT LEVELS, fr = 40 Kc/s



RELATION OF INPUT VS. OUTPUT VOLTAGE
FOR fm = 1 kc/s AND 3 kc/s

FIG. NO. 4·10



VARIATION OF SNR WITH PRF

FIG. NO. 4·11

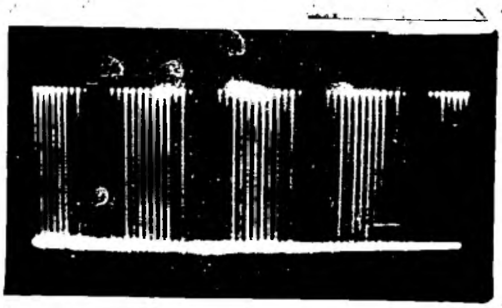
experimental curve is about 8.6 db/octave.

A photograph of the output pulses of the transmitter in the Binary SQ-PCM system is shown in Fig.4.12(a), and a photograph of the reconstructed signals at the output of the receiver (before the filter) is shown in Fig.4.12(b) and (c), for two signal input levels. It is clearly seen [from Fig.4.12(b) and (c)] that, as the signal level is reduced, the decrease of the cumulative slope is more and an extra pulse is generated to reduce the difference between the original and the approximated signal. Large error will be present at the output of the system if the cumulative slope is less than the negative slope of the signal, and the signal waveform is only reproduced by a block of +1 pulses followed by a gap when no pulses are present. Evidently, such a situation will be obtained when, (i) either the feedback voltage-amplifier gain is less or, (ii) the input signal is more than the 0 db reference, both resulting in an overload. As the input voltage is reduced, more and more pulses will appear where previously there was a gap and no pulses, and also, some of the pulses in the block of +1 pulses will be missing. Therefore, for the input level of about -5 db below the reference, the occurrence of the pulses is random and there is no continuous sequence of either +1 or 0 pulses. A photograph of the original and reproduced triangular waveform is shown in Fig.4.13, at a PRF of 40 Kc/s, and it is seen that the reproduction is fairly good.

The instantaneous compandor is now included in the system and similar tests, as given above, are performed. Fig.4.14 shows

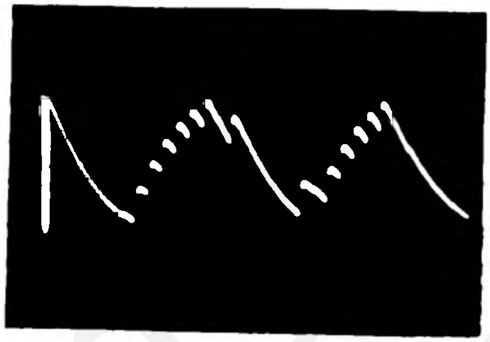
FIG 4.12

(a)



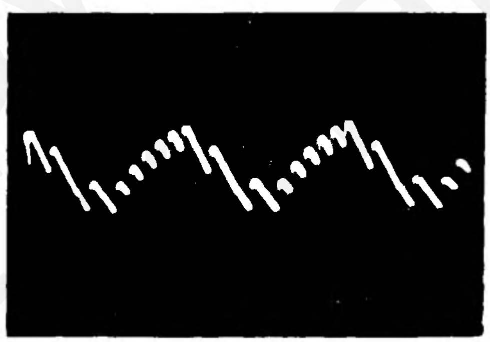
TRANSMITTER PULSES OF BINARY SQ – PCM

(b)



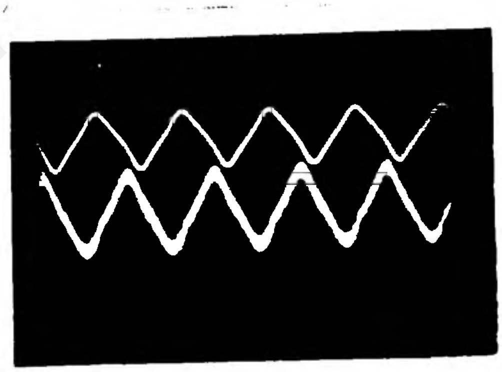
RECONSTRUCTED SIGNAL AT THE RECEIVER

(c)



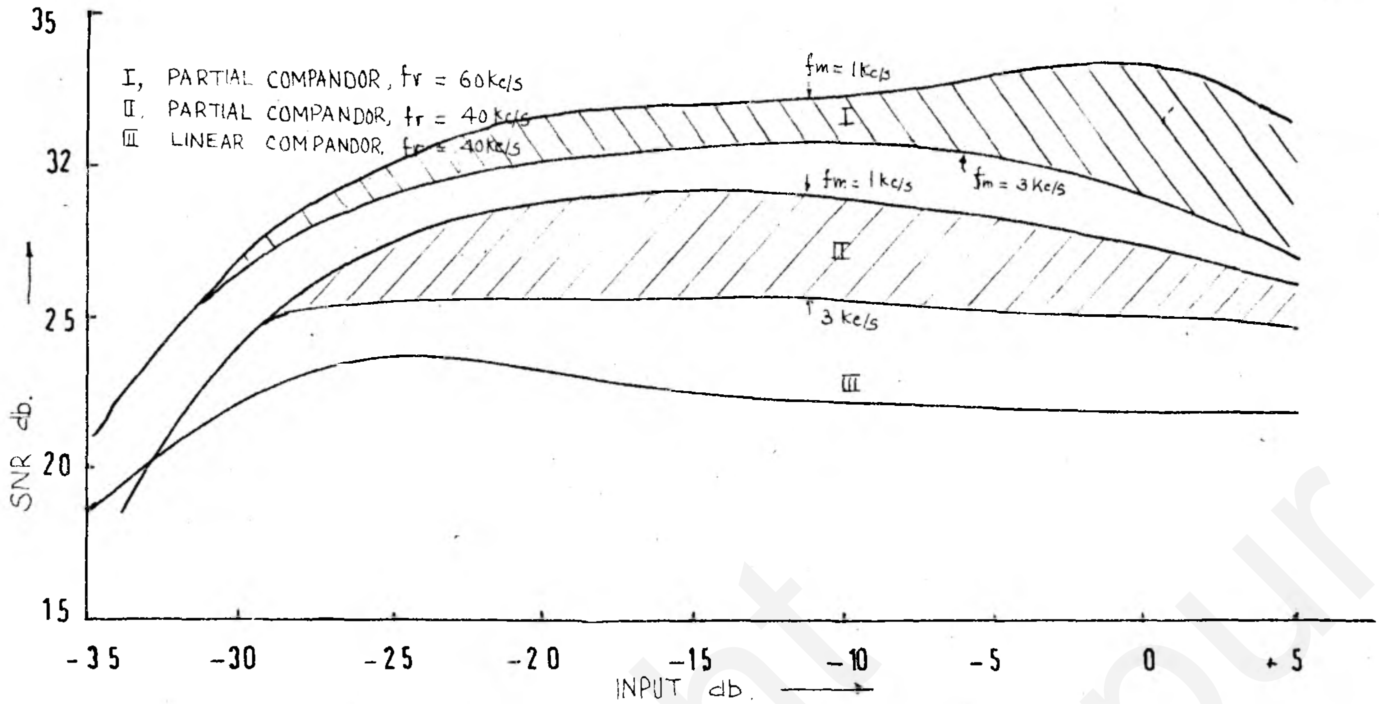
SAME AS (b) FOR A DIFFERENT INPUT LEVEL

FIG 4.13



TRIANGULAR INPUT (TOP) AND RECONSTRUCTED OUTPUT (BOTTOM) the variation of SNR with input signal level for the compandors at 60 and 40 Kc/s PRF. The shaded area 1 represents the SNR vs. input voltage of the Partial compandor at 60 Kc/s PRF, for the signal frequencies in the range of the audio band. The upper curve is for the 1 Kc/s signal and the lower curve is for the 3 Kc/s signal frequency. It is seen that the SNR is above 25 db for a dynamic range of 39 db and the best SNR is 32 db. Similarly, the compandor at 40 Kc/s PRF (shaded area II) has a dynamic range of 35 db for an SNR above 25 db, and the best SNR is 29 db. The companding advantage, as has been explained earlier in Section 3.1.1, is 20 db. The linear compandor, on the other hand, does not work very satisfactorily in the Binary Sq-PCM system, presumably because of the larger bandwidth required. An SNR vs. input variation for a linear compandor at 40 Kc/s is, however, shown in the same figure as curve III, where it is seen that the SNR is 22 db. The input-output voltage relation of the system, with the compandor, is shown in Fig.4.15. The curve has been drawn for the Partial compandor and shows the amount of compression still present at the output of the expandor because of the insufficient expansion. Because of the extra saturation or overloading effect obtained at the higher audio frequencies, the residual compression at 3 Kc/s is more than 1 Kc/s, as shown in the figure.

The output voltage vs. frequency of the input signal has been shown in Fig.4.16. The output at 3 Kc/s falls by about 13 db as compared to 1 Kc/s at 0 db input level. It has been shown that for the non-compandored system, the output at 3 Kc/s changes by 2.0 db for a 5 db change at the input, as compared to 1 Kc/s



VARIATION OF SNR WITH INPUT LEVEL, WITH COMPANDOR INCLUDED

FIG. NO. 4·14

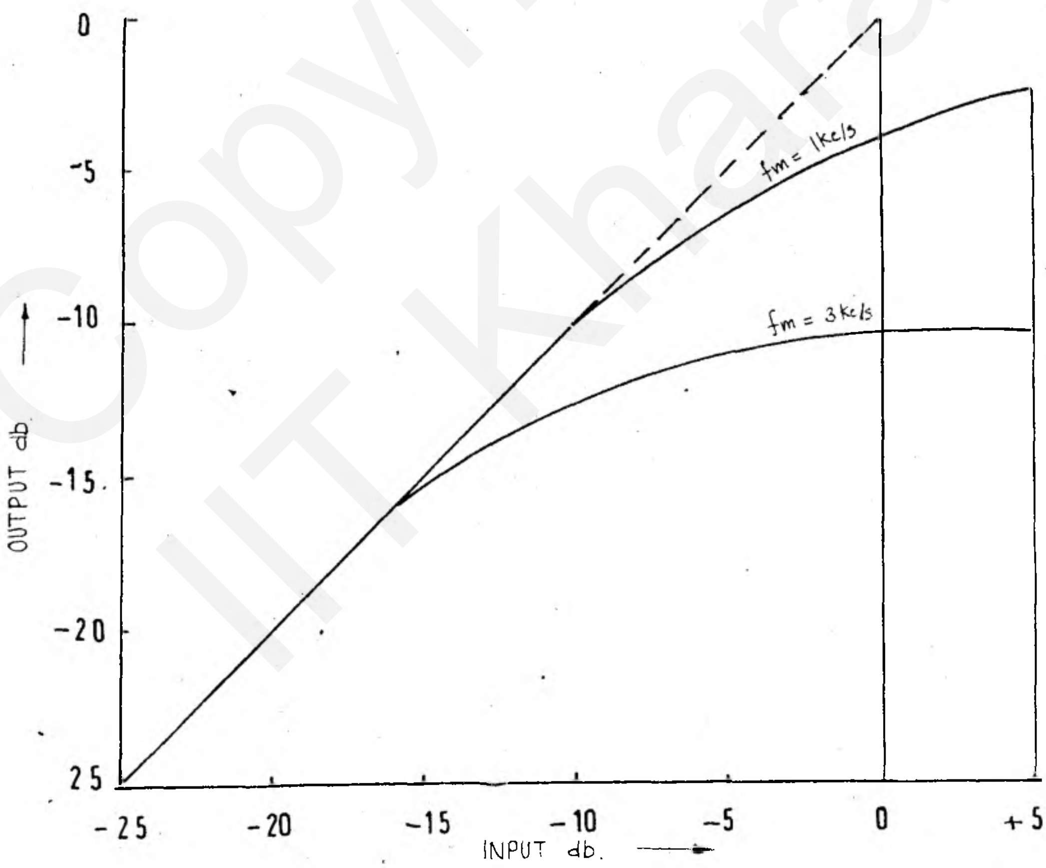


FIG. NO. 4·15

RELATION OF INPUT-OUTPUT VOLTAGE,
WITH PARTIAL COMPANDOR INCLUDED; fr = 40kc/s

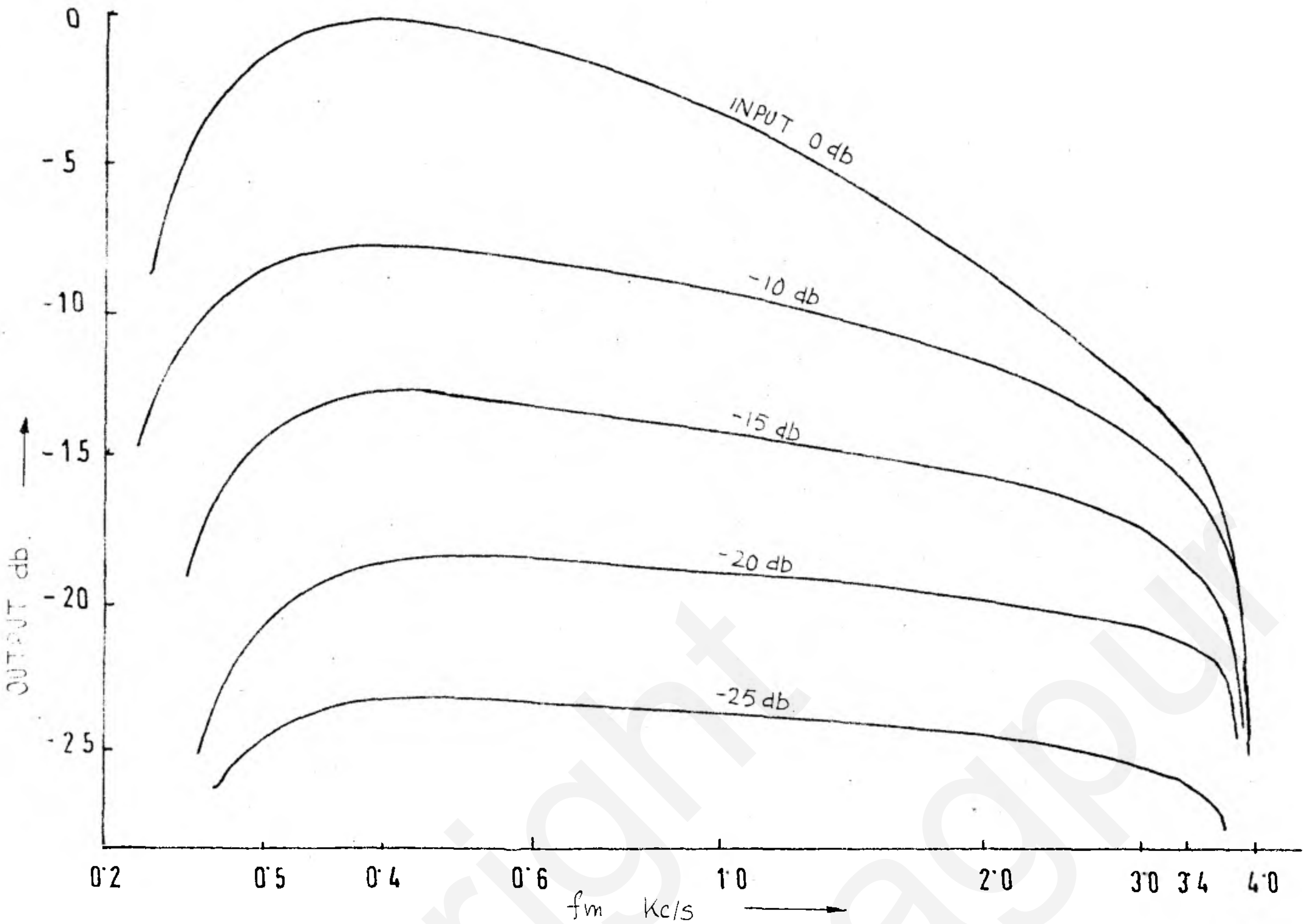
(Fig.4.8). The insensitive zone, where the feedback does not operate, of the 3 Kc/s signal is, therefore, about 3 db. Since the compression changes a 12 db input change to a 3 db change, this 3 db compressed and insensitive range at 3 Kc/s will therefore extend upto approximately 10 - 12 db when the compandor is used. For input levels below about -10 db, the system with compandor shows a tolerable frequency distortion. The curves in Fig.4.16 have been drawn only for 40 Kc/s PRF, but similar results are obtained at the PRF of 60 Kc/s.

(b) <u>FM Signals Test.</u>

The technique of measuring the response to the FM signals has been given in Section 3.1.1 (Fig.3.2.1). The SNR for the FM signals input is 32 db and 28 db, for the PRF's of 60 and 40 Kc/s, respectively, as shown in Fig.4.17. This SNR is the average of the measurements made when the signal is in Band I and then in Band II, and the noise is observed in Band II and Band I, respectively. For the narrow band FM, occupying only a band of 800 - 1700 c/s, the average SNR is 34 and 30 db, at the 60 and 40 Kc/s PRF, respectively. The SNR for the other PRF's follows the same general pattern.

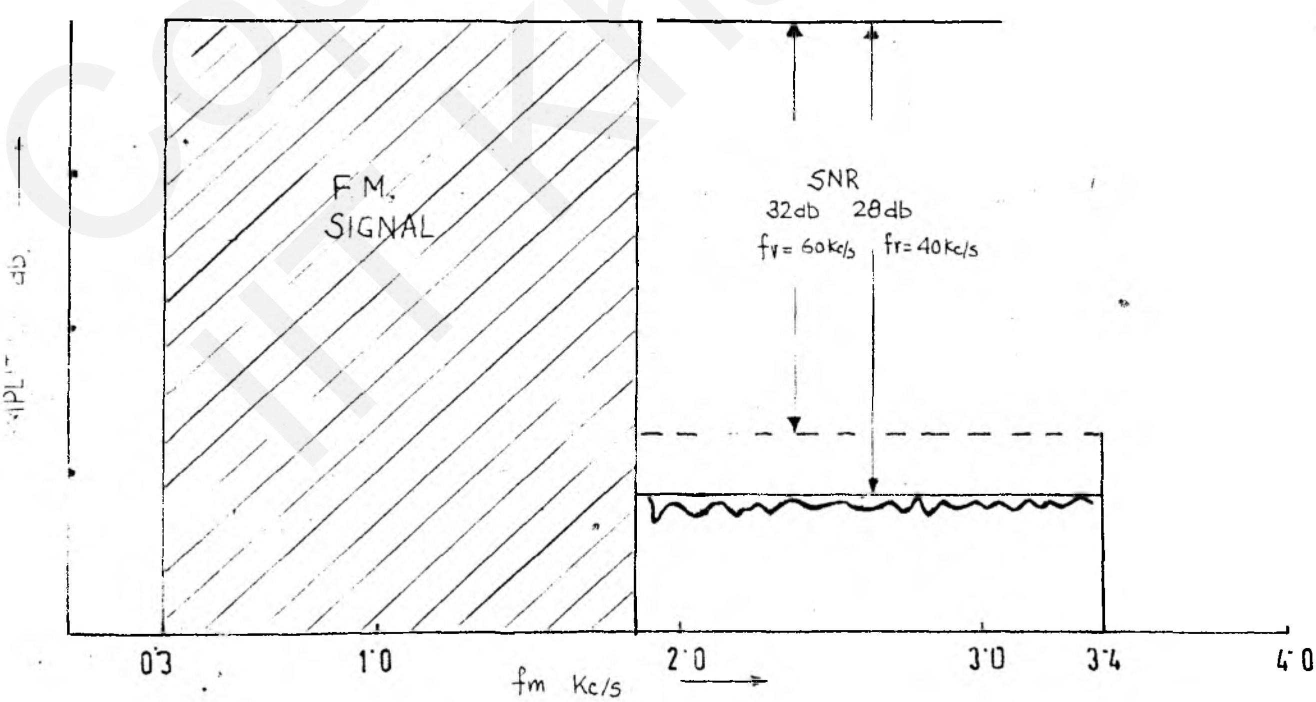
(c) <u>Noise Signals Test.</u>

The performance of the system for the noise signal input has been evaluated by the method given in 3.1.1 (Fig.3.25). The SNR at 60, 40 and 30 Kc/s PRF, for the half band noise signal, is 31, 27 db and 22 db, respectively, and is shown in Fig.4.18.



VARIATION OF OUTPUT WITH fm , WITH THE PARTIAL COMPANDOR INCLUDED; fr= 40 kc/s

FIG. NO. 4·16



SNR FOR 1/2 BAND FM SIGNAL

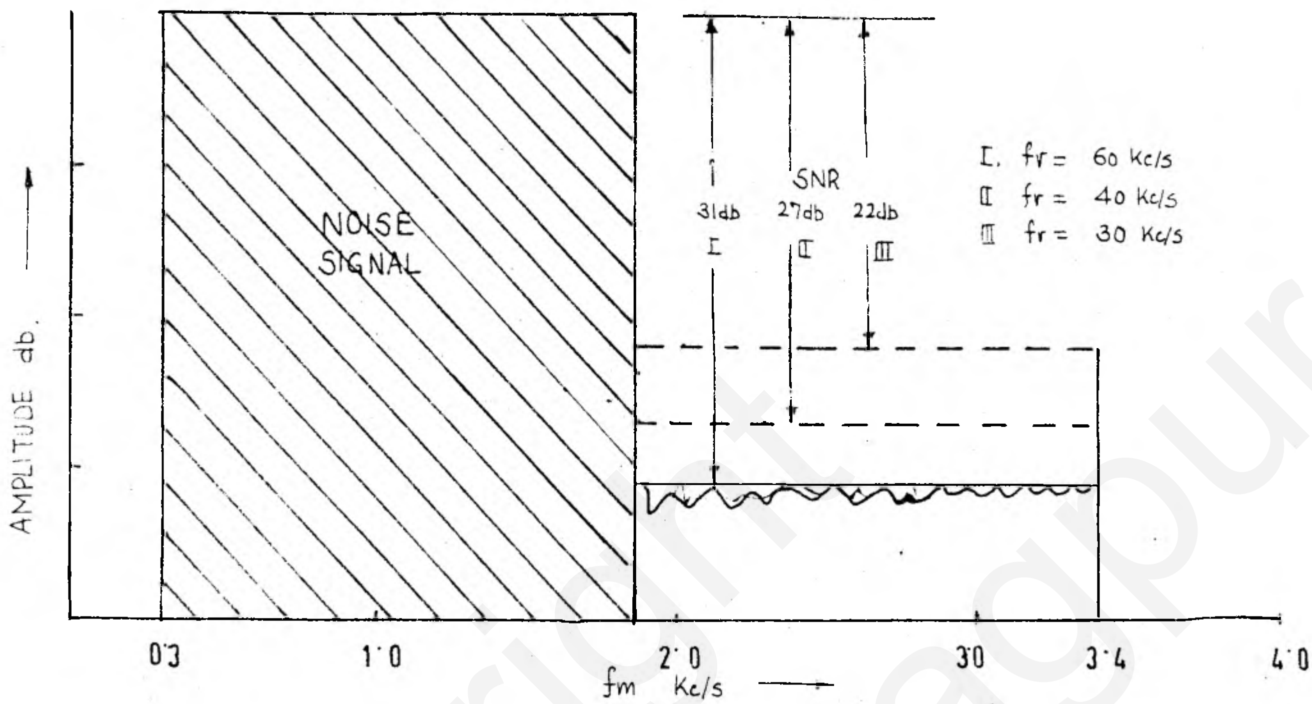
FIG. NO. 4·17

In the other method of measurement, where a band stop filter of about 800 c/s is used in the noise signal occupying the audio band, the equalised SNR (equalised to the bandwidth) is 30 db at a PRF of 60 Kc/s, as shown in Fig.4.19. This result agrees fairly well with the one obtained by the 1/2 band signal method of Fig.4.18. The use of the compandor deteriorates the SNR by about 2 db, for the reasons given earlier in Section 3.1.1.

(d) Speech Signals Test.

The quality and intelligibility of the reproduced speech signals is estimated by methods similar to those given in Section 3.1.1. Listening tests showed that the quality of the speech is excellent at a PRF of 60 Kc/s. The quality of the reproduced speech at 40 Kc/s is good, except that the dynamic range is small and the peaks of the input signals are allowed to overmodulate the system by 8-10 db. Use of the compandor is helpful in extending the dynamic range and the result at 60 Kc/s PRF is good. At 40 Kc/s PRF, however, the quality of the speech is fair and tolerable, though a slight noise in the background is audible all the time.

A record of the sentence "Pandit Jawahar Lal Nehru died on 27th May, 1964, Bharat Ki Jai", through the three 1/3 octave bandpass filters has been made, as has been explained in Section 3.1.1 (Fig.3.29). The reproduced sentence at the output of the system is again recorded through the same 1/3 octave bandpass filters and compared with the original, to show the quality of the speech processing. The recordings of the reproduced signals have been made at PRFs of 60 and 40 Kc/s, with and without com-



SNR FOR 1/2 BAND NOISE SIGNALS

FIG. NO. 4.18

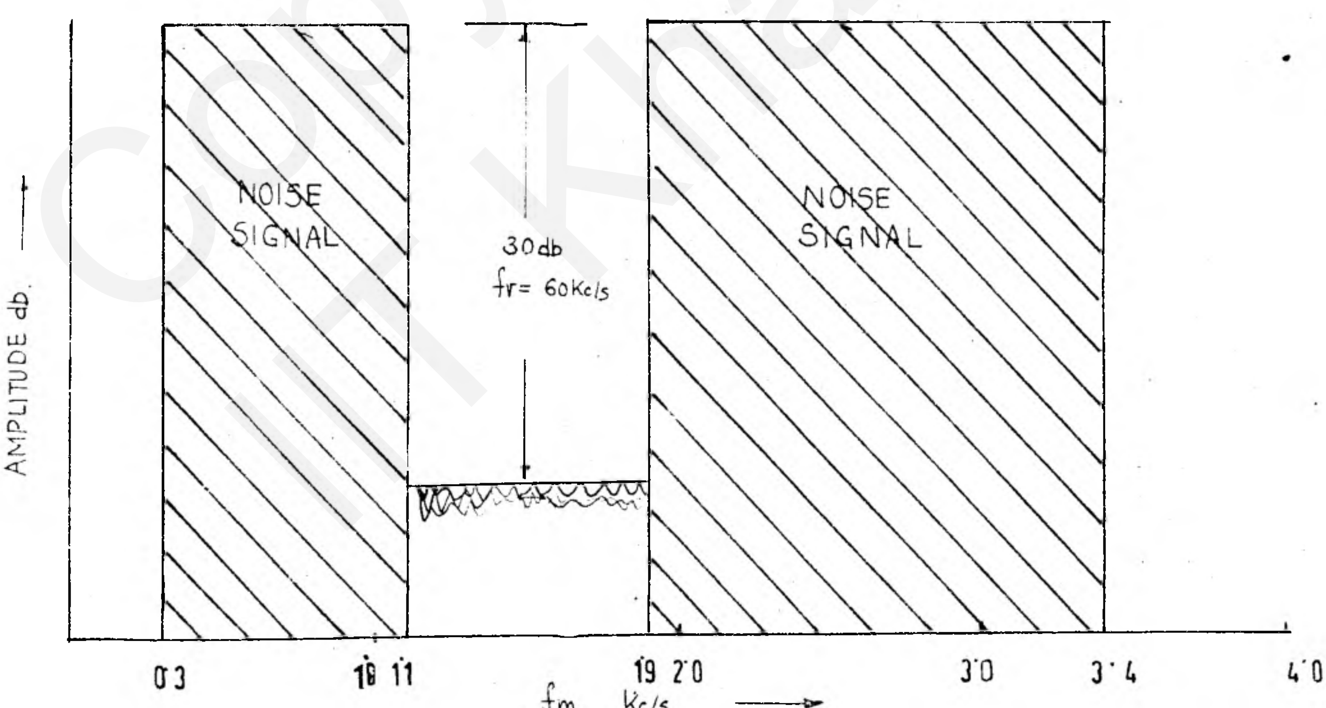


FIG. NO. 4.19

EQUALISED SNR FOR NOISE SIGNALS USING A BAND-STOP FILTER pandor. Fig.4.20 shows one such record at the centre frequency of 500 c/s with (i) original speech, (ii) reproduced speech at 60 Kc/s PRF, (iii) reproduced speech at 40 Kc/s PRF, and (iv) reproduced speech with the partial compandor at 40 Kc/s PRF. Fig.4.21 shows the same four recordings at a centre frequency of 1000 c/s, and Fig.4.22 shows the same at a centre frequency of 3150 c/s. It is seen that there is some compression of the input levels with the compandor at 40 Kc/s PRF, otherwise the reproduction is quite alright.

4.2.1. STABILITY.

The stability of the Binary SQ-PCM system will naturally depend upon the proper functioning of all the constituent circuits. However, disregarding any major breakdown of the system when it stops functioning altogether, the most sensitive parts of the circuit are the reference bias voltage of the quantizer (Schmitt trigger here) and the loop gain and phase characteristics.

The variation of the SNR, when the reference bias voltage of the Schmitt trigger is varied, is given in Fig.4.23. For an increase in bias, resulting in a higher threshold level, and therefore, fewer error voltages quantized as +1 pulse, the SNR falls by 3.4 db only for a voltage change of 10 volts. However, as the bias is increased further, the SNR falls rapidly till, at 120 V bias the trigger circuit stop functioning and there is no pulse output. A decrease of the bias from the reference level brings the triggering level so low that all the small noise

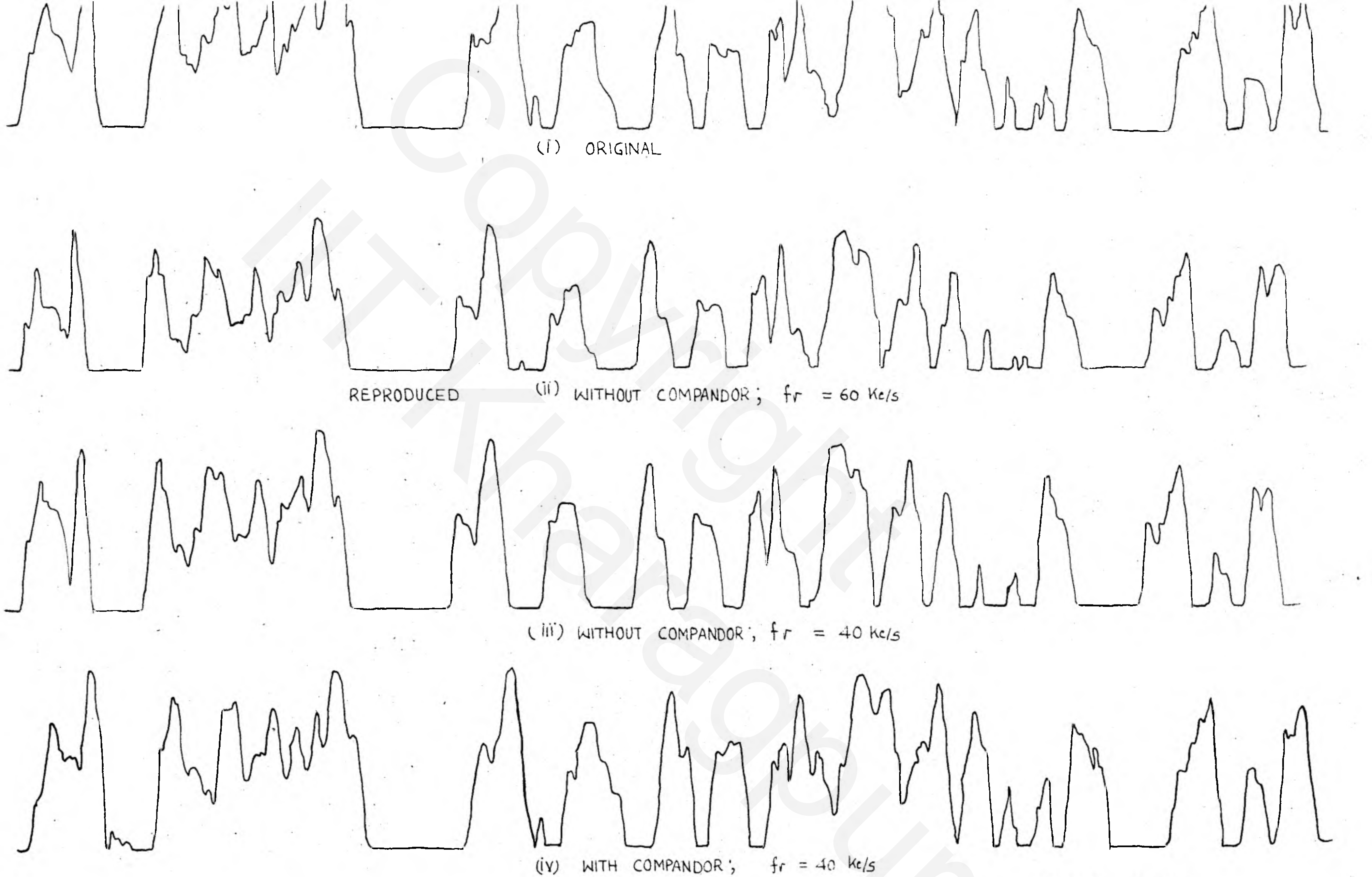


FIG. NO. 4·20

AMPLITUDE - TIME - FREQUENCY RECORDING OF THE 'SENTENCE'; CENTRE FREQUENCY 500 c/s

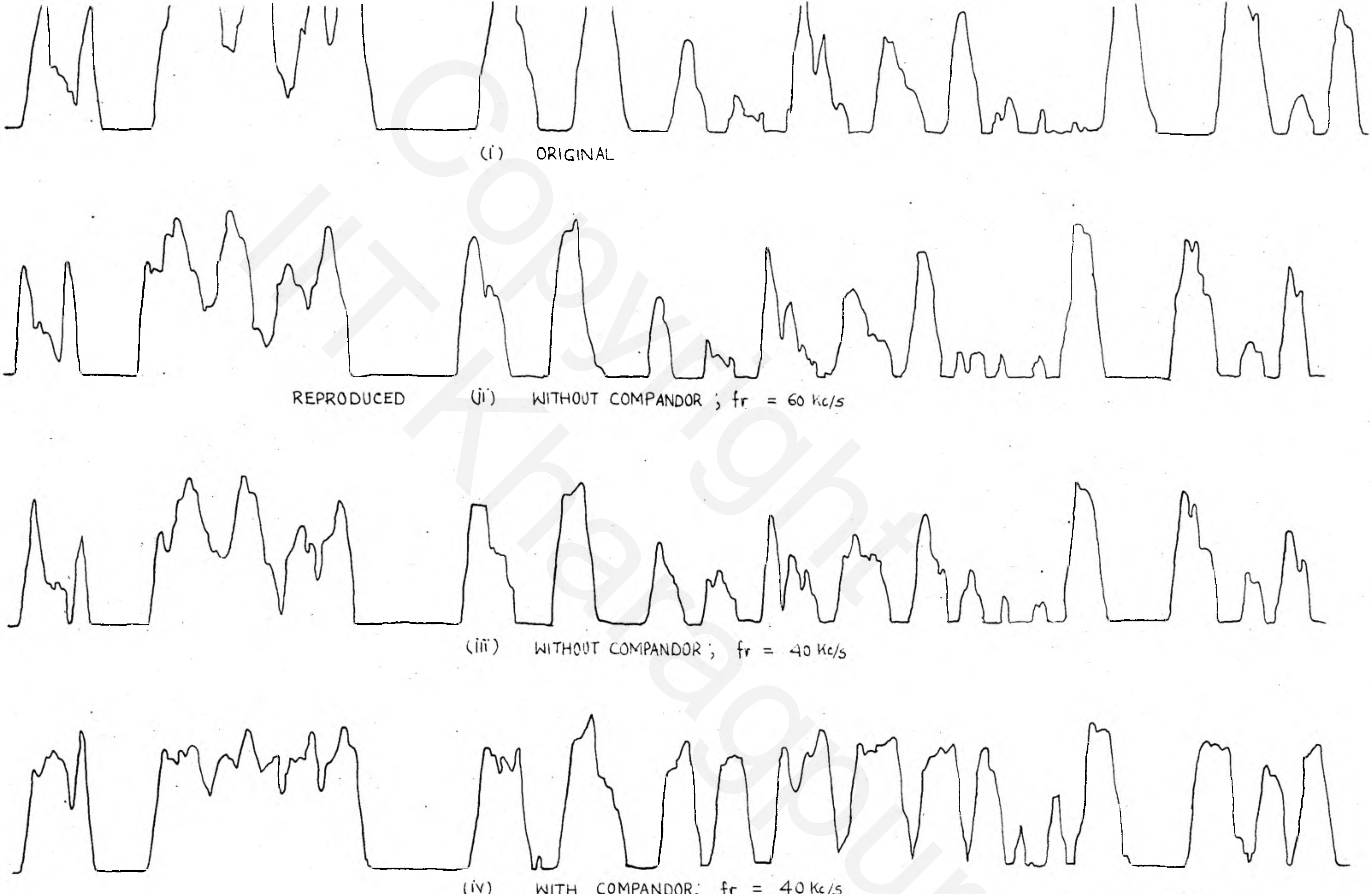


FIG. NO. 4·21

AMPLITUDE-TIME-FREQUENCY RECORDINGS OF THE 'SENTENCE'; CENTRE FREQUENCY 1000 c/s

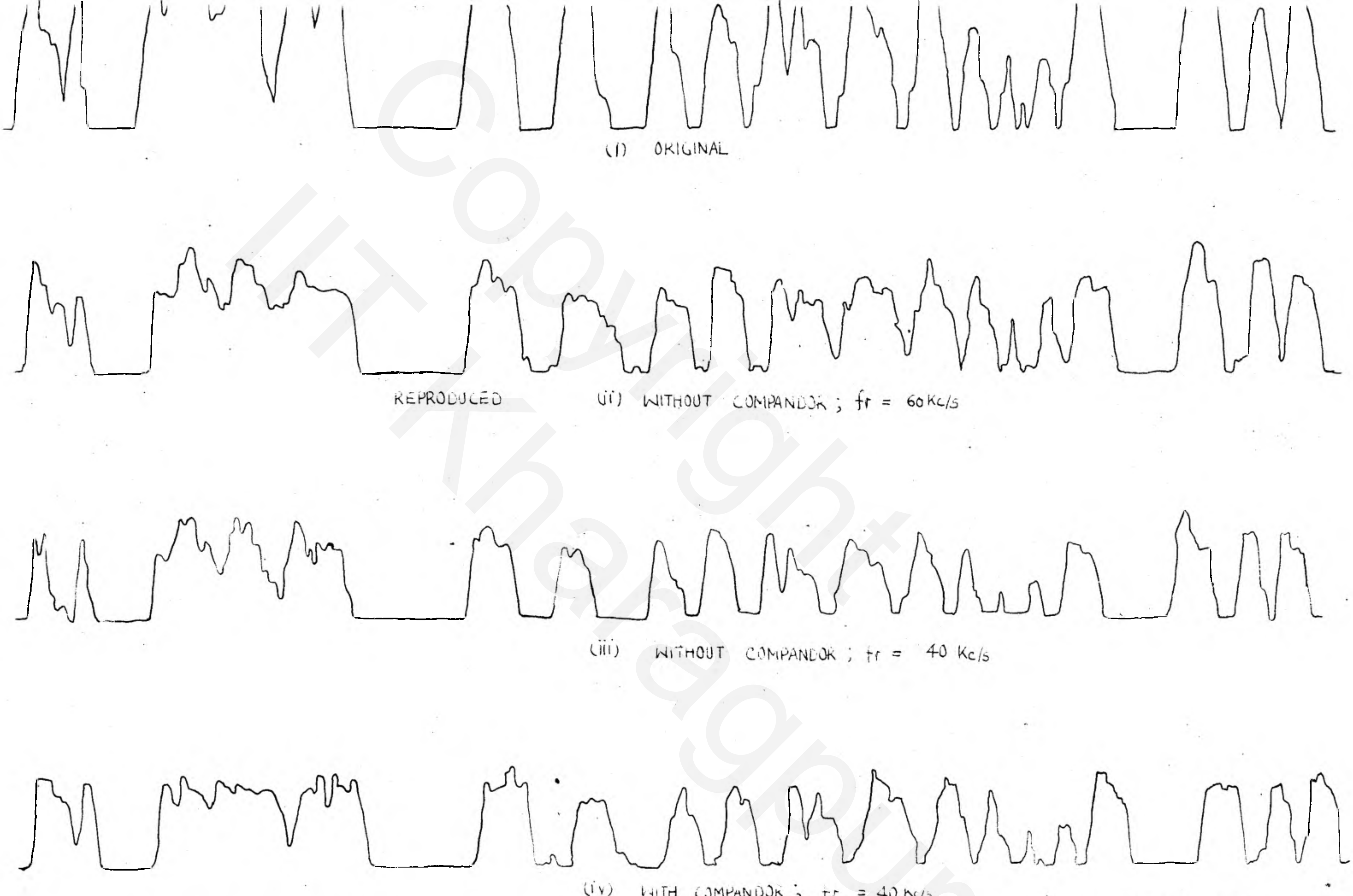
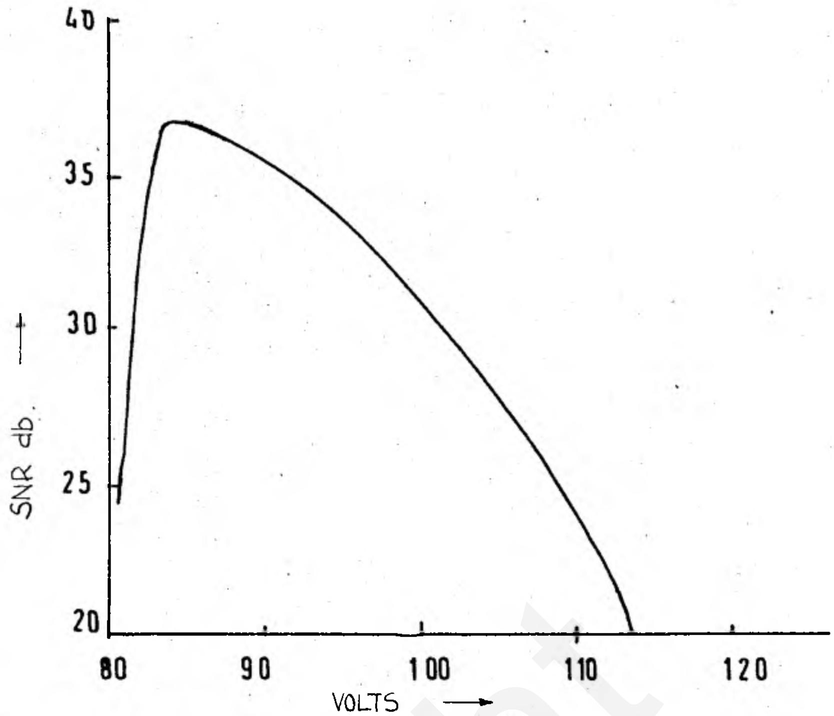


FIG. NO. 4·22

AMPLITUDE - TIME - FREQUENCY RECORDINGS OF THE 'SENTENCE' ; CENTRE FREQUENCY 3150 C/s



VARIATION OF SNR WITH BIAS VARIATIONS OF THE SCHMITT TRIGGER

FIG. NO. 4·23

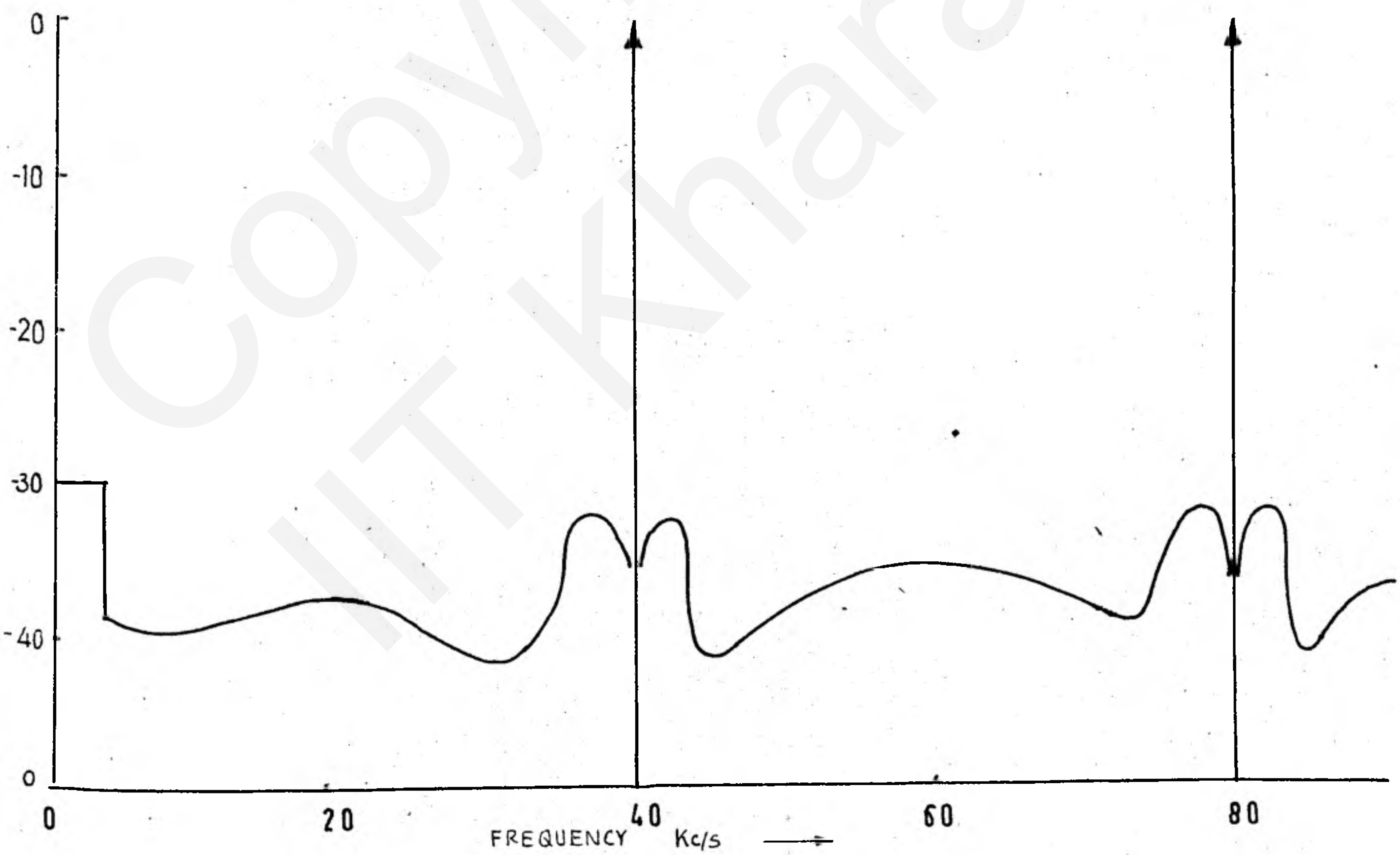


FIG. NO. 4·24

SPECTRUM OF THE PULSES AT THE OUTPUT OF THE TRANSMITTER

fr = 40 Kc/s

voltages at the sampling points trigger the circuit, and the SNR is poorer. A further decrease results in the trigger output being wider and continuous, and thus the trigger circuit loses its properties. The system is fairly stable to small changes in the bias voltage.

With the particular impulse response of the pulses, the cumulative negative slopes of these responses match the negative slopes of the signal, and hence, the feedback circuits' gain and phase characteristics are very significant parameters. It is important that the asymptotic slope of the loop transfer function is kept within a certain limit. If the slope increases, the circuit will oscillate and become unstable. It is seen experimentally that a maximum slope of about 9 db/octave is permissible in the feedback loop. With proper adjustments, however, the circuit can be made fairly stable against oscillations at a desired PRF, and for a specified requirement, there is no trouble of oscillations once the loop gain characteristics is chosen correctly.

4.2.2. SPECTRUM OF QUANTIZING-NOISE.

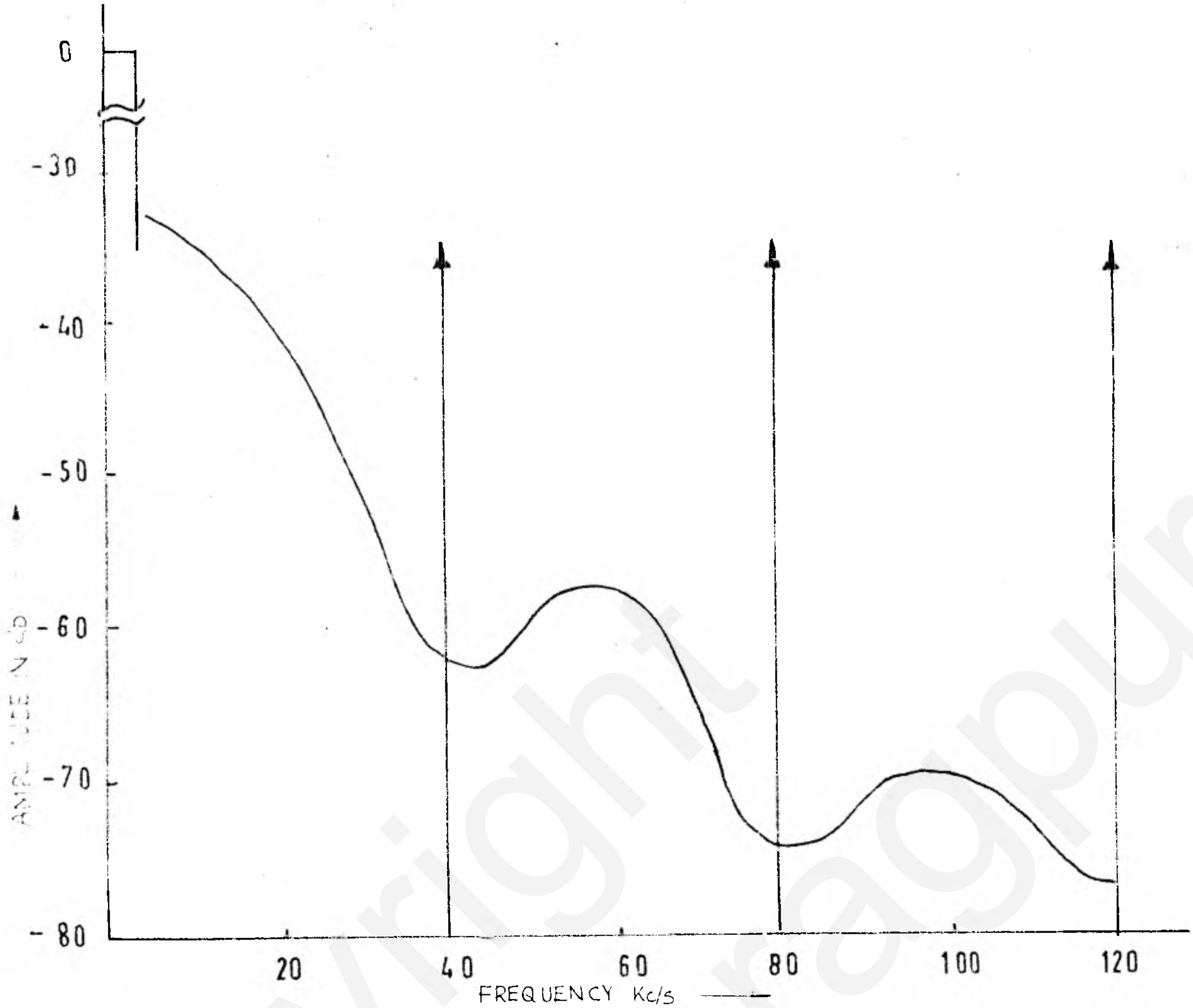
The quantizing noise characteristics of the Binary SQ-PCM system has been discussed in Section 2.4. The spectrum of the output pulses of the transmitter and the receiver output (before filter), has been experimentally determined in a manner similar to the one given in Section 3.1.3. The spectrum consists of a continuous spectrum and a line spectrum at the PRF, and its harmonics, as shown in Fig.4.24.

The continuous part of the spectrum is about -37 db compared to the line spectrum of the fundamental of the PRF. The sidebands around the fundamental and its harmonics are about -32 db, as compared to the carrier and the audio energy at the d.c. is also about -30 db as compared to the same reference. The timing pulses can be easily extracted here, as the line spectrum at the fundamental of the PRF is quite strong.

After integration, the noise spectrum is modified as shown in Fig.4.25, and it is seen that the shape of the spectrum matches the theoretically assumed noise spectrum in Section 2.4. The signal power in the audio band is equal to the reference 0 db, and the continuous noise spectrum outside the AF band is approximately 33 db below this. The fundamental of the PRF gets reduced to -34 db and the harmonics of the PRF are almost equal to it in amplitude. It is seen that quite a large amount of audio power energy is at the lower frequencies, as compared to the Ternary where more energy was in the sidebands of the PRF and its harmonics. Video transmission of these pulses through cables etc. will require a conversion of the Binary 1/0 pulses into some Pseudo Ternary code[37], so that the spectrum energy at the lower frequencies (AF) gets shifted to higher frequencies. Such a conversion is needed to economise in the cost of transformers etc., and is necessary, if the transmission of the spectrum requires a transmission of low frequencies.

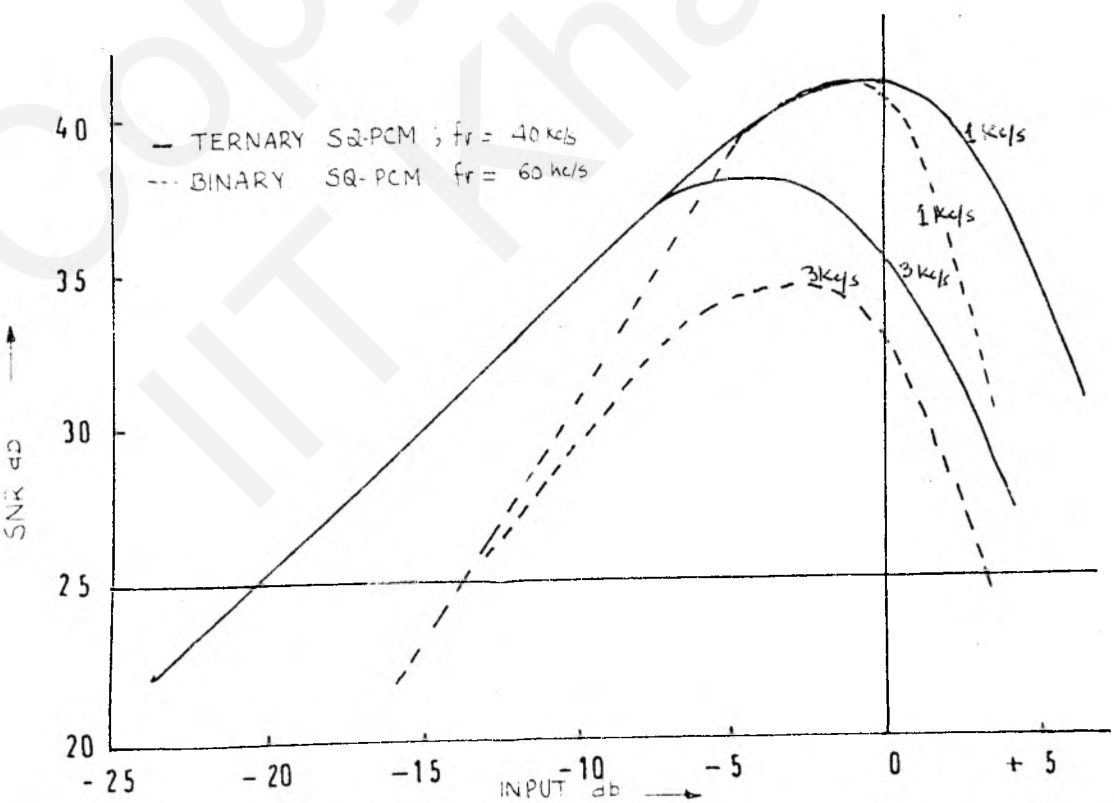
4.2.3. SUMMARY OF THE RESULTS.

The performance of the Binary SQ-PCM system has been given



SPECTRUM OF THE TRANSMITTER PULSES AFTER THE INTEGRATION ; fr = 40 kc/s

FIG. NO. 4·25



VARIATION OF SNR WITH INPUT LEVEL.

FIG. NO. 4·26

for the four different types of input signals like sinusoidal, FM, noise, and speech, and for three PRF's of 60, 40 and 30 Kc/s. For a sinusoidal frequency of 1 Kc/s, the best SNR of 41, 37 and 32 db is obtained at 60, 40 and 30 Kc/s PRF, respectively. The SNR falls linearly with inputs below the overload level, but falls sharply for the inputs above this. The reason is that the quantizing noise is random with a uniform power spectral density, and therefore, the SNR falls linearly with the input level, because the signal decreases. Above the overload level, there is a clipping and hence, large amount of harmonic distortion resulting in a sharp decrease in the SNR. The dynamic range for an SNR of above 25 db is 20, 17.5, and 12 db, for the PRF's of 60, 40 and the 30 Kc/s, respectively. For a sinusoidal frequency at the higher end of the audio band, the overload level of the input is at about -5 db as compared to the input at 1 Kc/s. The effect produced is also reflected in the decrease of the SNR and frequency distortion of the higher frequency signals at 0 db input, when the SNR and the output fall by about 8 db and 5 db, respectively at a PRF of 40 Kc/s. However, at lower input levels the SNR and the output are constant for a variation of the signal frequency. The input-output voltage relation is linear at 1 Kc/s signal frequency, but the output is slightly compressed at the 3 Kc/s signal frequency. The improvement of the SNR with PRF comes out to be 9 db/octave change of PRF.

An instantaneous compandor improves the dynamic range of the input considerably. Among the two types of the compandors, the partial compandor is more successful, where a companding advantage of 20 db is obtained. The dynamic range is extended to 39 db and 35 db, at 60 and 40 Kc/s PRFs, and the best SNR is 32 db and 29 db, respectively. As the overload for higher frequency is at 5 db below that for the centre frequency, a further frequency distortion occurs at the output of the compandor. The output at 3.2 Kc/s falls by about 14 db as compared to the lower frequencies at 0 db input level. However, the output becomes constant at about -22 db level when the frequency distortion is negligible.

The FM signals are reproduced with good fidelity at 60 Kc/s PRF and the SNR at 60 and 40 Kc/s PRF, is 32 db and 28 db, respectively for the 1/2 band wide signals. The narrow band (800 c/s) signals are reproduced with a SNR of 34 db and 30 db for the two PRF's, respectively. The results of the noise signals test are satisfactory and give a good general idea of the performance of the system. The SNR at 60, 40 Kc/s, and 30 Kc/s PRF, is 31, 27 and 22 db, respectively when the 1/2, band measuring technique is employed. With a band stop filter arrangement, the equalised SNR is 30 db at a PRF of 60 Kc/s.

Listening test at 60 Kc/s PRF gave excellent results, and the reproduction at 40 Kc/s PRF without compandor is good. With the compandor the results at 60 Kc/s PRF are good, and at 40 Kc/s, they are fair. A record of the "sentence" through the 1/3 octave filters shows the compression still present when a partial compandor is used. Otherwise, the quality of the reproduced signal matches fairly well with the original signal.

The stability of the reference voltage against drift is good, provided, the ultimate change in the bias is not very large. The gain and the phase characteristics of the feedback loop is to be carefully adjusted, so that the slope of the output vs. frequency response does not exceed about 9 db octave.

The spectrum of the quantizing noise, observed experimentally, agrees fairly well with the theoretical spectrum assumed. The RF energy is distributed in the audio band, and as sidebands around the carrier PRF and its harmonics. However, the energy at the lower frequencies is quite substantial as compared to the energy in the sidebands. A video transmission of the Binary 1/0 pulses through cables etc. will require a conversion to some Pseudo ternary pulses, to save in the cost of the equipment.

4.3. DISCUSSION.

From the results given above it is seen that the Binary Eq-PCM system has a satisfactory performance for the different types of input signals. A medium grade circuit with an average SNR of 25-30 db may be obtained by using a limited channel capacity of 40 - 50 Kbits/sec. It has been shown that the pulse response of the feedback network, and the spread obtained practically, bear out the theoretical assumption of the exponentially decaying interpolating filter, which together with its cumulative slopes matches the slopes of the signal. The approximation so obtained follows the waveform more closely and hence, the errors at the sampling points are reduced. A further minimisation of the errors is obtained by using a threshold in the quantizer with the idea that very small positive difference between the input and the approximated signal should not produce a +1 pulse, and thereby cause a larger error at that point. The cumulative slope will be more than the slope of the highest frequency signal at the peak input amplitude, if about 4-6 pulses (PRF 40 Kc/s) of +1 have already occured. This matching of the slopes together with the threshold in the comparator-quantizer form the major difference between SQ-PCM and the Δ-M and Δ-ΣM systems. The absence of a definite threshold in the pulse modulator of the Δ-M and Δ-ΣM systems results in some extraneous pulses, either positive or negative, which do not contribute to the minimisation of the error waveform, and thereby produce some extra error at the output.

Use of compandors has increased the dynamic range and a companding advantage of 20 db is obtained. Unfortunately, the instantaneous compandor also gives rise to some frequency distortion at the higher signal frequencies. It is seen that the output at the higher frequency is compressed, but it is felt that such a compression does not lead to much deterioration in the performance. The output is almost linear for input levels upto -10 db of the peak input level, and therefore, only the large amplitude signals, having amplitudes within 10 db of the peak, will be compressed. For signals like speech, this amount of compression is allowable without hampering the faithful reproduction of the signal.

The overall circuitry of the Binary SQ-PCM system is very simple, and with the use of the common instantaneous compandor and a common coder, many channels can be multiplexed and transmitted, as will be shown in Section 5.5. The cost of the complete system will be comparably less than that of an AQ-PCM system for the same grade of service. Even without companding, the dynamic range of the Binary SQ-PCM system is sufficient to be useful for processing FM signals, as used in FM-telemetring. It is seen that the binary SQ-PCM system at 40 Kc/s PRF is equivalent to a 6 digit (50 Kc/s) Binary AQ-PCM.

A comparison of the two systems, the Binary and the Ternary SQ-PCM, developed in the laboratory, shows that the Ternary system is very much superior to the Binary system. The dynamic range of the input (for the SNR to be above a particular value) is larger in the Ternary system, and the frequency distortion is also less. In Ternary SQ-PCM, better input-output voltage linearity is maintained at all frequencies, and the SNR at the higher signal frequency falls less than that in the Binary system. Since the bandwidth of the Ternary system is slightly more than that of the Binary system, a linear compandor could be used with advantage. The companding advantage is more in the Ternary system, and the dynamic range is also more after companding. The Ternary system is much more stable than the Binary system, and even if one of the quantizers does not work in the Ternary, the system degenerates into a Binary system and does not completely breakdown. The quantizing noise spectrum of the Ternary pulses does not have much energy at the lower frequencies and is, therefore, suitable for video transmission through cables, etc. The Binary system, on the other hand, will require a conversion into Pseudo ternary pulses before an efficient video transmission is possible.

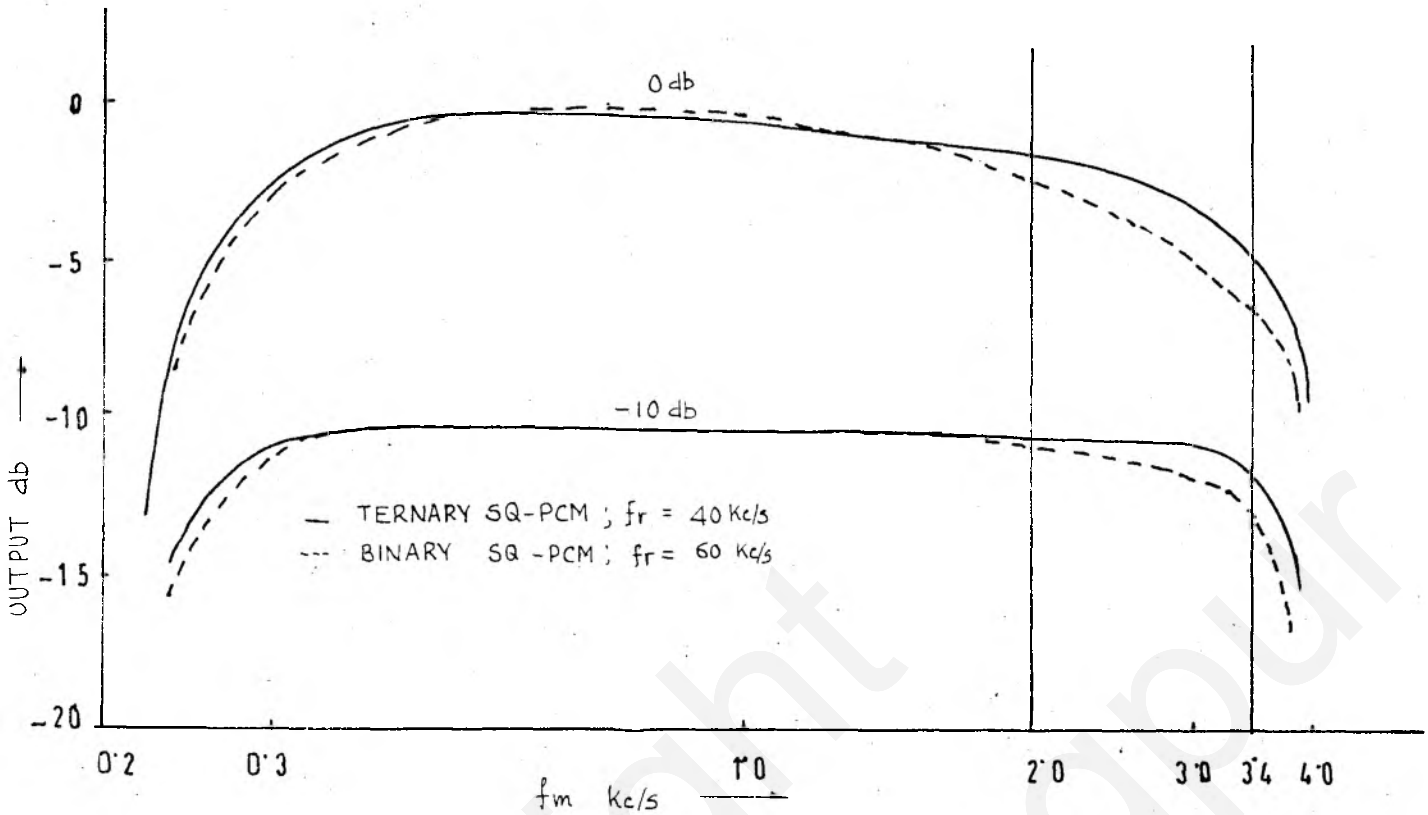
Both circuits are essentially very simple and, for a particular operation range, stable. To examine the situation in more detail, the performance of a Binary SQ-PCM will be compared in detail with an equivalent Ternary system.

At 40 Kc/s PRF, the Ternary SQ-PCM system has a signalling rate of about 63 K bits/second and therefore its performance will be compared with the Binary SQ-PCM system at 60 Kc/s PRF and signalling rate of 60 K bits/second. Fig.4.26 shows the variation of SNR with the input signal level variation for the Ternary SQ-PCM system at 40 Kc/s PRF (solid curve), and for the Binary SQ-PCM system at 60 Kc/s PRF (dotted curve). The SNR for both 1 Kc/s and 3 Kc/s signal frequency are shown in the figure. It is seen that the input dynamic range (for inputs below 0 db reference) is 20.0 db and 14.5 db, for the Ternary and the Binary systems, respectively. The peak SNR at 1 Kc/s is the same, but at 3 Kc/s the SNR in the Ternary system falls by about 6 db, and in the Binary system by 8-9 db, at 0 db input. In general, the performance of the Binary system at the higher signal frequencies is poorer than the performance of the equivalent Ternary system. Beyond the overload point, the SNR falls more rapidly in the Binary system than in the Ternary system. As the Ternary system is almost symmetrical and balanced, the harmonic distortion will be less than that in the Binary system which is unbalanced. The SNR deterioration above the threshold is mainly due to the saturation resulting in harmonics, as has been mentioned earlier, and hence, the sharper deterioration in the Binary SQ-PCM system.

The output voltage vs. signal frequency is plotted in Fig.4.27, where it is seen that the frequency distortion at the higher signal frequency is less in the Ternary system (solid curves) than in the Binary system (dotted curves), e.g. at 3 Kc/s, the output in the Ternary falls by 3 db, as compared to output at 1 Kc/s; and the output in the Binary falls by about 5-6 db, again compared to the 1 Kc/s output. However, at an input level of -10 db, the frequency distortion in both the Ternary and the Binary systems is negligible.

A comparison of the improvement in the dynamic range due to the compandor is shown in Fig.4.28, where the noncompandored SNR curves are also drawn for reference. The SNR is above 30 db for a range of 29 db in the Ternary system at 40 Kc/s PRF (solid curve), and 24 db in the Binary system at 60 Kc/s PRF (dotted curve). The total dynamic range, for the SNR to be above 25 db, is about 40 db and 36 db, against an uncompandored dynamic range of 20 db and 14.5 db, for the Ternary and the Binary systems, respectively. However, the Binary system suffers from another disadvantage of larger frequency distortion with the compandor included than the Ternary system.

FM signals are reproduced fairly well by both the systems, and the SNR for the wideband signal is 33 db and 32 db, for the Ternary and the Binary system, respectively. Noise signals are processed with an SNR of 32 db and 31 db by the Ternary and the Binary systems, respectively. Speech quality for both the systems is quite good, except that in the Binary system slight overmodulation of the system is purposely done to handle all the dynamic



VARIATION OF OUTPUT WITH fm TWO DIFFERENT INPUT LEVELS

FIG. NO. 4·27

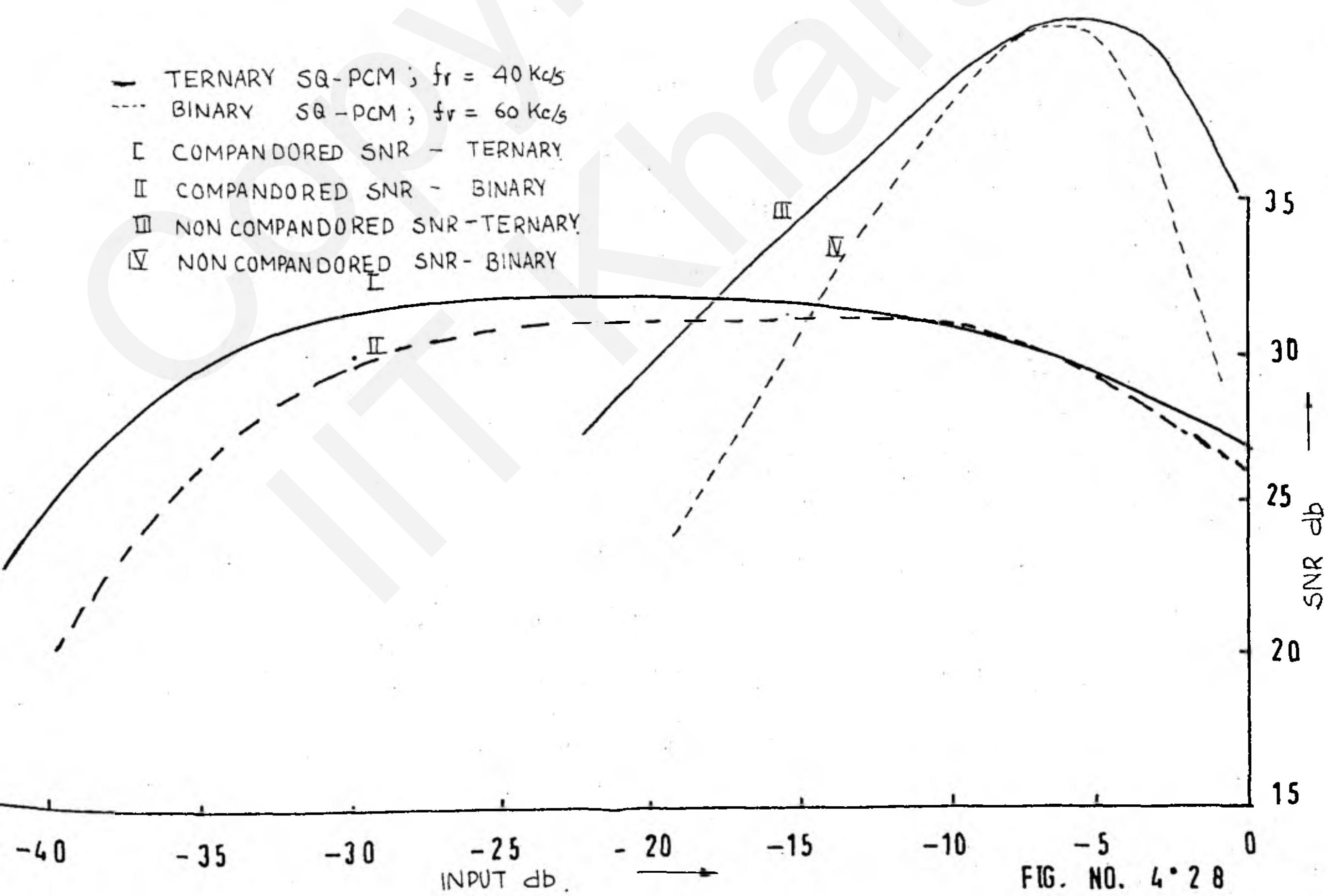


FIG. NO. 4·28

IMPROVEMENT OF DYNAMIC RANGE WITH A COMPANDOR IN THE TERNARY AND BINARY SQ-PCM SYSTEMS levels of the speech. In listening tests, this overmodulation is unrecognisable to an untrained ear. However, the necessity of the overmodulation can be successfully circumvented by using a syllabic or an instantaneous compandor. The results with the instantaneous compandor are quite satisfactory and since a multiplexing of channels is quite possible for both the systems (Section 5.5), the use of an instantaneous compandor has been experimentally made feasible.

The spectrum of the transmitter pulses in the Ternary SQ-PCM system contains the major part of the AF energy as the sidebands of the fundamental and harmonics of the PRF, and the carrier is partially suppressed as compared to the sidebands. The spectrum of the Binary SQ-PCM, however, shows that the AF energy at the lower frequency is of the same order as the sidebands energy around the fundamental and the harmonics of the PRF, and the carrier (line spectrum) is at a much higher level. The timing pulses in the Binary system can be easily extracted by simple filtering, but in the ternary system this method will produce jittering error in the timing pulses. However, the timing pulses in the Ternary system can be generated by first rectifying and converting the ternary into binary pulses, and then applying the same techniques as in the Binary SQ-PCM system.

Thus it is seen that, although the peak SNR is the same for both the systems, the overall performance of the Ternary SQ-PCM system working at 40 Kc/s PRF is superior to that of the Binary SQ-PCM system working at 60 Kc/s PRF. A comparison of both of these SQ-PCM systems with the other similar coded pulse modulation systems, like AQ-PCM, Δ-M and Δ-ΣM , is included in Section 6.1.

CHAPTER V.

TRANSMISSION CHARACTERISTICS OF SQ-PCM SIGNALS.

5.0 GENERAL.

The coding characteristics of the Ternary and the Binary SQ-PCM systems have been given in Chapters III and IV. The purpose of the present Chapter is to discuss the transmission characteristics of coded pulse signals. The signals could be transmitted as video pulses on cables, or modulated onto some RF carrier, before transmission. The channel will introduce some noise and interference during the transmission, and the correct detection of the signals in presence of the noise is of great significance. The signal power necessary to achieve this with a given amount of noise is the subject matter of discussion in Section 5.1 and 5.2. The bandwidth necessary in each case has been calculated in Section 5.3. Naturally, the systems are not ideal, and also, the different methods of sending the SQ-PCM pulses are not equally efficient. The coding efficiency, discussed in Section 5.4.1, compares the ideal rate of information transmission, H, to the actual rate. The minimum power required for a given output SNR in a certain bandwidth gives another figure of merit, called power efficiency, and is discussed in Section 5.4.2. It is seen that with the assumed input SNR, an ideal system will have a much larger SNR and hence, larger information rate in the message bandwidth. The input SNR required in the actual system, to have the same information rate in the message bandwidth, is another criteria of the efficacy of the transmission schemes, and has been called communication efficiency (discussed in Section 5.4.3). The SQ-PCM channels can be multiplexed on a time division basis and two schemes of multiplexing have been discussed in Section 5.5.

5.1.0. VIDEO TRANSMISSION.

It is well known that all wideband systems have a threshold below which the noise improvement properties of the system rapidly deteriorate. To demonstrate the existence and importance of this threshold level, it is assumed that Binary 0/1 video pulses of SQ-PCM system are being transmitted. At the receiver, fluctuation noise is added to the incoming group of pulses, and there is a possibility of an error in ~~the~~ decoding the signal. The error will occur, either, if noise in the absence of signal has an instantaneous amplitude comparable to that of a pulse when present; or, if noise in the presence of a signal has a large instantaneous negative amplitude which destroys the pulse.

It is assumed that the receiver, consisting of a slicer circuit with a threshold at Y volt, will interpret an occurrence of a pulse if the instantaneous voltage (signal plus noise) exceeds the threshold, and an absence of a pulse, if the instantaneous voltage is less than the threshold. To calculate the probability of error (which depends upon the statistical distribution of noise), it is assumed that a "zero" is sent, so that there is no pulse at the input of the slicer circuit. The probability of an error in this case will just be the probability that the noise will exceed

Y volts in amplitude and be mistaken for a pulse by the slicer; this is the probability that the input to the slicer, $v(t)$, will appear between Y and infinity. The noise is assumed to be completely random with a Gaussian distribution of amplitudes. Hence, if the noise alone is present, the probability density of its amplitudes is

$$p(v) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-v^2/2\sigma^2}$$

where σ is the standard deviation of the Gaussian distribution assumed, and the probability of an error is the area under the curve $p(v)$, shown in Fig.5.1(a), from Y to infinity.

$$P_{rob}\,(v > Y) = \int_Y^{\infty} \frac{e^{-v^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \cdot dv \qquad \ldots (5.1).$$

It is now assumed that a "one" is transmitted and appear as a pulse of amplitude 2V plus superimposed noise at the input of the slicer. At the particular instant, v(t) is a signal plus noise voltage of average value 2V, and its probability density is

$$p(v) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-(v-2V)^2/2\sigma^2}$$

The probability of error now will correspond to the chance that the signal plus noise voltage drops below the threshold voltage Y, and be mistaken as a "zero" signal by the slicer. This probability of error is just the area under the $p(v)$ curve from minus infinity to Y, shown in Fig.5.1(b), and is

$$P_{rob}\,(v < Y) = \int_{-\infty}^{Y} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-(v-2V)^2/2\sigma^2} \cdot dv \qquad \ldots (5.2).$$

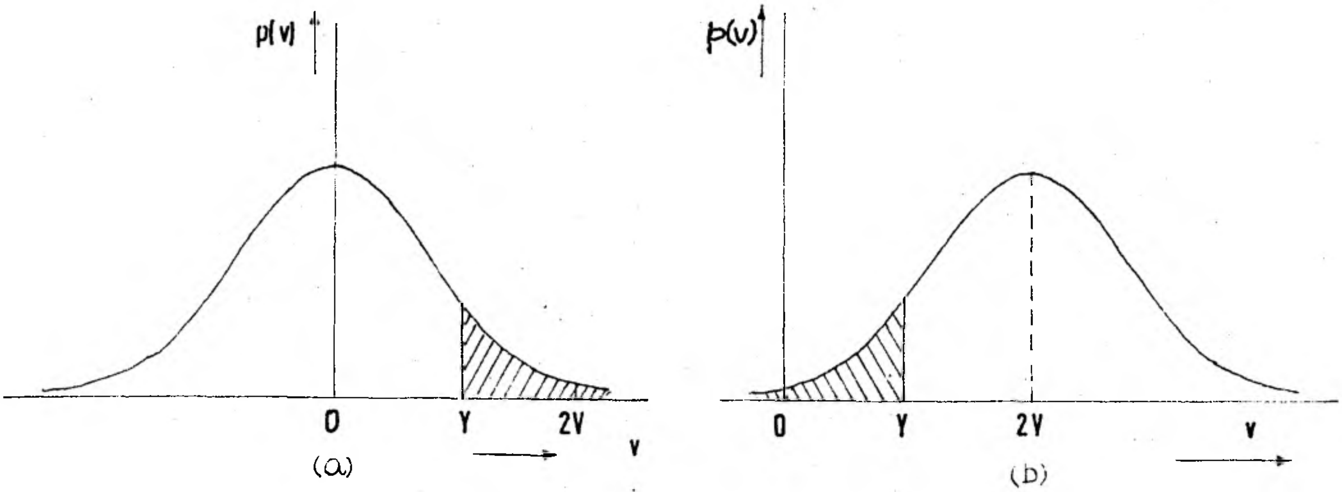


FIG. NO. 5·1

PROBABLITY DENSITIES IN BINARY PULSE TRANSMISSION
(a) NOISE ONLY, (b) PULSE PLUS NOISE

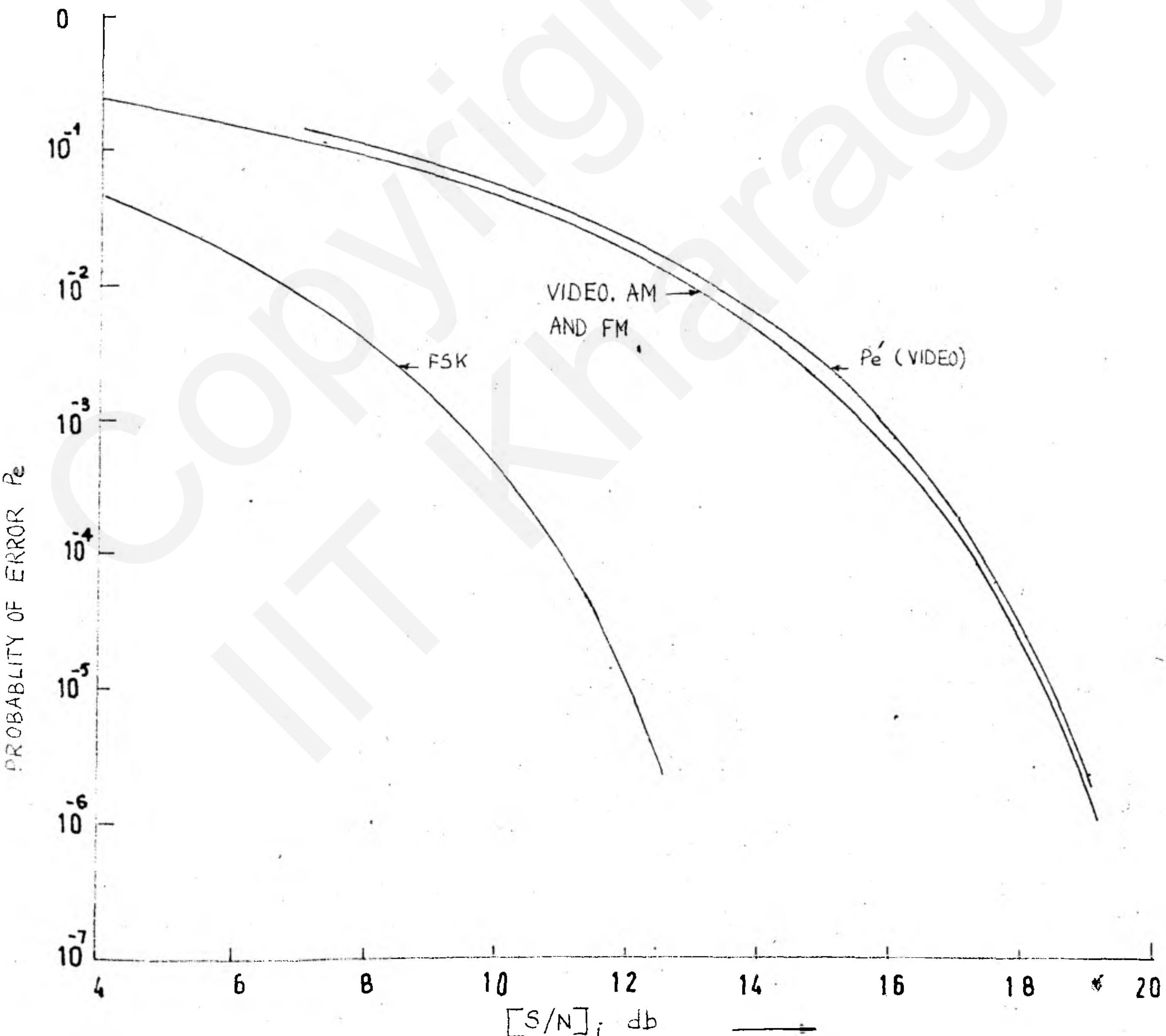


FIG. NO. 5·2

PLOT OF Pe VS. PEAK SNR (c)

The level Y of the slicer circuit has to be adjusted to a value such that, the probability of recognising the signal is maximised. The optimum level of the threshold between "zero" and "one" is such that,

$$P(0/1) = P(1/0) = P_e$$

where $P(0/1)$ = probability of receiving a "zero" when "one" was transmitted.

$P(1/0)$ = probability of receiving a "one" when "zero" was transmitted.

P_e = probability of error.

that is

$$P_e = \int_Y^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-v^2/2\sigma^2} \, dv = \int_{-\infty}^{Y} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(v-2V)^2/2\sigma^2} \, dv$$

For the P_e to be minimum, it can be shown that the threshold level Y should be approximately equal to V (the optimum threshold level), and for large SNR.

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{V}{\sigma}} e^{-x^2/2} \, dx \qquad \ldots (5.3(a))$$

$$= \frac{1}{2} - erf(x)$$

also

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-1/2\sqrt{S_{pi}/N_i}} e^{-x^2/2} \, dx \qquad \ldots (5.3(b))$$

where x is a variable V/σ

and $$erf(x) = \frac{1}{\sqrt{2\pi}} \int_0^{V/\sigma} e^{-x^2/2}\, dx$$

S_{pi} = peak signal power at the input = $4V^2$.

N_i = mean Noise power at the input = σ^2

Fig.5.2 shows a plot of P_e for variation of peak channel SNR (S_{pi}/N), calculated from equation (5.3(b)). If the average power is the limiting factor in the channel, then the average signal power S_i for 0/1 pulses is 3 db below the peak signal power. The variation of P_e with (S_i/N_i) is shown in Fig.5.3. However, instead of transmitting 0/1 pulses, one can send +1/-1 (Bipolar) pulses of amplitude V without losing the noise margin. The peak power of a +V and -V pulse is V^2 and is 6 db lower as compared to 0/1 transmission. The average power is equal to the peak power in this case. The relation of the P_e with the input SNR for such a Bipolar Video transmission is also shown in Fig.5.2.

The input SNR, in the peak power limited video Unipolar-transmission-SQ-PCM system, is 18.2 db for an output SNR of 40 db, and is 12.2 db in the Bipolar Video transmission case for the same output SNR. The noise power N_o, at the output of the threshold circuit in the receiver, can be calculated with the knowledge of the probability of error as follows. If P_e is the probability of receiving a digit erroneously, then $(1 - P_e)$ is the probability of receiving the digit correctly, and for a "word" of n digits, $(1 - P_e)^n$ is the probability of receiving it correctly. Hence,

$(1 - (1 - P_e)^n)$ gives the error probability in a group of n digits. Now, in the SQ-PCM system, each pulse has equal weightage, i.e., every pulse, of height Δh received increases the signal power built up by just one unit.

The noise pulse occuring in any of the n positions of the "word" does not have an equal effect in disturbing the true signal height built up by the pulses. An error pulse in the first digit position of the word will have more effect on the signal than an error in the last digit position. However, if a large input SNR is assumed, the probability P_e is very small, and the probability that two error pulses will occur in the same "word" is even smaller[28]. Therefore, for a very small P_e on an average, the weightage of the position of the noise pulse on the signal can be assumed to be equal, and hence, the mean square change due to a noise pulse will be $(\Delta h)^2$. The output mean noise power is, therefore,

$$N_o = (\Delta h)^2 \times \left[1 - (1 - P_e)^n\right]$$

where n = no. of digits in a "word"

$= f_r/2f_m$ (as illustrated in Section 2.4)

f_r = PRF.

f_m = Signal frequency.

Except for P_e, which is dependent on the RF modulation scheme and the detector used, the eq.(5.4), giving the mean noise power at the output, will apply in all the SQ-PCM-modulated transmission systems.

5.1.1. BINARY SQ-PCM.

The peak to peak signal built up by a group of n binary 1/0 pulses is ($n.\Delta h$) and the rms sinusoidal signal voltage at the output is

$$V_o = n\Delta h / 2\sqrt{2}$$

In the Binary SQ-PCM system, the voltage built up is approximately half of this amplitude, as has been shown in Section 2.4 and therefore, the signal power is

$$\begin{aligned}[S_o]_{BINARY} &= [n.\Delta h/4\sqrt{2}]^2 \\ &= (\Delta h)^2/32 \cdot (f_r/2f_m)^2\end{aligned} \quad \ldots (5.5).$$

The SNR at the output is obtained by dividing eq.(5.5) by eq.(5.4) and is given by,

$$[S_o/N_o]_{BINARY} = \frac{1}{32}\cdot\left(\frac{f_r}{2f_m}\right)^2\cdot\frac{1}{[1-(1-P_e)^n]} \quad \ldots (5.6).$$

For a sufficiently small probability of error P_e, the equation (5.6) is reduced to

$$[S_o/N_o]_{BINARY} = \frac{1}{32}\cdot\left(\frac{f_r}{2f_m}\right)^2\cdot\frac{1}{nP_e}$$

$$\sim [S_o/N_o]_{BINARY} = \frac{1}{32}\cdot\left(\frac{f_r}{2f_m}\right)\cdot\frac{1}{P_e} \quad \ldots (5.7).$$

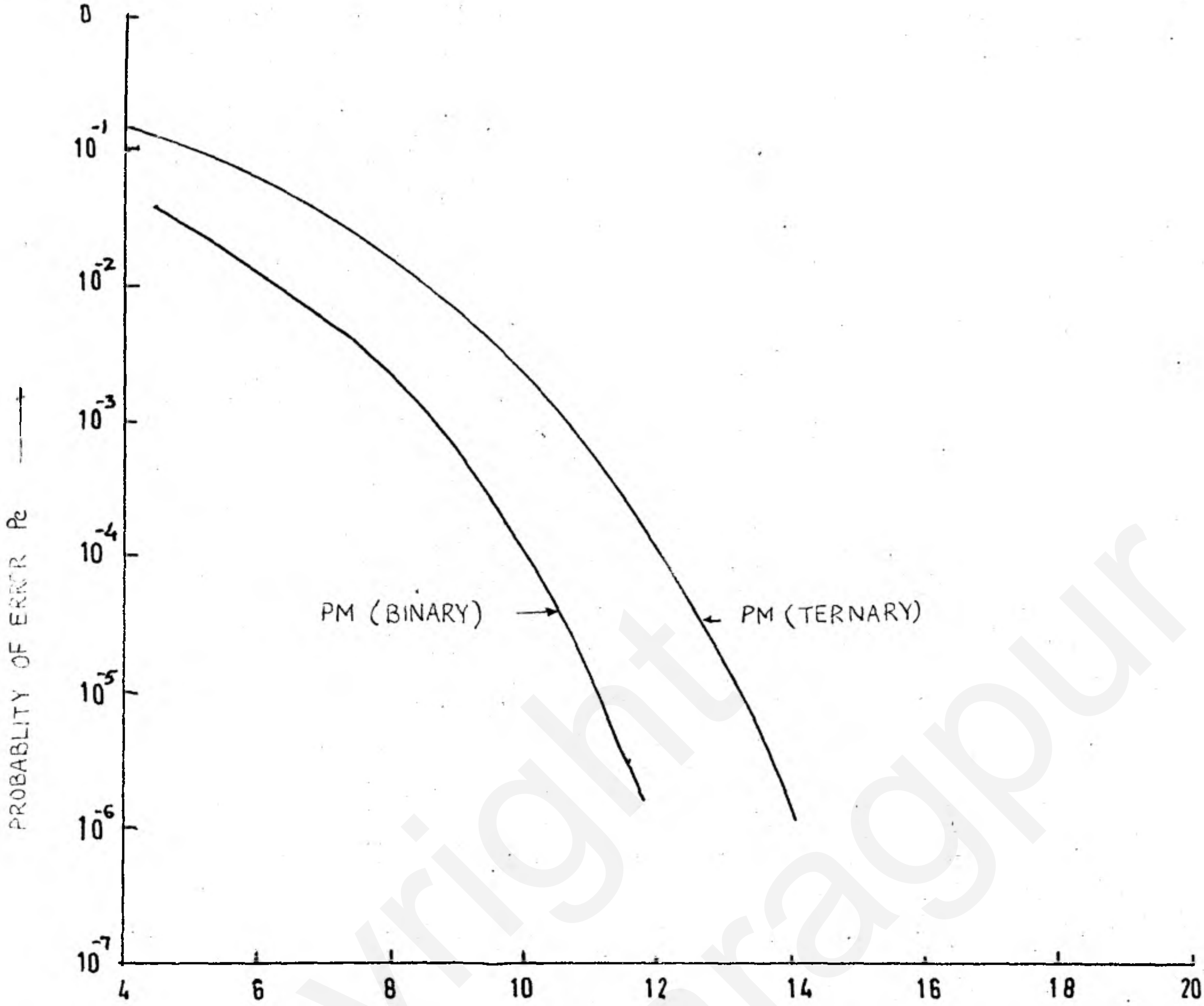
The eq.(5.7) shows that the SNR is dependent on f_r, f_m and P_e. Choosing a pulse repetition frequency of 40 Kc/s and f_m of 3.33 Kc/s, $f_r/2f_m = 6$, and

$$[S_o/N_o]_{AVERAGE} = \frac{6}{32}\cdot\frac{1}{P_e} \quad \ldots (5.8).$$

Since P_e depends upon the input SNR $[S_i/N_i]$, as shown in Fig.5.3, the output SNR can be calculated from eq.(5.8). For different input SNR the output SNR is plotted in Fig.5.4 and Fig.5.5. The Fig.5.4 shows the variation of the S_o/N_o for the average S_i/N_i and Fig.5.5 shows the same for the peak S_i/N_i. For a peak S_i/N_i of 18.2 db, or average S_i/N_i of 15.2 db, the output SNR is seen to be about 40 db, which is normally acceptable for a satisfactory transmission.

5.1.2. TERNARY SQ-PCM.

For the video transmission of the +1, 0, -1 pulses of the Ternary SQ-PCM system, the pulse heights of the transmitted signals will be 2V, as shown in the Fig.5.6(a), with two thresholds at +V and -V. Every time the received signal crosses either +V or -V volts threshold, a "plus one" or a "minus one" pulse is generated, otherwise, a "zero" is interpreted. To calculate the average probability of error P_e' for this system, it is seen that P_e' will be greater than P_e, which is the probability of error in the binary case. Assuming noise bursts of amplitudes greater than $\pm V$, the probability of error is not equal for all the three levels of +1, 0, -1. If a '0' was transmitted, noise bursts of magnitudes $V \leq |\upsilon| \leq \infty$, or $-V \leq |\upsilon| \leq -\infty$, will give rise to an error, whereas if '+1' was transmitted, a noise burst of amplitude $V \leq |\upsilon| \gtrless \infty$ will not effect the identification at the receiver, but a noise burst of amplitude $-V \leq |\upsilon| \leq -\infty$ will destroy the pulse. Similarly, if a '-1' was transmitted, noise bursts of magnitude $-V \leq |\upsilon| \leq -\infty$ will not effect the reception, but a noise burst of magnitude $V \leq |\upsilon| \leq \infty$ will obliterate the pulse.



A PLOT OF Pe VS. AVERAGE SNR (c)

FIG. NO. 5·3

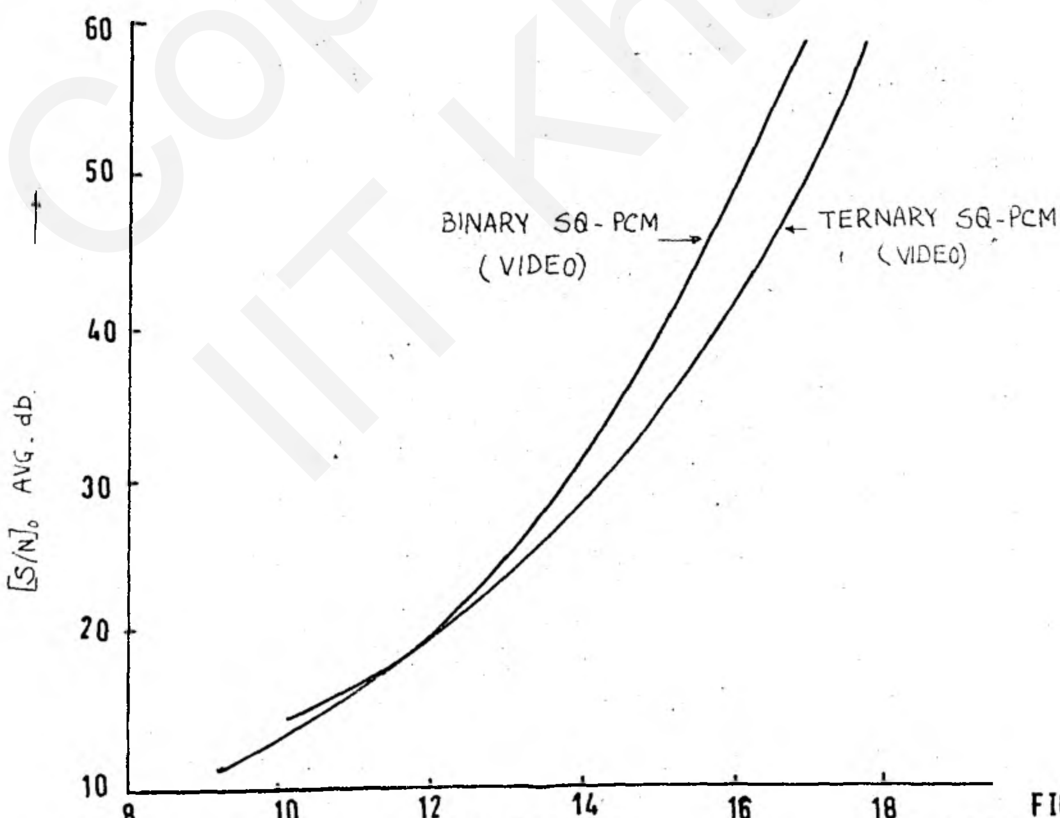


FIG. NO. 5·4
VARIATION OF So/No FOR

The error probability of '0' will, therefore, be twice that of a '+1' or a '-1', and the probability of error in the Ternary system is

$$P_e' = \frac{4}{3} P_e$$

Sanders[7] has shown that for a multilevel transmission scheme, the probability of error P_e' is $= \left[2(L-1)/L\right] \cdot P_e$, where L is number of levels transmitted. For L = 3, the P_e' of the Ternary system is $4/3\ P_e$, which is the result obtained above. P_e is the probability of error when only 0 or 1 is being transmitted and has been calculated before (eq.5.3). The peak power is $4V^2$ as in the binary case, but the average power, assuming an equal probability of +1, 0, -1, is $(4V^2 \times 2)/3$ or $8V^2/3$, that is, 1.75 db below the peak.

The mean noise power for a word of n digits is,

$$N_o = (\Delta h)^2 \cdot \left[1 - (1 - P_e')^n\right]$$
$$\text{or} \quad N_o = (\Delta h)^2 \cdot \left[1 - (1 - \tfrac{4}{3} P_e)^n\right] \qquad \ldots (5.9).$$

The peak-to-peak signal built up by the n digits of Ternary pulses is $n \cdot \Delta h$, as shown in Section 2.4, and the rms signal output for a sinusoidal waveform is

$$V_o = (n \Delta h) / 2\sqrt{2}$$

The signal (average) power at the output of the receiver is

$$S_o = \frac{(\Delta h)^2}{8} \cdot \left(\frac{f_r}{2f_m}\right)^2 \qquad \ldots (5.10).$$

and the average SNR is

$$\left[S_o/N_o\right]_{\text{TERNARY}} = \frac{1}{8} \cdot \left(\frac{f_r}{2f_m}\right)^2 \cdot \frac{1}{\left[1 - (1 - \tfrac{4}{3} P_e)^n\right]}$$

which, for small P_e, is reduced to

$$\left[S_o/N_o\right]_{TERNARY} = \frac{3}{32} \cdot \frac{f_r}{2f_m} \cdot \frac{1}{P_e} \qquad \ldots (5.11).$$

Choosing a PRF of 40 Kc/s and f_m = 3.33 Kc/s, the average SNR is

$$\left[S_o/N_o\right]_{TERNARY} = \frac{18}{32} \cdot \frac{1}{P_e} \qquad \ldots (5.12).$$

As P_e depends upon the input SNR, the variation of S_o/N_o for different S_i/N_i, peak and average, is plotted in Fig.5.5 and 5.4, and it is seen that a peak input S_i/N_i of 18.2 db is required for an output S_o/N_o of 45 db.

5.2. R.F. TRANSMISSION.

Often it happens that the communication link is used over an R.F. medium and that necessitates the modulation of a suitable R.F. carrier with the SQ-PCM pulses. Many modulation schemes are possible, and in the present Section, four of the most commonly used modulation schemes have been discussed with a view to ascertain the requirements of transmitter power for a desirable SNR at the output of the receiver. They are SQ-PCM-AM, SQ-PCM-FM, SQ-PCM-FSK and SQ-PCM-PM.

5.2.1. AMPLITUDE MODULATION.

Among many methods of radio frequency transmission, the amplitude modulation method is possibly the simplest. The trans-

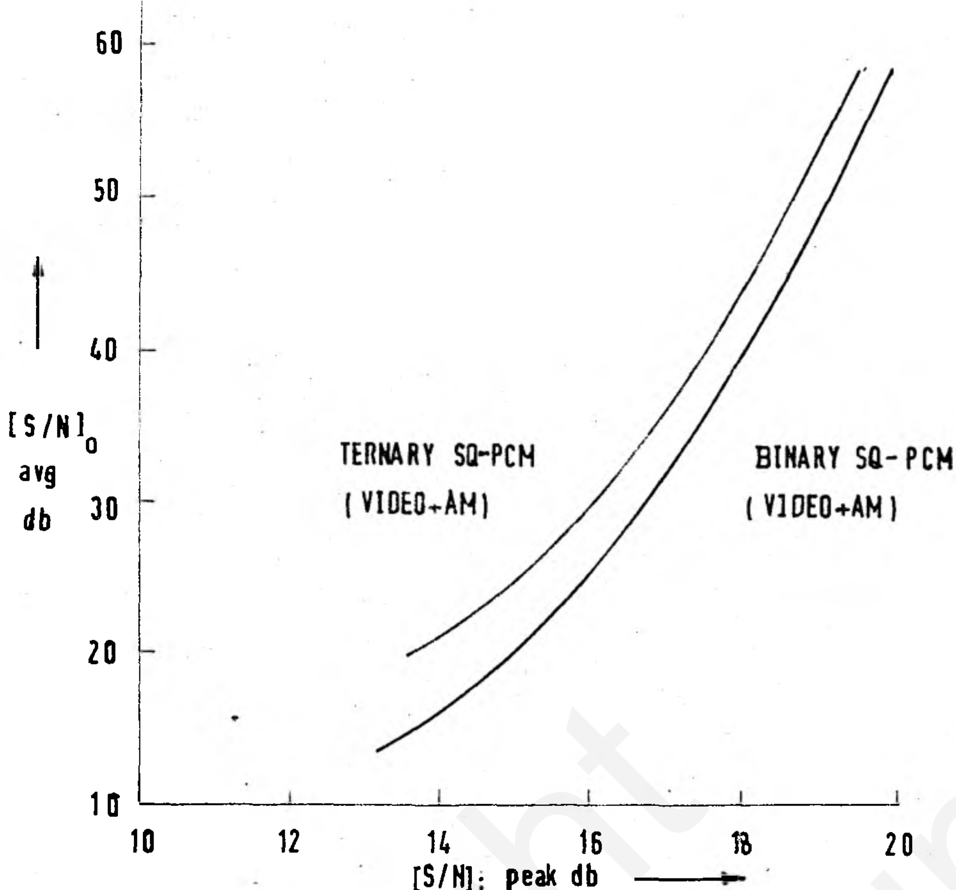
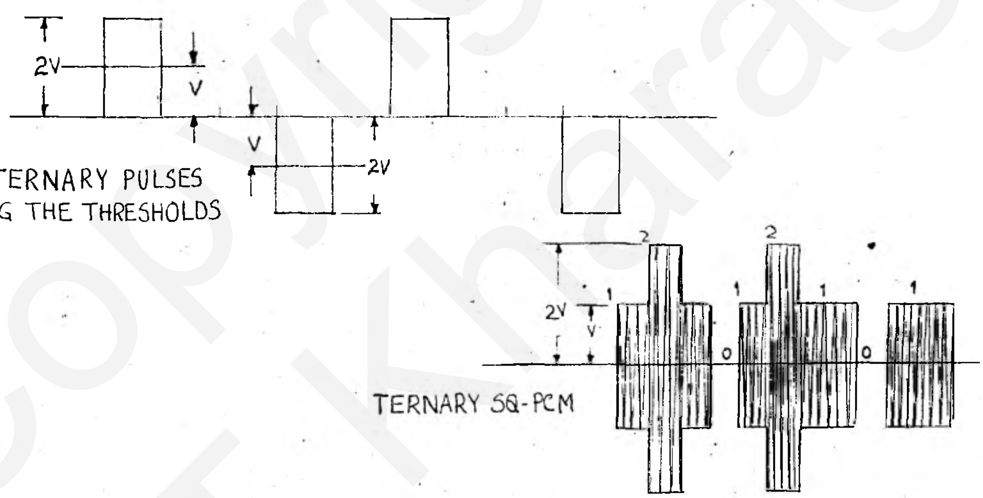


FIG. NO. 5·5

VARIATION OF S_o/N_o FOR AVERAGE S_i/N_i



TERNARY PULSES SHOWING THE THRESHOLDS

FIG. NO. 5·6 (a)

TERNARY SQ-PCM

(b)

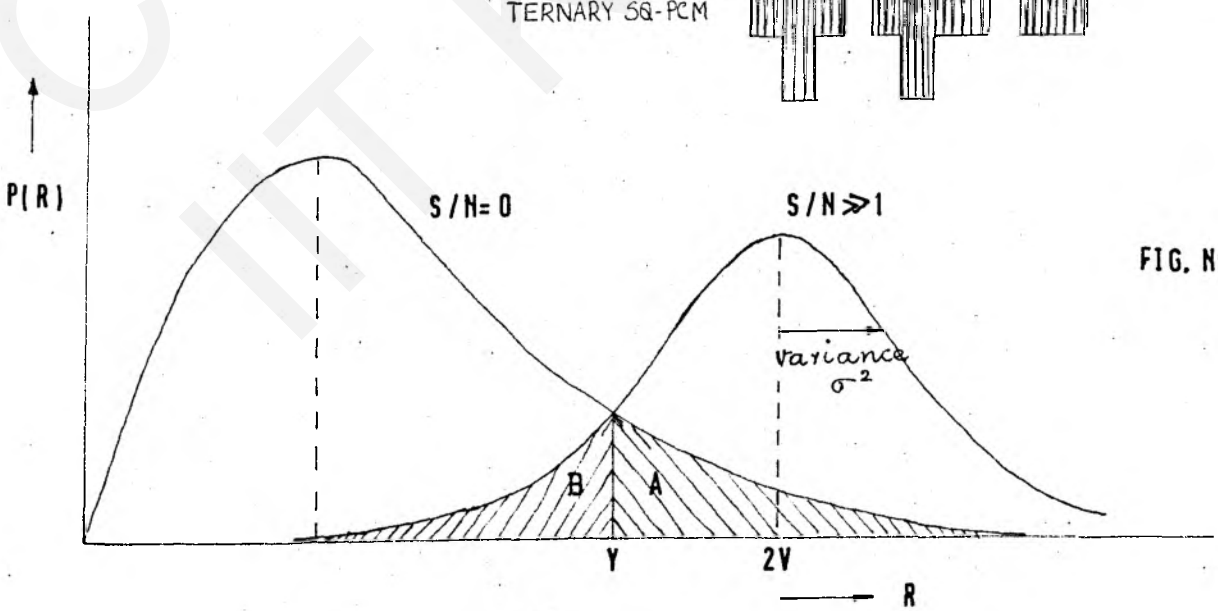


FIG. NO. 5·7

PROBABLITY DENSITY DISTRIBUTION, FOR NOISE ALONE AND FOR SIGNAL PULSE NOISE mission of Binary SQ-PCM pulses involves the transmission of 2 levels, a "zero" or "one", and the pulses therefore could amplitude modulate a carrier such that the full power is transmitted for a "one" and no power for a "zero". At the receiver an envelope detector would be required to demodulate these AM signals. The amplitude modulation of Ternary SQ-PCM, however, involves some problems, as three levels, 0, 1, 2 (for -1, 0, +1), require to be transmitted, the receiver again being an envelope detector. The envelope detector is followed by a threshold device in all cases. If the output of the detector exceeds the threshold, a "one" is read out, and if it is less than the threshold, a "zero" is interpreted, and so on. It will now be shown that for large SNR at the input of the envelope detector, the probability density distribution of signal plus noise is Gaussian at the output of the detector. First, it is assumed that noise alone is present at the input of the detector, and then, signal plus noise is present at the input. The amplitude distribution of the random noise at the input is Gaussian, with a mean value of zero, and a signal of amplitude 2V shifts this distribution such that the average amplitude is now 2V, and the curve peaks around 2V. The probability distribution of the signal plus noise at the input of the detector is

$$p(v) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-(v-2V)^2/2\sigma^2}$$

To calculate the probability density of the amplitude of signal plus noise at the output of the detector, the following procedure is adopted.[19] If a white noise is passed through a narrow band filter the output v(t) could be represented by a sine wave of frequency ω_c, with slowly varying amplitude and phase i.e.

$$v(t) = R(t).\cos\left[\omega_c t + \theta(t)\right]$$

where R_t represents the slowly varying envelope of the amplitudes and $\theta(t)$, the slowly varying phase. R(t) would be the voltage at the output of an envelope detector, and $\theta(t)$ would be at the output of a phase-sensitive detector. The probability density function of the envelope R(t) is P(R) and is

$$P(R) = R\, e^{-R^2/2\sigma^2} . \frac{1}{\sigma^2} \qquad \ldots (5.13).$$

This is the Rayleigh Distribution with its peak at R = σ and a peak value of $e^{-1/2}/\sigma$. As σ the rms input noise increases, the distribution flattens out, the peak decreasing and moving to the right. The envelope can have only positive values. Now if signal plus noise, signal of the form $2V\cos\omega_c t$, is applied to the envelope detector, the probability density function can be written as

$$P(R) = \frac{e^{-(2V)^2/2\sigma^2} . R . e^{-R^2/2\sigma^2}}{2\pi\sigma^2} \int_0^{2\pi} e^{2VR\cos\theta/\sigma^2} . d\theta.$$

The integral in the above equation can be evaluated by noting that it has the same form as the Bessel function of the first kind and zero order i.e.

$$I_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{z\cos\theta} . d\theta$$

In terms of this modified Bessel function, the probability density function for case of signal plus noise is

$$P(R) = \frac{R}{\sigma^2} \cdot e^{-(R^2+4V^2)/2\sigma^2} \cdot I_0\left(\frac{2VR}{\sigma^2}\right) \qquad \ldots (5.14).$$

If $2V \rightarrow 0$, the eq.(5.14) reduces to the Rayleigh distribution, checking with the result of zero signal case. According to the previous definitions, the mean R.F. signal power is $2v^2$ and mean noise power is σ^2 and therefore, $2V^2/\sigma^2$ is the SNR at the input of the envelope detector. For sufficiently large values of SNR ($2V \gg \sigma$), the eq.(5.14) will give rise to the Gaussian distribution, as can be seen below. For large values of Z, the modified Bessel function can be approximated as

$$I_0(z) = \frac{e^z}{\sqrt{2\pi z}} \qquad \ldots (5.15)$$

and hence, if it is assumed that $2VR \gg \sigma^2$ the eq.(5.14) can be written by substituting in the value of $I_0\left(\frac{2VR}{\sigma^2}\right)$ according to eq.(5.15).

and $$P(R) = \frac{R}{\sqrt{2\pi \cdot 2V \cdot R \cdot \sigma^2}} \cdot e^{-(R-2V)^2/2\sigma^2} \qquad \ldots (5.16).$$

The function of eq.(5.16) peaks sharply about the point R = 2V, and in this range of R, one can let $R \approx 2V$ in the nonexponential portions of P(R) and get,

$$P(R) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(R-2V)^2/2\sigma^2}$$

Therefore in the vicinity of the point R = 2V, which is essentially the average value of R, the distribution at the output of the detector approximates a Gaussian distribution, with a mean value of $\sqrt{4V^2/\sigma^2} = \sqrt{\frac{S_p}{N}}$ and variance σ^2 , and is valid only for large input SNR. This normal distribution is thus identical with that at the input to the detector, as it has the same average amplitude 2V and the same variance σ^2 . Hence, it can be said that the envelope detector does not alter the probability density of the signal plus noise. The probability density distribution for the case of noise alone and for signal plus noise is shown in Fig.5.7.

The level Y of the threshold circuit after the detector has to be adjusted to an optimum value such that

$$P(1/0) = P(0/1) = P_e$$

as in the case of the Video transmission. That is,

$$P_e = \int_Y^{\infty} \frac{R\, e^{-R^2/2\sigma^2}}{\sigma^2}\, dR = \int_0^Y \frac{R}{\sigma^2} \cdot e^{-(R^2+4V^2)^2/2\sigma^2} \cdot I_0\left(\frac{2VR}{\sigma^2}\right) \cdot dR$$

For the P_e to be minimum, it can be shown that Y, the threshold level, is approximately equal to V (the optimum threshold level), and for large SNR,

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-V/\sigma} e^{-x^2/2} \cdot dx \qquad \ldots (5.17(a)).$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{1}{2}\sqrt{S/N}} e^{-x^2/2}\, dx \qquad \ldots (5.17(b)).$$

$$= \frac{1}{2} - \operatorname{erf}\left(V/\sigma\right)$$

where x is the variable V/σ

and

$$erf(x) = \frac{1}{\sqrt{2\pi}} \int_0^{V/\sigma} e^{-x^2/2} \cdot dx$$

As seen from Fig.5.7, the probability of interpreting "zero" as "one" is the area A, and the probability of interpreting a "one" as "zero" is the area B. These two areas are approximately equal for large SNR.

The average power in the AM is 3 db below peak the peak power and the variation of P_e with the peak and average (S_i/N_i) is similar to the video transmission case, shown in Fig.5.2 and 5.3.

BINARY SQ-PCM-AM.

The output noise power N_o will be given by,

$$N_o = (\Delta h)^2 \times \left[1 - (1 - P_e)^n\right] \qquad \ldots (5.4).$$

and the output signal power S_o by,

$$\left[S_o\right]_{BINARY} = \frac{(\Delta h)^2}{32} \cdot (f_r/2f_m)^2 \qquad \ldots (5.5).$$

and therefore, the output SNR for a PRF of 40 Kc/s, and f_m = 3.33 Kc/s is

$$\left[S_o / N_o\right]_{BINARY} = \frac{1}{32} \cdot \frac{1}{P_e} \times \frac{f_r}{2f_m} = \frac{6}{32} \cdot \frac{1}{P_e} \qquad \ldots (5.8).$$

The P_e will depend on the input SNR and the plot of peak input SNR vs. output SNR is shown in Fig.5.5.

TERNARY SQ-PCM-AM.

The amplitude modulation of carrier by +1, 0, -1 pulses, converted to 0, 1, 2 pulses, will be as shown in Fig.5.6(b). The peak power is still $4V^2$ and the average power, for 0, 1, 2, being equally probable, is $5V^2/3$ or 3.8 db below the peak power. The probability of error is $P_e' = 4/3\ P_e$ (as given in Section 5.1.2) where P_e is the probability of error in the 0/1 transmission. The signal power and the noise power can be calculated in a manner similar to the one given in Section 5.1.2. The output SNR for large input SNR is

$$[S_o/N_o]_{TERNARY} = \frac{3}{32}\cdot\left(\frac{f_r}{2f_m}\right)\cdot\frac{1}{P_e} \qquad \text{(eq.5.11).}$$

The variation of S_o/N_o for peak S_i/N_i is shown in Fig.5.5.

5.2.2. FREQUENCY MODULATION.

The binary and the ternary pulses of the SQ-PCM systems could be transmitted by frequency modulating a carrier. At the receiver a discriminator type of detector will give two voltage levels in the binary and three voltage levels in the ternary system. The discriminator circuit first converts the frequency modulated signals into amplitude modulated signals and then detects the amplitude modulated signals in the usual way. The analysis of this type of FM detection is therefore, an extension of the AM detection case. The output of the discriminator is passed through similar threshold circuits to recover the original pulses. For sufficiently large signal to noise ratio at the input of detector, the output could be assumed to have a Gaussian distribution, and the probability of error is the same as for SQ-PCM-AM and is

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-V/\sigma} e^{-x^2/2}\, dx = \frac{1}{2} - erf(V/\sigma).$$

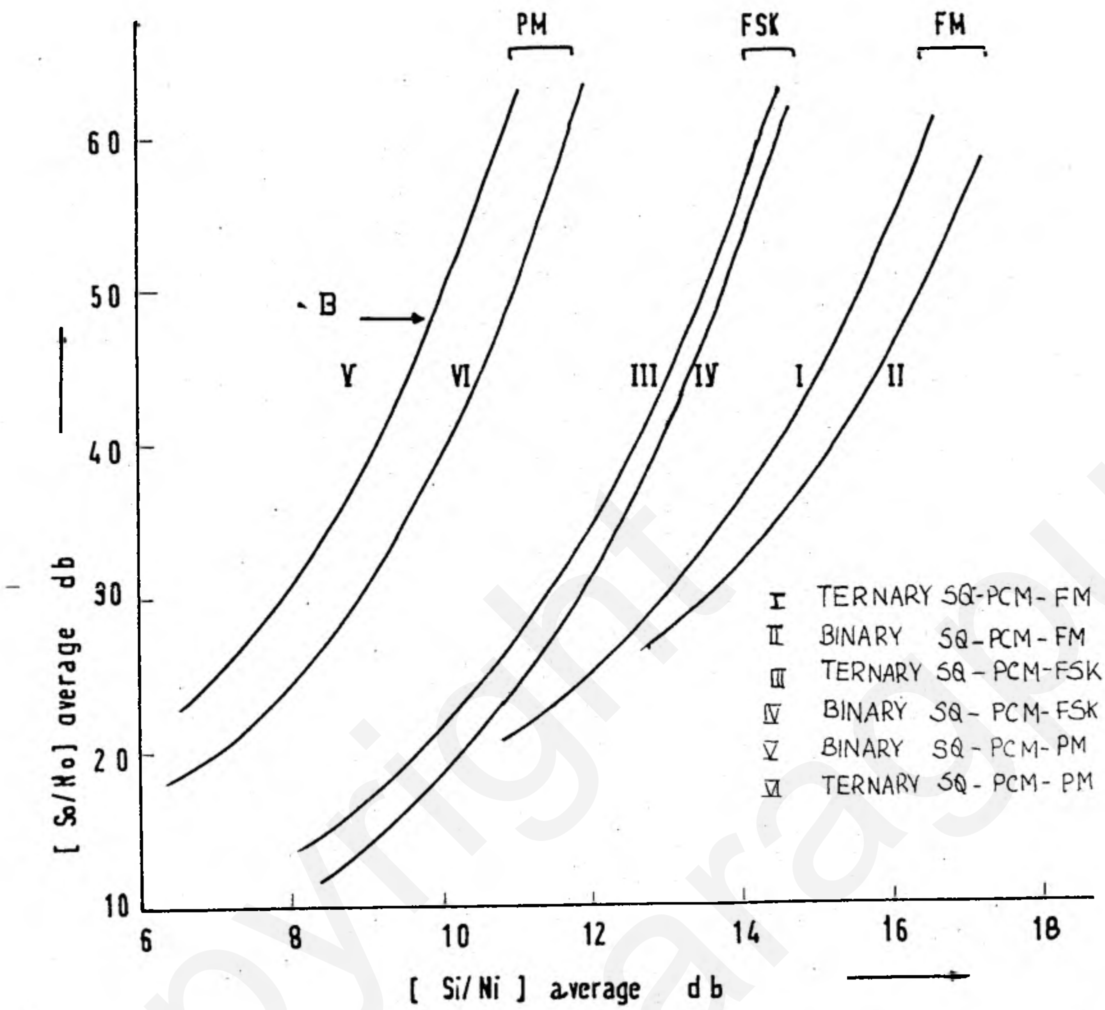
where $$erf(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-x^2/2}\, dx.$$

All the advantages of an -FM system over an -AM system will be obtained and the calculation of the output SNR will be similar to the AM case, except that a threshold effect is obtained in the FM system which will give better S_o/N_o. As before the Ternary SQ-PCM will have two threshold circuits and the probability of error will be 4/3 P_e.

The amplitude of the modulating pulses is chosen such that a deviation ratio of unity is produced. It is known that for a unity deviation ratio the SNR advantage of FM is 4.8 db over AM. Therefore, to each value of S_o/N_o, calculated with the help of eq.(5.18) and (5.19) for a certain S_i/N_i, 4.8 db is added to obtain the output SNR in the FM modulation scheme. The variation of output SNR with input SNR is shown in Fig.5.8 for both the Binary and Ternary SQ-PCM systems.

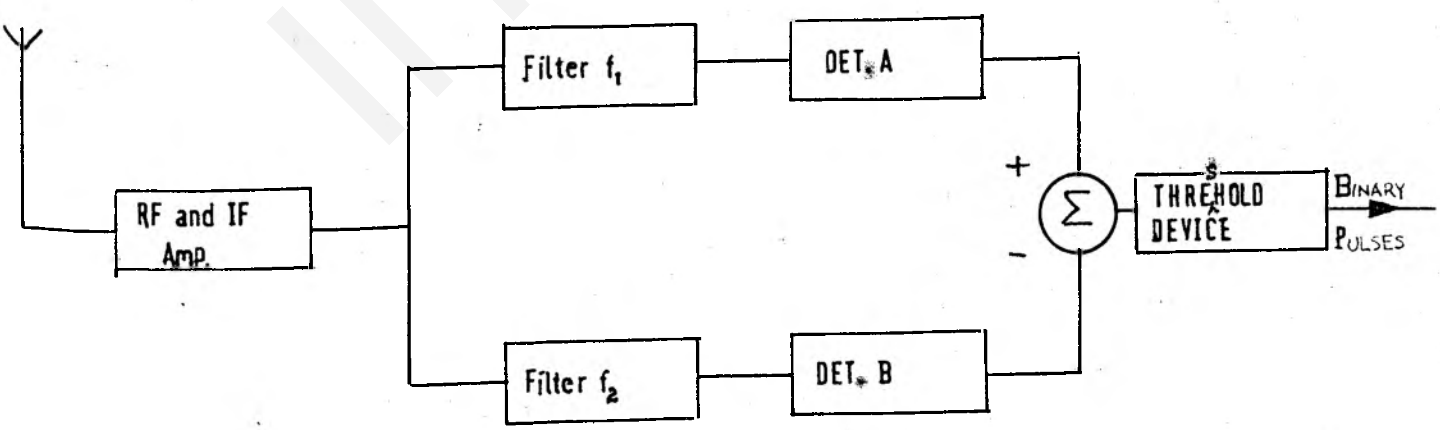
5.2.3. FREQUENCY MODULATION (FSK).

The Frequency Modulation (FSK) differs from the scheme considered in Section 5.2.2, only in the type of detector used at the receiver. In the simple FM case a discriminator type of detector has been assumed, but it will be shown here that a slight modification in the detection leads to much better results.



VARIATION OF So/No FOR AVERAGE Si/Ni

FIG. NO. 5·8



RECEIVER FOR TWO TONE BINARY SQ-PCM-FSK

FIG. NO. 5·9

Two-tone Frequency Shift Keying could be employed for the transmission of the Binary SQ-PCM pulses[29]. The carrier jumps between two frequencies f_1 and f_2, f_1 representing the "one" and f_2 the "zero" of the pulses. At the receiver, Fig.5.9, two tuned circuits, tuned to f_1 and f_2, will be used followed by two detectors, a difference circuit and a threshold device. The threshold circuit reads out "one", if the output of the detector, A is greater than the output of the detector B, and a "zero", if the reverse is true. The probability of an error at the output of the threshold circuit will now be modified because of the differencing circuit. The analysis proceeds in a manner similar to AM case, where the probability density distribution of the output of the detector when noise alone is present at the input is P(N), where,

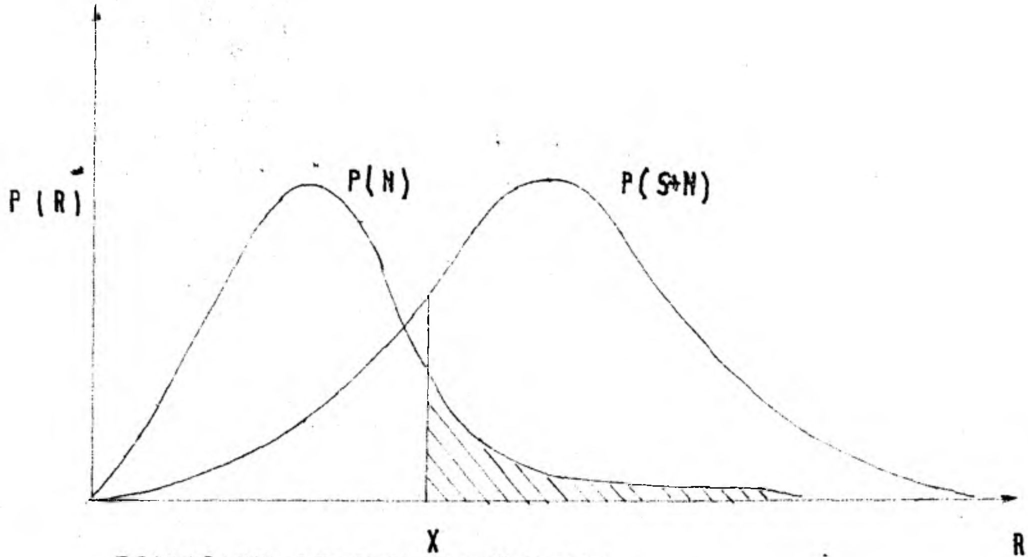
$$P(N) = P(R) = \frac{R}{\sigma^2} \cdot e^{-R^2/2\sigma^2} \qquad \ldots (5.20).$$

and the distribution when signal plus noise is applied to the detector is P(R'), and is,

$$P(R') = P(S+N) = \frac{R'}{\sigma^2} e^{-(R'^2+4V^2)/2\sigma^2} \cdot I_0\left(\frac{2VR'}{\sigma^2}\right) \qquad \ldots (5.21).$$

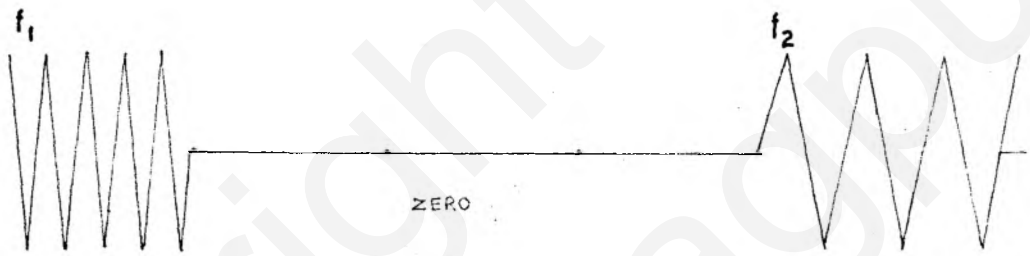
A plot of P(N) and P(S + N) is shown in Fig.5.10.

Now if it is assumed that a "one" was transmitted as a frequency f_1, it will be accepted by the filter tuned to f_1, detected by the Det.A and will have a distribution corresponding to eq.(5.21). At the same time the output of the Det.B, which will have noise only at its input and therefore, a distribution following eq.(5.20), will also be applied to the difference circuit. The



PROBABLITY DENSITY DISTRIBUTION
FOR NOISE ALONE AND SIGNAL PLUS NOISE

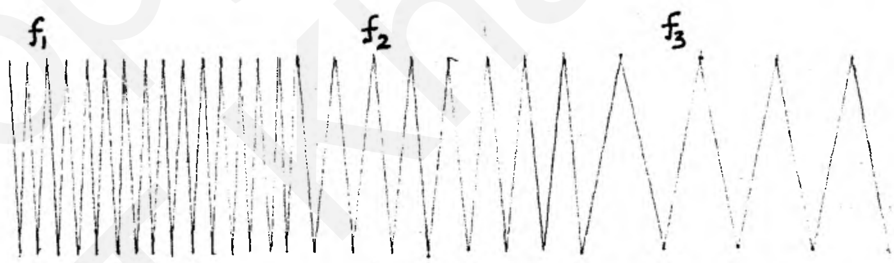
FIG. NO. 5·10



Two tone

TERNARY SQ-PCM-FSK
a. TWO TONE KEYING

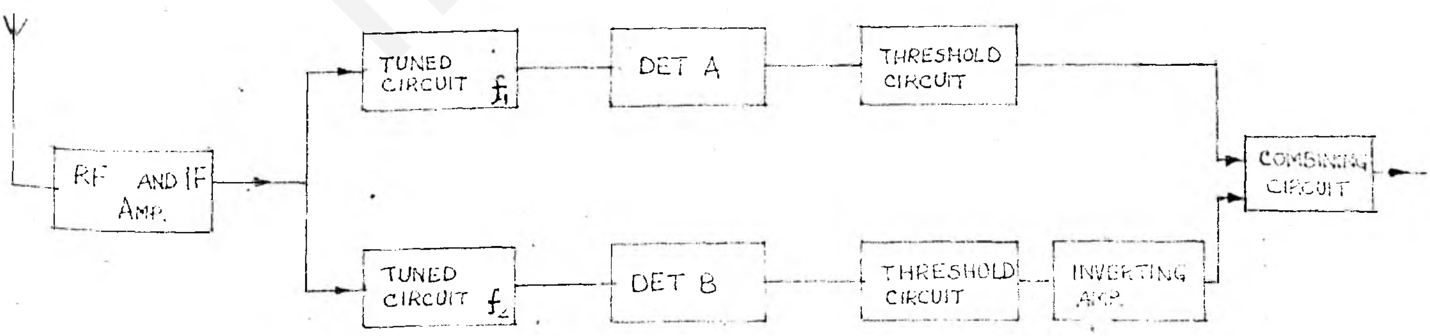
FIG. NO. 5·11 (a)



Three tone

TERNARY SQ-PCM-FSK
b. THREE TONE KEYING

FIG. NO. 5·11 (b)



RECEIVER FOR TWO-TONE TERNARY SQ-PCM-FSK

FIG. NO. 5·12

probability of an error will then be the probability that, the noise in the B channel is more than the signal plus noise in A channel. The probability that the noise will exceed the level set at X volts (Fig.5.10) is

$$P(\text{error, if } R' = X) = \int_X^\infty P(R)\,dR$$

where P(R) is given by eq.(5.20). The input of the difference circuit has both noise and signal plus noise, and therefore, the joint distribution at the input is

$$P(\text{error}, R') = P(R').\int_X^\infty P(R)\,dR$$

and the probability of an error

$$P_e = \int_0^\infty \left[P(R') \int_X^\infty P(R)\,dR \right] dR' \qquad \ldots (5.22).$$

The integral in eq.(5.22) can be evaluated to give the probability of error

$$P_e = \frac{1}{2} \cdot e^{-V^2/\sigma^2} \qquad \ldots (5.23)$$

Because the average power is $2V^2$, the eq.(5.23) can be rewritten as

$$P_e = \frac{1}{2} \cdot e^{-S/2N} \qquad \ldots (5.24).$$

A plot of the variation of P_e with input SNR is shown in Fig.5.2.

The output SNR of the Binary SQ-PCM-FSK system will be

$$[S_o/N_o]_{BINARY} = \frac{1}{32} \left(\frac{f_r}{2f_m}\right)^2 \cdot \frac{1}{[1-(1-P_e)^n]} \qquad \ldots (5.25).$$

With P_e as given by eq.(5.24). Fig.5.8 shows the relationship between the input SNR and output SNR of the modulation scheme.

For the transmission of the Ternary SQ-PCM pulses either a two-tone or a three-tone-FSK could be used. In the two-tone scheme, a '+1' is transmitted as a frequency f_1, a '-1' is transmitted as a frequency f_2 and a "zero" is no transmission, as shown in Fig.5.11(a). In the three tone FSK scheme, three frequencies f_1, f_2 & f_3, could be Keyed to represent +1, 0, and -1 pulses, as in Fig.5.11(b).

At the receiver, in the two-tone scheme, there will be two tuned circuits-tuned at f_1 and f_2, followed by two detectors, two threshold circuits and a combining circuit, as shown in Fig.5.12. When a "zero" is being transmitted, none of the threshold circuits will give any output and hence, at the output of the combining circuit there will be a zero. The +1 and -1 pulses will be obtained directly. The probability of error in reception will follow exactly as in the case of AM and FM, and since, the error chances for a "zero" are double that of the error chances of '+1' or '-1', the weightage factor will be 4/3, as before. The output S_o/N_o is given as,

$$[S_o/N_o]_{TERNARY} = \frac{1}{8}\cdot\left(\frac{f_r}{2f_m}\right)^2\cdot\frac{1}{[1-(1-P_e')^n]}$$

$$\text{or} \quad \approx \frac{3}{32}\cdot\left(\frac{f_r}{2f_m}\right)\cdot\frac{1}{P_e}$$

This is similar to the Ternary SQ-PCM-AM scheme, given in Fig.5.5.

For the 3-tone-FSK scheme, the receiver (Fig.5.13) consists of three tuned circuits, tuned to f_1, f_2 and f_3, followed by three detectors A, B and C. The outputs of detectors A and C are compared with the output of the detector B in two difference circuits as shown. If the output of detector A is greater than detector B, a '+1' pulse is read out by the threshold circuit; if the output of detector C is greater than that of detector B, the threshold circuit reads out a '-1' pulse. In case the outputs of detector A and C are smaller than that of B, the threshold circuits do not produce any output and a "zero" is read out. It is assumed that the chance of a '+1' being wrongly read as '-1' is negligible and therefore no comparison between the outputs of detector A and C is required.

As there are two decision circuits, on a heuristic basis the probability of getting an error is twice that of given in eq.(5.24), and P_e'' given by,

$$P_e'' = e^{-S/2N} \qquad \ldots (5.26).$$

The output SNR is, therefore,

$$\left[S_o/N_o\right]_{\text{TERNARY}} = \frac{1}{8} \cdot \left(\frac{f_r}{2f_m}\right)^2 \cdot \frac{1}{\left[1 - (1 - P'')^n\right]} \qquad \ldots (5.27).$$

where P_e'' is given by eq.(5.26). A plot of input vs. output SNR for the three tone FSK modulation scheme is also given in Fig.5.8.

5.2.4. PHASE MODULATION.

The pulses of the Binary and Ternary SQ-PCM systems could be transmitted by the Phase modulation of a carrier. Suppose an m-level code is to be transmitted by phase modulation, then the levels may be represented by m-vectors spaced equally apart in a 360^0 phase space. The spacing between the phase vectors will be $2\pi/m$ radians. At the detector, either due to the noise in the detector itself, or due to additive noise in the channel, the phase of the vector will depart from its nominal value. So long as the phase of the signal plus noise is within the range of $\pm\,\pi/m$ radians of its original phase, it will be taken as the signal vector itself. If all levels are equally likely, this decision threshold will give the minimum probability of error.

Assuming a narrow band receiver again, the probability density function of the phase difference between signal alone, and signal-plus-Gaussian-noise[30], is given as,

$$P(\theta) = \frac{1}{2\pi} e^{-S/N} \left[1 + \sqrt{\frac{4\pi S}{N}}\, e^{(S/N)\cos^2\theta} \cdot \cos\theta \cdot \Phi\left(\sqrt{\frac{2S}{N}} \cdot \cos\theta\right) \right] \quad \ldots (5.28).$$

where $\Phi(x)$ is the probability integral.

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy$$

If the reference phase is known i.e. locally generated, the probability of error is

$$P_e = 1 - \int_{-\pi/m}^{+\pi/m} P(\theta) \cdot d\theta \quad \ldots (5.29).$$

For large signal to noise ratios in the vicinity of $\theta = 0$ (if $\theta = 0$ is the assumed phase angle of the signal), the of eq.(5.28) will approach a Gaussian distribution,

$$P(\theta) \simeq \frac{e^{-(S/N)\cdot\theta^2}}{\sqrt{\pi(S/N)}} \qquad \text{if } S/N \gg 1,\ \theta < 5^\circ$$

with a mean value of zero and variance $\sigma^2 = \frac{1}{2(S/N)}$

For the Binary SQ-PCM system the number of levels is m = 2, and the value of $P(\theta)$ and hence P(e), is evaluated from eqs.(5.28) and (5.29). In Fig.5.3, P_e is plotted as a function of input SNR; the value of the output SNR for different input SNR is calculated from a equation similar to the eq.(5.25) and plotted in Fig.5.8.

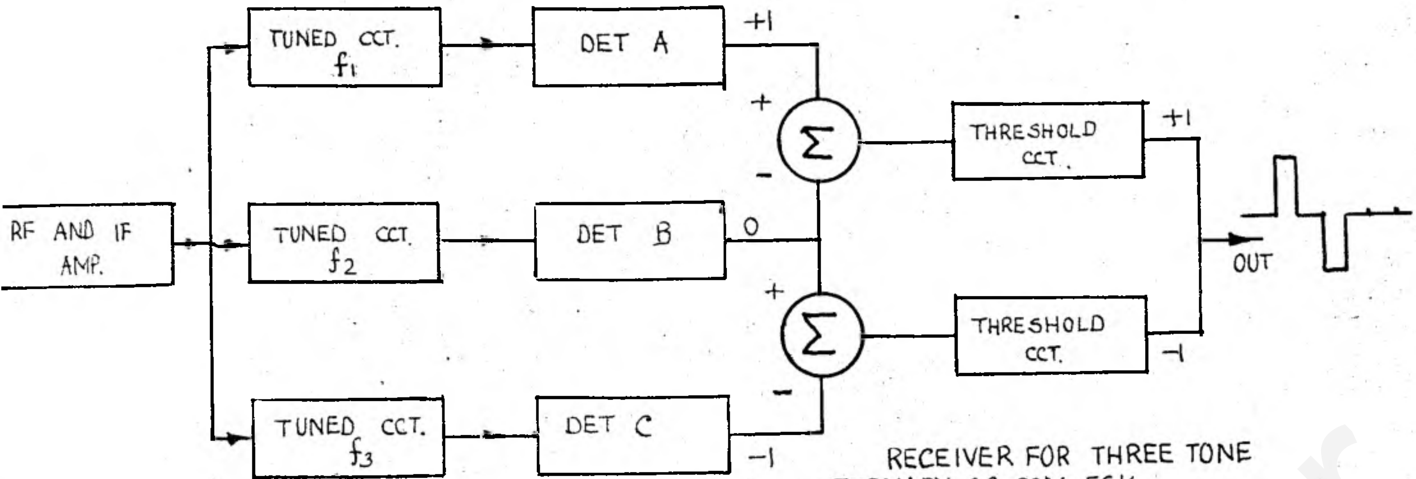
For the Ternary SQ-PCM system, the number of levels to be transmitted are three and therefore, with m = 3, $P(\theta)$ and hence P_e are evaluated from eq.(5.28) and (5.29). Fig.5.3 also shows a plot of P_e vs. input SNR and Fig.5.8 shows a plot of output SNR vs. input SNR calculated from an equation similar to eq.(5.27).

Both the curves of PM shown in Fig.5.3 and 5.8 have actually been calculated for an input SNR upto 10 db only, and then extrapolated to have the same general similarity with the curves given for the FSK system and others. This limit in calculation was unavoidable because the integrations of eqs.(5.28) and (5.29) were performed with the IBM 1620 Computer, using "Fortran" programming, and an accuracy of more than six significant figures was not possible.

A scheme of phase modulating the sinusoidal carrier with the Ternary SQ-PCM pulses can be as follows. Three phase vectors spaced 120^{o} apart are as shown in Fig.5.14(a) i.e., if it is assumed that '0' pulse has a zero phase, then '+1' and '-1' pulses have a phase difference of 120^{o} with it and with each other. The '0' pulse has a zero phase and therefore it is straightaway generated. To produce a phase difference of 120^{o} for a '+1' pulse, the carrier signal is inverted, reduced to half and then added in quadrature with $\sqrt{3}/2$ times the carrier as shown in Fig.5.14(b). Similarly the phase change of (-120^{o}) could be produced for '-1' pulse. In Fig.5.14(c) a block schematic diagram of a possible phase modulating circuit is given. Of all modulation schemes discussed in this Section, the phase modulation with a locally generated phase reference requires minimum signal power for a given output SNR. FSK, FM and AM require larger signal power in the same order.

5.3. BANDWIDTH REQUIREMENTS.

In earlier Sections some modulation schemes for the transmission of SQ-PCM pulses have been discussed, without regard to the bandwidth necessary for such transmission. Clearly, if the signal power is reduced for a particular communication with an undue increase in bandwidth, the system may not be as efficient as made out from consideration of the signal power alone. In the present Section a rough estimate has been made for the value of the minimum theoretical bandwidth necessary in each modulation scheme.



RECEIVER FOR THREE TONE TERNARY SQ-PCM-FSK

FIG. NO. 5.13

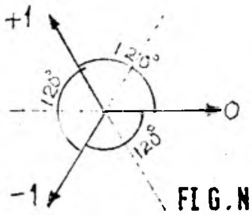
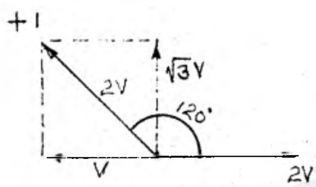
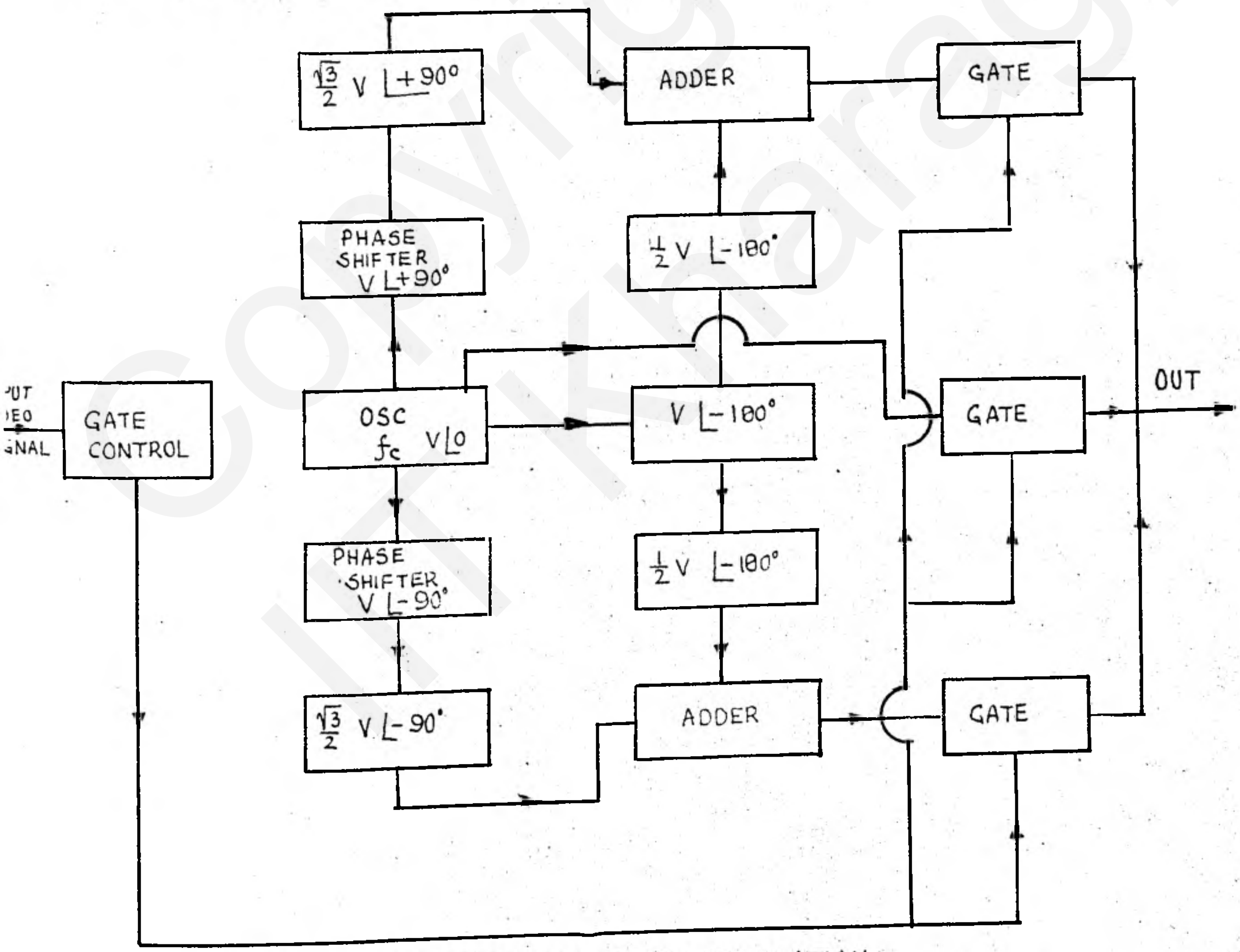


FIG. NO. 5.14 (a)



(b)



A SCHEME OF PRODUCING PHASE MODULATION

a. THREE PHASE VECTORS

b. GENERATION OF THE PHASER FOR +1 PHASE, AND

c. SCHEMATIC BLOCK DIAGRAM OF GENERATING TERNARY SQ-PCM-PM

FIG. NO. 5.14 (c)

The transmission of the Binary SQ-PCM is done by the presence or absence of a pulse, and if the pulses and spaces are uncorrelated, it is known that the power spectral density of the pulse train[27] is P(f), given by,

$$P(f) = \frac{|G(f)|^2 \, p(1-p)}{\tau} + |G(f)|^2 \cdot \frac{p^2}{\tau^2} \sum_{n=-\infty}^{\infty} \delta(f - nf_r) \quad \ldots (5.30).$$

where p is the probability of a pulse

$1 - p$ is the probability of no pulse

G(f) is the fourier transformer of the pulse shape

τ is the reciprocal of the PRF $= 1/f_r$

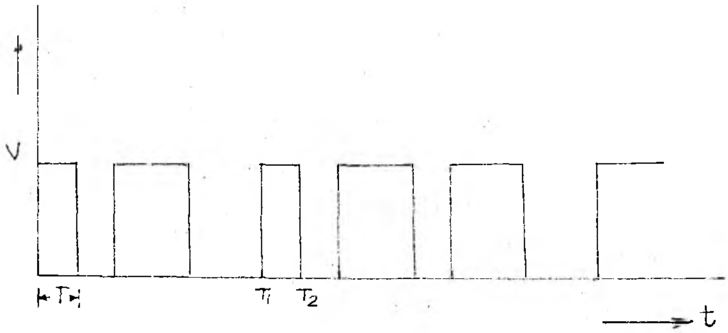
$\delta(f)$ is the impulse function.

The spectrum consists of a continuous and a line spectrum. The information can only be in the continuous part of the spectrum, because the line spectrum has ideally no bandwidth, and information cannot be transmitted in zero bandwidth.

A simple and straight forward method of determining the Bandwidth required for the video transmission of the pulses shown in Fig.5.15(a) will consist of finding the shape of these pulses after they have been passed through a low pass filter of cut-off frequency f_s c/s, and a linear phase shift = -gw, where g is a constant. Goldman[31] has shown that the output of the filter for a pulse of width $(T_2 - T_1)$ will be

$$G(t) = \frac{k}{\pi}\left\{ Si\left[\omega_s(t-g-T_1)\right] - Si\left[\omega_s(t-g-T_2)\right]\right\} \quad \ldots (5.31).$$

and for a fixed pulse width $(T_2 - T_1)$, if the bandwidth is reduced below $f_s = \frac{1}{2(T_2-T_1)} = \frac{1}{2\tau}$, the output signal



VIDEO PULSES

FIG. NO. 5.15 (a)

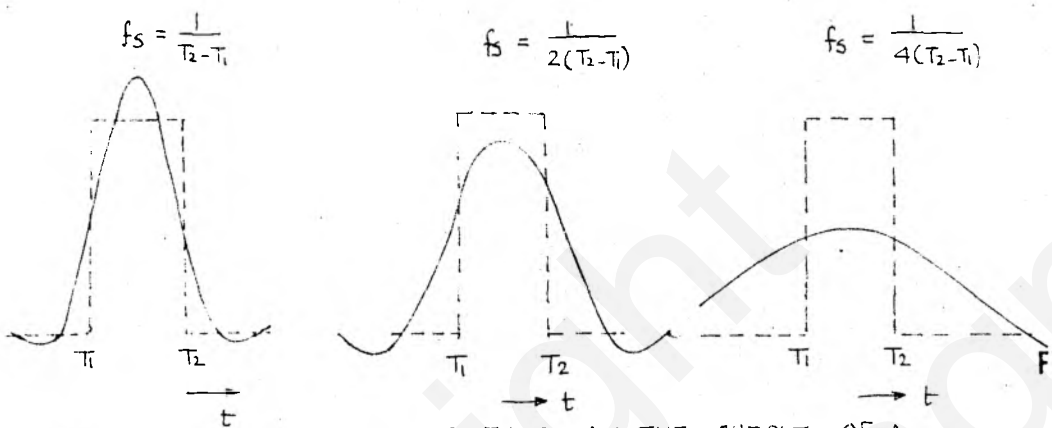
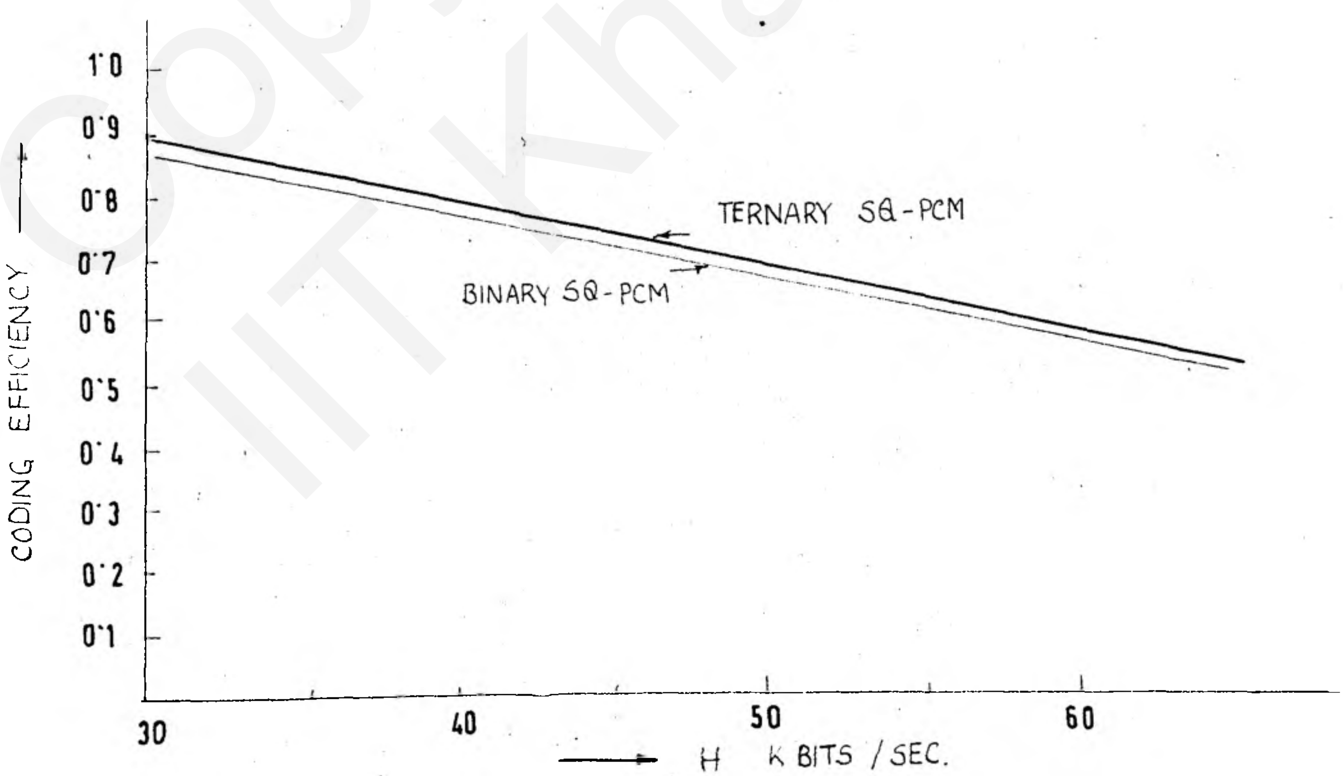


FIG. NO. 5.15 (b)

PULSE AMPLITUDE AT THE OUTPUT OF A FILTER OF VARIABLE CUT-OFF FREQUENCY



A PLOT OF CODING EFFICIENCY WITH THE INFORMATION RATE H.

FIG. NO. 5.16

amplitude decreases rapidly, as shown in Fig.5.15(b). Since the signal must exceed the noise level, a minimum bandwidth requirement therefore, will be

$$f(s) = \frac{1}{2T} = \frac{f_r}{2} \qquad \ldots (5.32).$$

where $\frac{1}{T}$ is the reperition rate of the pulses = f_r.

Now if the pulses are used to amplitude modulate a carrier of frequency f_c c/s, it has been shown (Goldman), that the output will be

$$G'(t) = \frac{K}{4\pi}\left\{Si\left[\omega_m(t-g-T_1)\right] - Si\left[\omega_m(t-g-T_2)\right]\right\} \times \cos\omega_c t \qquad \ldots (5.33).$$

where $\omega_m/2\pi$ is half the width of the pass band in the amplitude modulator circuit. The eq.(5.33) shows a carrier, amplitude modulated by a signal of the same form as eq.(5.31). Thus for a symmetrical double sideband transmission of the amplitude modulated pulses, twice the amount of the video bandwidth (eq.5.32) will be required.

$$(\text{Bandwidth})_{AM} = f_r = 2\, f_s$$

The FSK modulation is obtained by switching on an oscillator of frequency f_1, when a "one" is to be sent, and switching on another oscillator of frequency f_2, when '0' (-1 in the ternary case) is to be sent. The bandwidth will therefore be a sum of two SQ-PCM-AM-bandwidth and the two-tone FSK will occupy double the bandwidth of the AM case

$$(\text{Bandwidth})_{FSK} = 2\, f_r$$

Three-tone keying in Ternary SQ-PCM-FSK will have, similarly, a $(\text{Bandwidth})_{FSK} = 3\, f_r$.

If, however, the shift between f_1 and f_2 frequencies is obtained by modulating a single oscillator, there will be a phase independence between the two frequencies and the modulated waveform may be viewed as the sum of two independent waveforms, one centered at f_1, and the other at f_2. Hence the bandwidth will again be twice of that in the amplitude modulation case. For a unity deviation ratio, $\Delta f / f_s = 1$, the bandwidth required in the FM case is $4.\Delta f$, where Δf is the peak deviation, and therefore, the bandwidth in the SQ-PCM-FM will be

$$(\text{Bandwidth})_{FM} = 4f_s = 2\, f_r \,.$$

For the Binary coded pulses, the relationship between the PM and AM is that, the phase modulated signal can be derived from AM signal of the same peak amplitude by doubling the amplitude of the AM signal and suppressing the carrier.[32] In fact, a PM signal is a suppressed carrier AM signal and is sometimes referred to as Bipolar AM. Because of this close relationship between the PM and AM, the frequency spectrum of a PM signal is similar to that of the equivalent AM signal, and the PM signal can also be transmitted as a double sideband signal. Therefore the Bandwidth of the SQ-PCM-PM is the same as SQ-PCM-AM.

5.4. EFFICIENCIES.

So far, the consideration has been the input signal power required for a satisfactory S_o/N_o with a given channel noise. For the communication of a message from one point to another, the main features in the system are the coding, the channel, and the decoding at the other end. The coding process introduces an inevitable quantizing noise, and the noise, thus, brought in is taken for granted. The transmitting system therefore sends the signals in the channel at a certain rate and it is assumed, for the purpose of this section, that this rate is optimum. The information rate at the receiver, however, is different, and the coding efficiency parameter has been used to give a picture of the coder-decoder combination. With a certain channel noise already existing, the different schemes of modulation used for effective transmission over the channel are compared on the basis of their power efficiency. The ideal modulation-demodulation scheme will have an information rate much higher than the actual system, and the Communication efficiency gives the departure from the ideal.

5.4.1. CODING EFFICIENCY.

The communication system consists essentially of a coder at the transmitter and a decoder at the receiver. After the coding, the rate of information, H, of the transmitter is governed by the PRF, and the number of signal levels used for transmission. However, the rate of information at the output of the decoder, H', for a most general message signal, is not the same as at the output of the coder. The coding efficiency is a measure of the efficacy of information transmission with this coder and decoder combination, when the channel between them is assumed to be noiseless. The information rate at the output of the decoder is

$$H' = W_m \log_2 (1 + S_1/N_0)$$

where S_1/N_0 = mean SNR at the decoder, for a most general message signal e.g., noise signal.

Hence the coding efficiency, ζ, is

$$\zeta = H'/H$$

$$\text{or} \quad \zeta = (W_m/H) \log_2 (1 + S_1/N_0) \qquad \ldots (5.34).$$

The coding efficiency, ζ, depends on H and S_1/N_0, which in turn depends upon the PRF. A sample calculation is given below.

For the Ternary SQ-PCM system, the PRF of 40 Kc/s is equivalent to an information rate of 63.2 K bits/sec. The post detector SNR for a noise type of signal is 32 db when W_m is restricted to a bandwidth of 3.5 Kc/s (see Chapter III). Therefore, from eq.(5.34)

$$H' = 3.5 \times 10^3 \log_2 (1 + 1585) = 3.5 \times 10^3 \times 10.6302$$

$$\text{and} \quad \zeta = \frac{3.5 \times 10^3 \times 10.6302}{63.2}$$

$$= 0.59$$

Similarly, the calculation of ζ is done for the other PRF's, and the results are shown in a tabular form in Table 1. The calculation of ζ for the Binary SQ-PCM is also similar and these results are also given in Table 1.

TABLE 1.

PRF	Ternary SQ-PCM system.			Binary SQ-PCM system.		
	S_i/N_o db	H Kbit/sec.	ζ	S_i/N_o db.	H Kbit/sec.	ζ
20 Kc/s.	23 db.	31.6	0.87	-	-	-
30 Kc/s.	26 db.	37.55	.806	22	30	.854
40 Kc/s.	32	63.2	0.59	26	40	0.761
50 Kc/s.	33 db	79.0	0.37	27.5	50	0.66
60 Kc/s.	-	-	-	29	60	0.562

The coding efficiency is plotted against the ideal information rate H in Fig.5.16. It is seen from the graph that the Ternary system coding efficiency is higher than that of the Binary system. The efficiency becomes poorer with increase in PRF, as can be expected in all SQ-PCM sys tems, because of the slow improvement in S_i/N_o with PRF.

5.4.2. POWER EFFICIENCY.

It has been shown in the previous sections, (Section 5.1 and 5.2), that the RF modulation schemes require different signal to noise ratios at the detector input for a particular signal to noise ratio at the output of the receiver. The maximum output SNR required would of course be limited to the minimum quantizing noise inherently present in the SQ-PCM system. A basis of comparison of these schemes evidently has to take into account the channel bandwidth required for the transmission of the RF-modulated signals. A simple criterion of comparison would be the minimum received signal energy required for each information bit transmitted in the presence of uniform Gaussian noise spectral density. Power efficiency β is defined by Sanders[7] as

$$\beta = \frac{E_{min}}{\epsilon^2}$$

where E_{min} = minimum received energy required per bit.

ϵ^2 = noise spectral density.

The above equation could be rewritten in terms of the minimum power required P_{min} as

$$\beta = \frac{P_{min}}{\epsilon^2 H}$$

where H = information rate in bits/second. For purposes of computation, the equation for β is written in a slightly modified form as follows:

$$\beta = \frac{P_{min}}{\epsilon^2 H} = \frac{P_{min}}{\epsilon^2 B} \cdot \frac{B}{H} = \frac{S_i}{N_i} \cdot \alpha \qquad \ldots (5.35).$$

where B = signal bandwidth in c/s.

$\epsilon^2 B = N_i$ = mean noise power in the bandwidth B,

$\alpha = \frac{B}{H}$, a variable parameter depending on the modulation bandwidth.

and S_i = mean signal power.

For any modulation scheme, the input SNR required for the optimum output SNR is found from the curves given in Section 5.1 and 5.2, the bandwidth is estimated from the equations in Section 5.3, and the value of β is calculated, knowing the information rate H in bits/sec. As an example, the power efficiency of the Ternary SQ-PCM-FM system would be calculated below:

In the Ternary case, at a PRF of 40 Kc/s, the quantizing noise limits the SNR to 45 db. The input SNR for an output SNR of 45 db is 15.85 db (seen from Fig.5.8). The information rate for the 3 level +1, 0, -1 pulses is,

$$H = 40 \log_2 3 = 63.2 \text{ Kbits/sec.}$$

and the bandwidth B from Section 5.3 is

$$B = 80 \text{ Kc/s.}$$

Therefore

$$\beta = 38.46 \times 80/63.2$$

or $$\beta = 48.7$$

In the Binary SQ-PCM, however, the information rate for a 40 Kc/s PRF will be 40 Kbits/sec.

The β for all the schemes proposed has been calculated in a similar fashion, and given in a tabular form in Table No.2. The basis of comparison of the different schemes has been that the S_0/N_0 is 45 $\left[= \text{SNR}(Q) \right]$ db for the Ternary SQ-PCM at 40 Kc/s PRF, and 40 db for the Binary SQ-PCM at 40 Kc/s PRF. A low value of β indicates a more efficient scheme because the power required is less for the same performance. The SQ-PCM-PM seems to be the most efficient scheme among the different schemes illustrated.

The power efficiency, β , gives an idea of the minimum transmitted power required in a certain bandwidth for a particular grade of communication, but the actual rate of information obtained in the link with this received power is not known. Communication efficiency, in the next section, will be defined as one such measure of the performance of the actual system and its departure from the ideal.

TABLE 2.

Mode or Method of Transmission.	Ternary $S\eta$-PCM SNR(Q)=45db PRF=40Kc/s.			Binary $S\eta$-PCM PRF = 40 Kc/s.		
	H information rate in Kbit/sec	B bandwidth in Kc/s.	β	H	B	β
VIDEO.	63.2	20	13.8	40	20	15.81
AM.	"	40	28	40	40	31.62
FM.	"	80	48.7	40	80	55.72
2-tone FSK.	"	80	55.4	40	80	39.0
3-tone FSK.	"	120	39.2	-	-	-
PM.	"	40	7.26	40	40	8.13

5.4.3. COMMUNICATION EFFICIENCY.

It has been shown that the Power efficiency defines the minimum power requirement of a transmitter signalling in a particular bandwidth and with a definite information rate, and that it does not indicate the ideal rate which could be achieved for the same power etc. The communication efficiency to be considered here gives an idea of the departure of the actual system from the theoretically possible rate of information. The information rate has been defined by Shannon[8] as

$$H = B \log_2 (1 + S_i/N_i) \qquad \ldots (5.36)$$

where H = information rate in bits/sec.

B = Bandwidth in c/s.

S_i/N_i = signal to noise ratio in the bandwidth B.

In a particular radio communication system transmitting in a bandwidth of B c/s, there will be a certain amount of power (P_r) at the input of the receiver, and if the noise power spectral density is ϵ^2, the noise power at the input to the receiver will be $\epsilon^2 B$. The signal to noise ratio is

$$S_i/N_i = P_r / \epsilon^2 B$$

With this SNR, the ideal information rate in bandwidth B is given by eq.(5.36). However, at the output of the detector, a certain signal to noise ratio will be obtained (refer to the curves in Section 5.1) in a bandwidth of W_m c/s, where W_m is the message bandwidth. The information rate in the message bandwidth W_m is H_M, given by

$$H_M = W_m \log_2 (1 + S_o/N_o)$$

where S_o/N_o = post detector output SNR.

It is expected that the same information rate as at the input of the detector will be obtained at the output also i.e.,

$$H_M = H \text{ for an ideal communication system}$$

or $$W_m \log_2 (1 + S_o/N_o) = B \log_2 (1 + S_i/N_i)$$

In actual communication systems, $H_M \ll H$. If the communication system has to have the same information rate as the ideal system, the SNR in the bandwidth W_m has to be increased, so that, with this increased (S_o/N_o), the new information rate H_M' is equal to H. This means that, in the actual system the received power P_r is to be increased to P_r'. In other words, the transmitted power is increased, and because the output SNR depends upon the input SNR, the amount of power increase is such that,

$$H_M' = W_m \log_2 (1 + (S_o/N_o)') = H. \quad \ldots (5.37).$$

A communication efficiency (η) could therefore be defined as the ratio of transmitter power required in an ideal system to the transmitter power required in an actual system. The information rate in bits/second, and the noise spectral density are assumed to be the same in both cases.

Therefore, according to Wright et al,[33]

$$\eta = \frac{P_r}{P_r'} = \frac{\text{Received power in an ideal system}}{\text{Received power required in an actual system for the same information rate.}}$$

$$= \frac{P_T}{P_T'}$$

To compute the values of η , eq.(5.37) can be written

$$W_m \log_2 (1 + S_o/N_o) = B \log_2 (1 + S_i/N_i)$$

$$\text{or} \quad (S_o/N_o)' = [1 + S_i/N_i]^{\gamma} - 1 \quad \ldots (5.38).$$

where $\gamma = B/W_m$.

Now, if it is assumed that the input SNR is S_i/N_i, then in an ideal system the output SNR will be as given by eq.(5.38) and will be $[(S_o/N_o)']$. But in an actual system, this output SNR $[(S_o/N_o)']$ can only be obtained if the signal power is increased, so that the new input SNR is $(S_i/N_i)'$. For the same information rate H, therefore,

$$\eta = \frac{[S_i/N_i]_{IDEAL}}{[S_i/N_i]_{ACTUAL}}$$

A sample calculation for a Binary SQ-PCM-AM system will be shown here. At a 40 Kc/s PRF, the bandwidth for amplitude modulated signal is 40 Kc/s. The input SNR is chosen as 14 db, W_m = 4 Kc/s, and therefore from eq.(38).

$$(S_o/N_o)' = (1 + 25.16)^{10} - 1 = 141.69 \text{ db.}$$

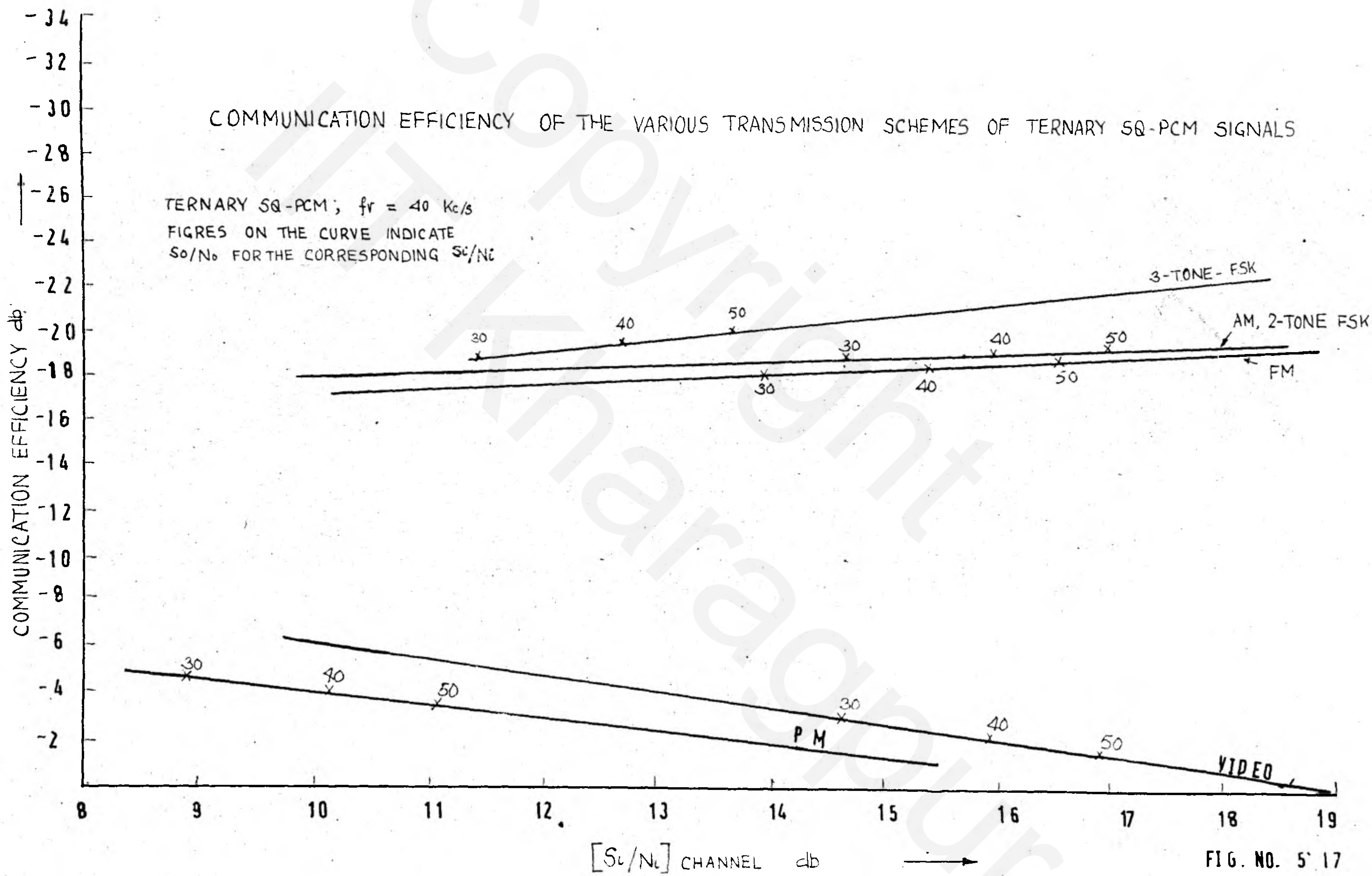
An SNR of 141.69 db in the message bandwidth of 4 Kc/s will therefore have the same information rate as the ideal system. Next, from the actual system curves given in Section 5.1 (Fig.5.4) relating the input and output SNR, $(S_i/N_i)'$ to give an output SNR of 141.69 db is estimated as +22.63 db. This is obtained by extrapolating the given SNR curve for large values by using the equation,

$$(S_o/N_o)' = 70 + 15 \left[(S_i/N_i)' - 17.85\right]$$

Therefore, the same information rate as an ideal system, with input SNR of 14 db, is obtained if the signal power is increased, so that, the input SNR in the actual system is 22.63 db. The communication efficiency is

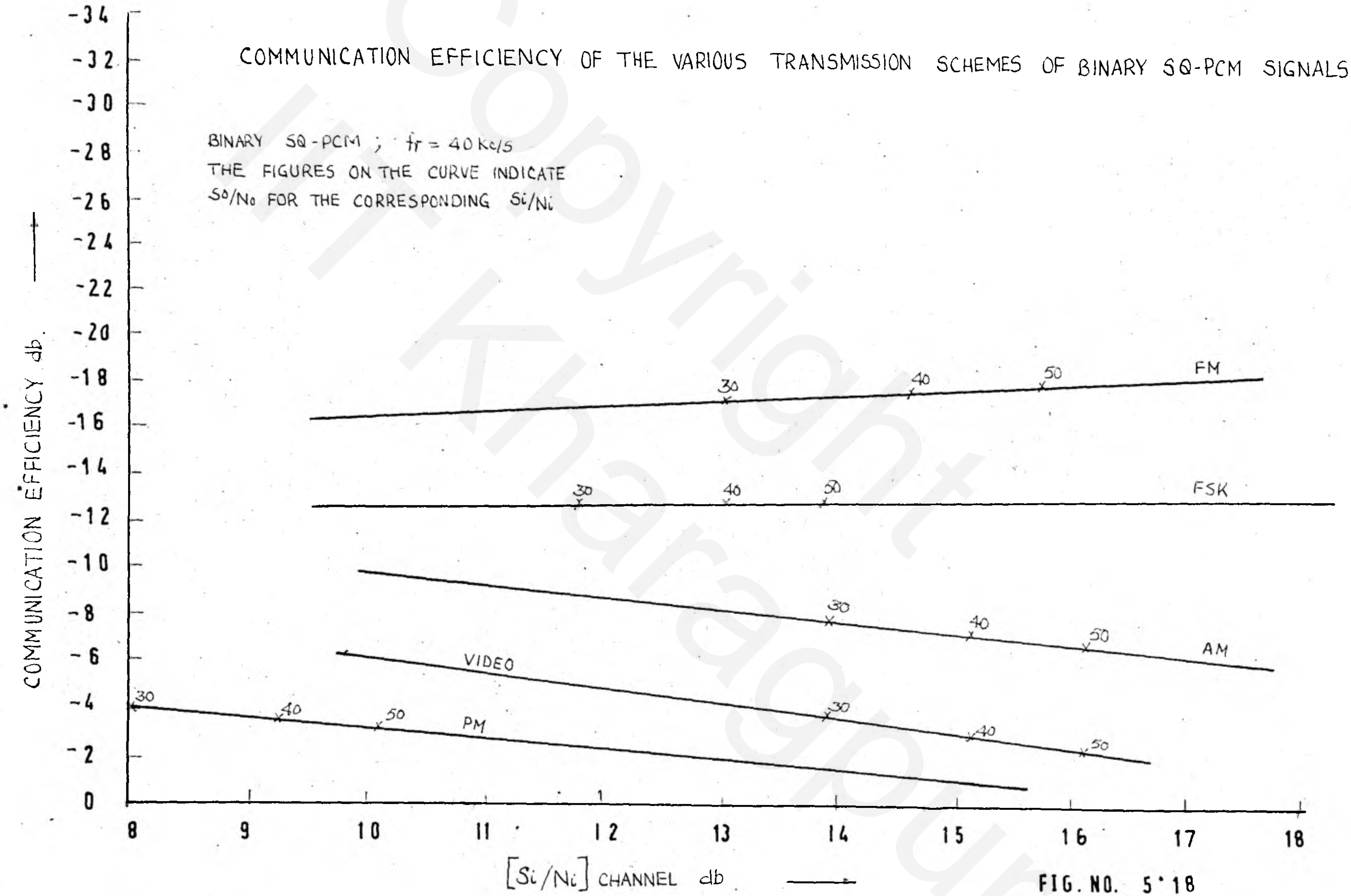
$$\eta = -22.63 + 14 = -8.63 \text{ db.}$$

Similar calculations of η have been made for the Binary SQ-PCM and Ternary SQ-PCM-AM, -FM, -FSK, -PM and video transmission



COMMUNICATION EFFICIENCY OF THE VARIOUS TRANSMISSION SCHEMES OF TERNARY SQ-PCM SIGNALS
TERNARY SQ-PCM; fr = 40 Kc/s
FIGRES ON THE CURVE INDICATE
So/No FOR THE CORRESPONDING Si/Ni
3-TONE-FSK
AM, 2-TONE FSK
FM
PM
VIDEO
COMMUNICATION EFFICIENCY db
[Si/Ni] CHANNEL db

FIG. NO. 5.17



COMMUNICATION EFFICIENCY OF THE VARIOUS TRANSMISSION SCHEMES OF BINARY SQ-PCM SIGNALS
BINARY SQ-PCM ; fr = 40 Kc/s
THE FIGURES ON THE CURVE INDICATE
So/No FOR THE CORRESPONDING Si/Ni
COMMUNICATION EFFICIENCY db
-34
-32
-30
-28
-26
-24
-22
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
8
9
10
11
12
13
14
15
16
17
18
[Si/Ni] CHANNEL db
FM
FSK
AM
VIDEO
PM
30
40
50

FIG. NO. 5·18

systems. The curves of efficiency with the input SNR are plotted in Fig.5.17, for the Ternary, and in Fig.5.18, for the Binary systems. The SQ-PCM-FM has the lowest η , and the video transmission and PM have the highest. The communication efficiency as defined and calculated above gives an account of the departure of a system from the ideal in terms of information transmission.

5.5. MULTIPLEXING OF SQ-PCM SIGNALS.

Multichannel time-division multiplexing of SQ-PCM is feasible. Two alternative schemes have been proposed and shown in Fig.(5.19) and (5.20). In scheme No.1 (Fig.5.19), each speech channel is passed through a low-pass filter and compressor to the sampler, and the 2 or 3 level quantizer (depending on whether a Binary or a Ternary system) issued. The quantizer is the only common equipment. The feedback loop of each channel is completed by gating the transmitter pulses with the channel pulses and integrating the lengthened pulses. The compressor will be of the syllabic type as it is directly acting on the speech, It requires a smaller bandwidth, to the extent that, it requires a bandwidth not larger than the message bandwidth. At the receiver, after the slicer, each channel is gated out and the message recovered as usual.

In scheme No.2 (Fig.5.20), the speech signals in each channel are filtered and sequentially sampled at 8 Kc/s rate. The output, thus obtained, is added in an adder circuit, and passed through a compressor which is followed by a sinx/x filter. Ideally, the output of the filter will be a complex signal of bandwidth m W_m

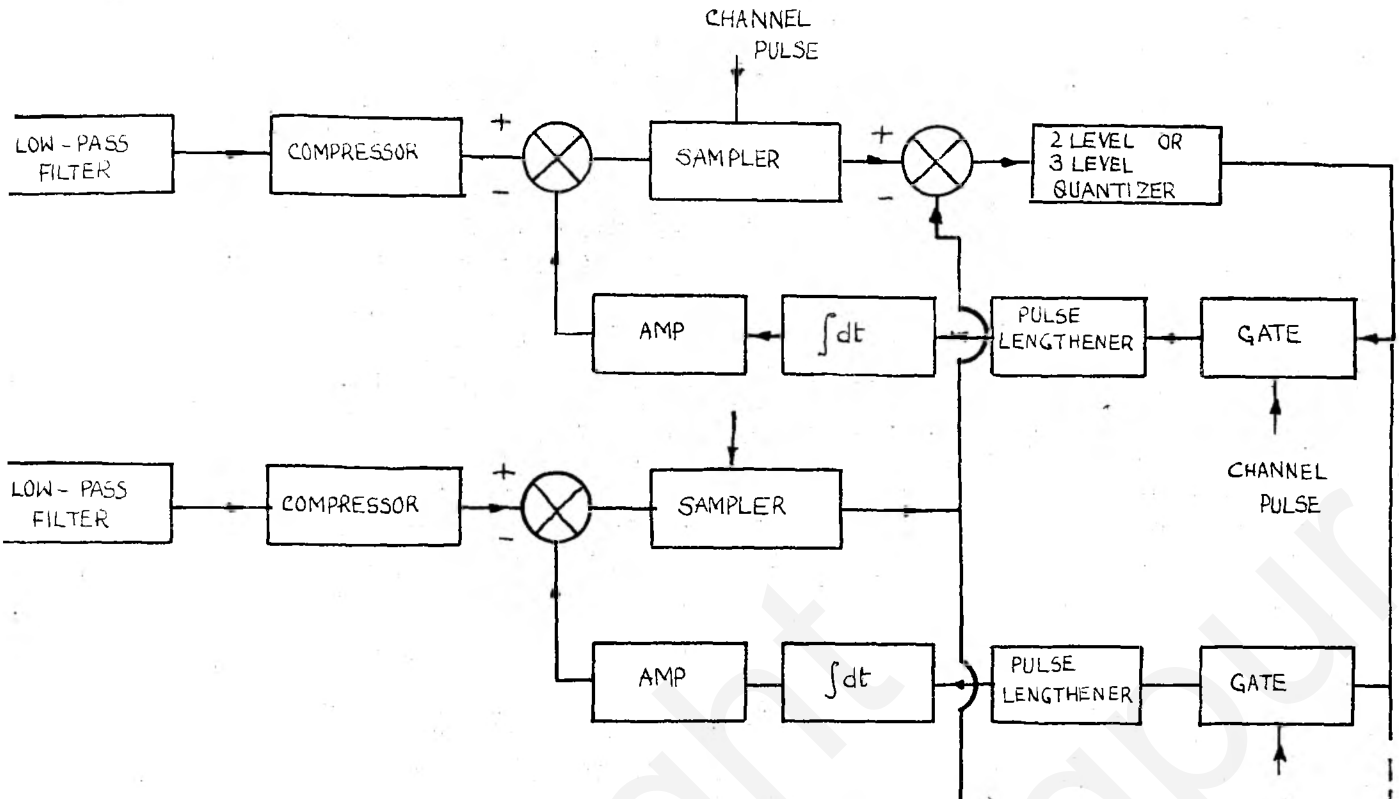
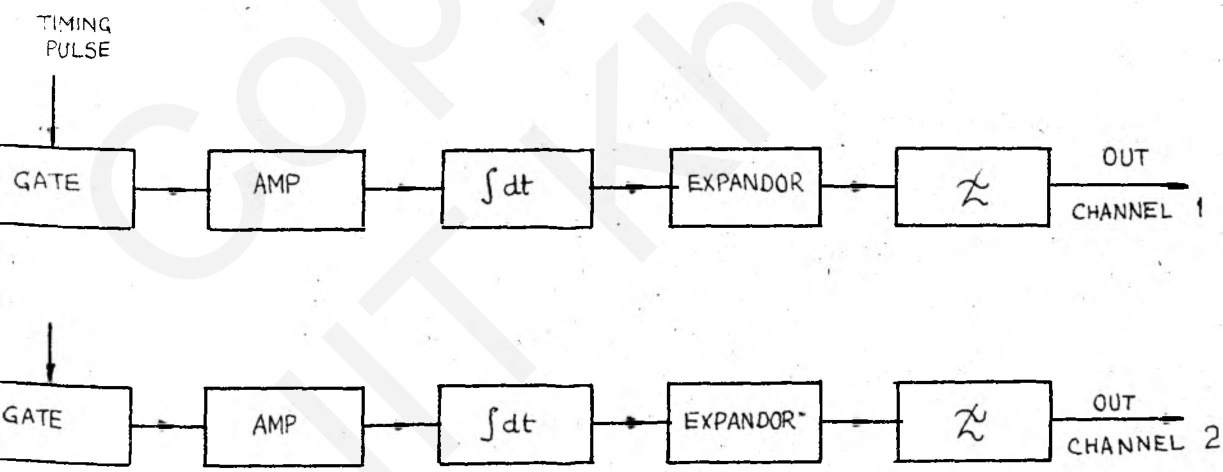


FIG. NO. 5.19(a)

TDM TRANSMITTER



TDM RECEIVER

FIG. NO. 5.19(b)

SCHEME 1 FOR TDM OF SQ-PCM

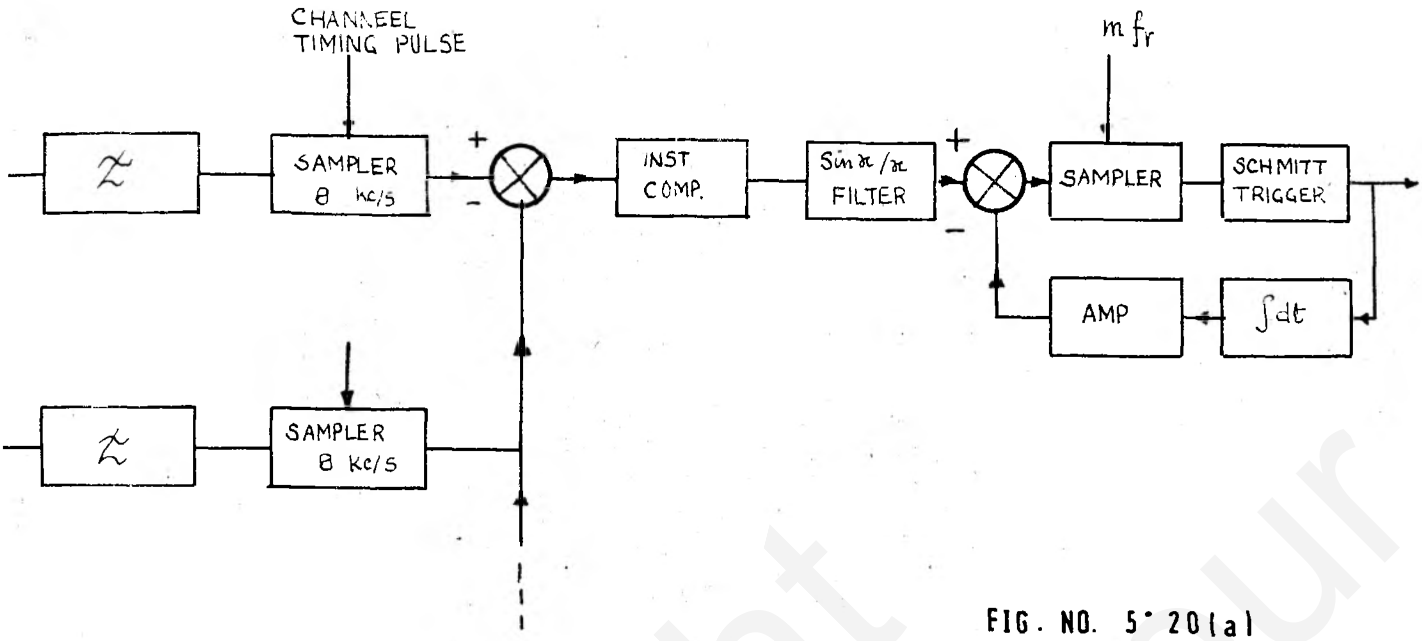
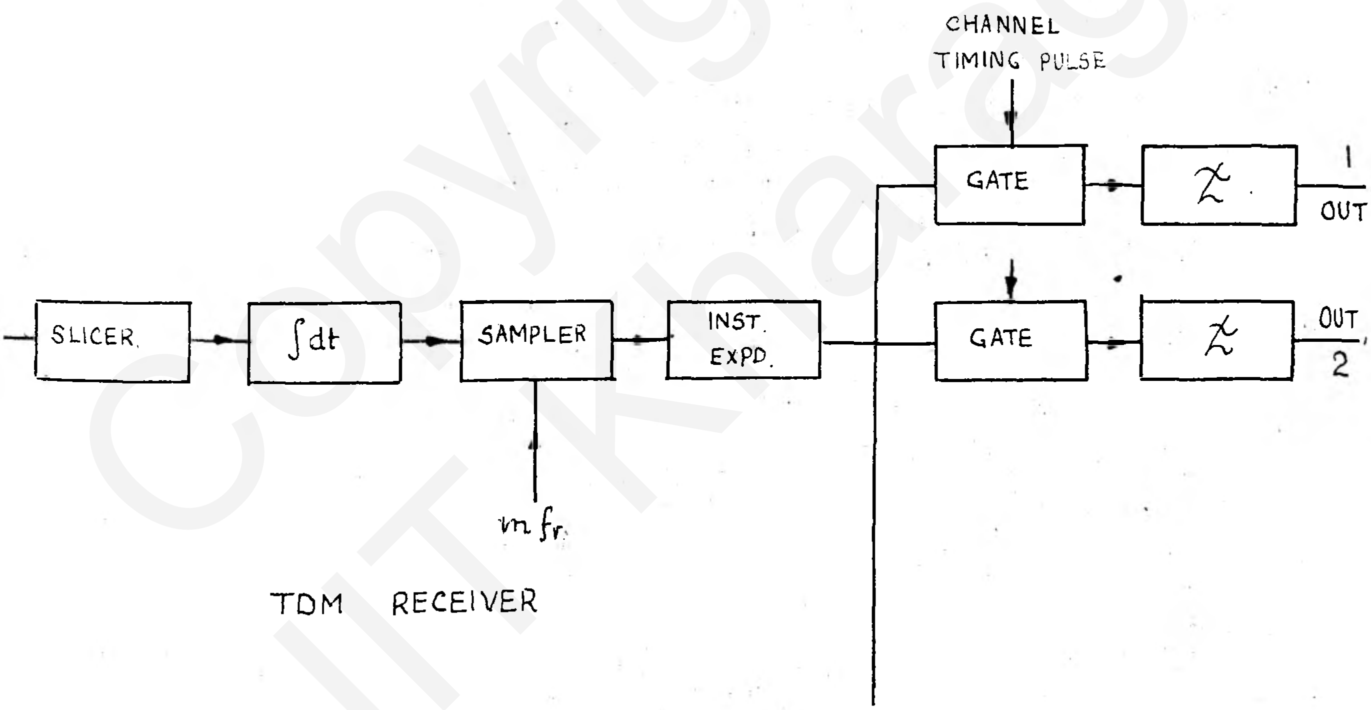


FIG. NO. 5·20(a)

TDM TRANSMITTER



TDM RECEIVER

FIG. NO. 5·20 (b)

SCHEME 2 FOR TDM OF SQ-PCM

where m is the number of speech channels being multiplexed, and f_m is the bandwidth of one channel (f being less than 4 Kc/s). This signal is now processed by the SQ-PCM system using a sampling rate of mf_r, where f_r is the PRF used for a single channel. The compressor, because it acts on the instantaneous amplitudes of the pulses, is of the instantaneous type. At the receiver, the pulses are passed through a slicer circuit and then through an integrator followed by a sampler. The sampling frequency is mf_r, as before. The output of the sampler is expanded by an instantaneous expandor. The m channels are separated by the m gates and then passed through a low pass filter to recover the message signal.

In any multiplexing scheme, the number of common equipment should be as large as possible and scheme No.1 suffers on this count. The proposed scheme No.2, however, uses quite a large amount of common equipment and therefore should have almost the same performance as the multichannel TDM in the AQ-PCM.

5.6. SUMMARY AND DISCUSSION.

The purpose of this Chapter has been to assess the transmission characteristics of the SQ-PCM pulses. It is seen that for the video transmission limited by the peak power, the input SNR for a 45 db output SNR is 18.2 db for the Ternary system and 18.7 db for the Binary system. More often than not, the average power of the transmitter is the limiting factor, and then, the input SNR is 16.4 db for the Ternary and 15.7 db for the Binary system for the same S_o/N_o. Bipolar transmission of the Binary pulses reduces the S_i/N_i further to 12.7 db for the same S_o/N_o.

At a 40 Kc/s PRF, both the systems require a minimum of 20 Kc/s bandwidth for transmission. RF modulation of the pulses with AM, FM and PM has been considered. In AM, on the peak power basis, the input S_i/N_i for a 45 db S_o/N_o is the same as the video case (peak power). In FM, FSK and PM, the input S_i/N_i is 15.85 db, 12.2 db and 10.6 db, respectively for the Ternary; 15.2 db, 12.4 db and 9.7 db, respectively for the Binary system (the S_o/N_o in each case being 45 db). The bandwidth for the FM is twice that of AM, for the FSK thrice that of AM and for PM equal to AM in the Ternary case, and the same relationship holds in the Binary case also, except that the bandwidth of the FSK is twice that of AM. The coding efficiency of the Ternary system is slightly better than that of the Binary system. The power efficiency, β , and the communication efficiency, η , for the different transmission schemes has been worked out showing that the Ternary system is more efficient than the Binary system.

The transmission scheme to be used should naturally be the most efficient, i.e. it must be able to communicate in the least bandwidth with minimum Power. In most of the cases an effort to decrease the power requirement results in an increase in the bandwidth. The efficiencies β and η both give an idea of this power and bandwidth balance. The β gives the power required in a certain bandwidth for a particular quality of transmission, and η describes the quality of transmission if the system was ideal. It would seem therefore, that η would give a more comprehensive picture of the situation.

The efficiency of the Bipolar AM, better called PM, is the highest, because it has the same bandwidth as the AM but requires 6 db less power. The type of detector used in FM gives rise to two different efficiencies, and it is seen that the tuned filter-difference circuit type of detection is more efficient than the simple discriminator type. But the β and η of the FM is less than that of the PM because the bandwidth requirement is more, and due to this alone, it is less efficient than AM. The video transmission has been assumed to have the Nyquist bandwidth of ½ f_r but in most of the practical cases, the bandwidth used is 3/4 f_r , and AM bandwidth is 3/2 f_r.

CHAPTER VI.

COMPARISON OF SQ-PCM WITH OTHER SYSTEMS.

6.0 GENERAL.

The theory and performance of the SQ-PCM systems have been discussed in the previous Chapters, II, III, IV and V. It would be interesting, therefore, to compare their performance with other coded pulse modulation systems - notable among them being the AQ-PCM, Δ-M, and Δ-ΣM systems. A broad outline of the principles of operation underlying all the coded pulse modulation systems has been given in Section 2.0. It is seen there that the basic mechanism of the systems is the same, and only the details of coding and the quantization process differ. There are two bases of comparison of the performance of these systems, one being the coding characteristics, and the other being the transmission characteristics.

The coding characteristics really determine the quality of approximation of the input signals, and therefore, form the basic core of any system. A comparison of the coding characteristics will include a comparison of the SNR(Q) obtained for a variation of input signal level, SNR(Q) obtained for a variation of the signal frequency, SNR(Q) variation with PRF, and the input-output voltage linearity in the systems. The dynamic range of input of all the systems is small and for complex signals, like speech, the SNR(Q) will be quite small for weak signals. A compandor is used to increase the dynamic range, and the advantage obtained in practice forms another important comparison between the systems.

A communication system, properly utilized, should transmit ensembles of functions, and not certain special signals such as a pure sine wave only, as has been pointed out by Shannon. A speech signal, or a band limited noise signal, will be one such ensemble of functions. The transmission of such continuous signals with the help of the systems under consideration can only be done within a certain accuracy. This leads to the question of the quality of transmission, and the effect of the channel noise etc. on the systems. As defined in Section 5.4, the coding efficiency, the power efficiency and the communication efficiency are also important bases of comparison.

The pulses of the SQ-PCM systems, like those of the AQ-PCM system, are capable of being repeatored and the transmission is synchronous. The coding is such that the repeater or the distant end receiver is required to make only a distinction between a pulse and no pulse. As long as the noise in the channel is below a certain threshold, the synchronous transmission will give a correct identification of the pulses at the repeater stations, and moderately small transmitter power could be used in a reliable communication link. Multiplexing of many channels of messages can be done by the two methods indicated in Section 5.5. Out of the two schemes suggested there for the SQ-PCM-TDM, the first scheme clearly requires too much equipment for any satisfactory solution of multiplexing large number of channels. The second scheme, which uses a common compandor and coder, is similar to the multiplexing scheme of the AQ-PCM system (Appendix A), and will be more suitable as far as economy of equipment is concerned.

The stability of the Ternary SQ-PCM system has been given in Section 3.1.2, and the system has been shown to be fairly insensitive to the variation of the reference voltages in the quantizer and other such voltage variations. It is seen that even if one of the quantizers stops functioning, the systems becomes a Binary SQ-PCM system and continues to be useful. For all the parameters, the effect of whose drift has been experimentally determined, the Ternary SQ-PCM system is very stable and the minimum SNR under any such large variations is 32 db at a PRF of 40 Kc/s. The stability of the Binary SQ-PCM system against the drift of the reference voltage is also good for small voltage variations. The SNR(Q) in the Ternary SQ-PCM system deteriorates by less than one db for reference voltage variation of about 1% (Fig.3.34). The regulation of the power supplies providing the reference bias is certainly better than this. Referring to Appendix A, it is seen that in the practical AQ-PCM system, the noise uncertainty in the coder, of as large a magnitude as the size of one quantum step, results in a deterioration of SNR(Q) by only 3 db. Otherwise, for normal operations, the SNR impairment due to the noise in the coder is less than one db.

Section 6.1 deals with the comparison of the coding characteristics of the different systems, Section 6.2 deals with the transmission characteristics of the different systems. Section 6.3

gives a comparison of the coding, power, and communication efficiencies of the systems, and Section 6.4 gives a summary of all the comparisons.

6.1.0. CODING CHARACTERISTICS.

To estimate the relative qualities of the processed signal, the Ternary SQ-PCM, Binary SQ-PCM, AQ-PCM, Δ-M and Δ-ΣM systems are compared on the basis of SNR(Q) obtained for the sinusoidal, FM and the noise signals. As has been explained earlier in Section 4.3, the Ternary SQ-PCM system at 40 Kc/s PRF will be compared with the 7-digit AQ-PCM system (an equivalent PRF of 60 Kc/s), and the Binary SQ-PCM system at 40 Kc/s PRF will be compared with the 6-digit AQ-PCM system of an equivalent PRF of 50 Kc/s. The results of the Δ-M and Δ-ΣM , which are chosen here for comparison, are at 60 Kc/s PRF.

The variations of the SNR(Q) with different input signal levels for the different systems are shown in Fig.6.1. The 0 db level is the full load sinusoidal input at 1 Kc/s signal frequency, and all inputs above this lead to a serious overloading distortion in all systems. The peak SNR is 42, 41, 37, 37, 33 and 29 db, for the AQ-PCM (60 Kc/s PRF), Ternary SQ-PCM, Binary SQ-PCM, AQ-PCM (50 Kc/s PRF), Δ-M and Δ-ΣM systems, respectively. The dynamic range of input, when the SNR is above 25 db and maximum input level below the overload, is 20 db for the AQ-PCM (60 Kc/s PRF) and the Ternary SQ-PCM system, and is 15 db and 12.5 db for the AQ-PCM (50 Kc/s PRF) and the Binary SQ-PCM, respectively. The dynamic range for the Δ-M and Δ-ΣM systems is 10 db and 7 db respectively.

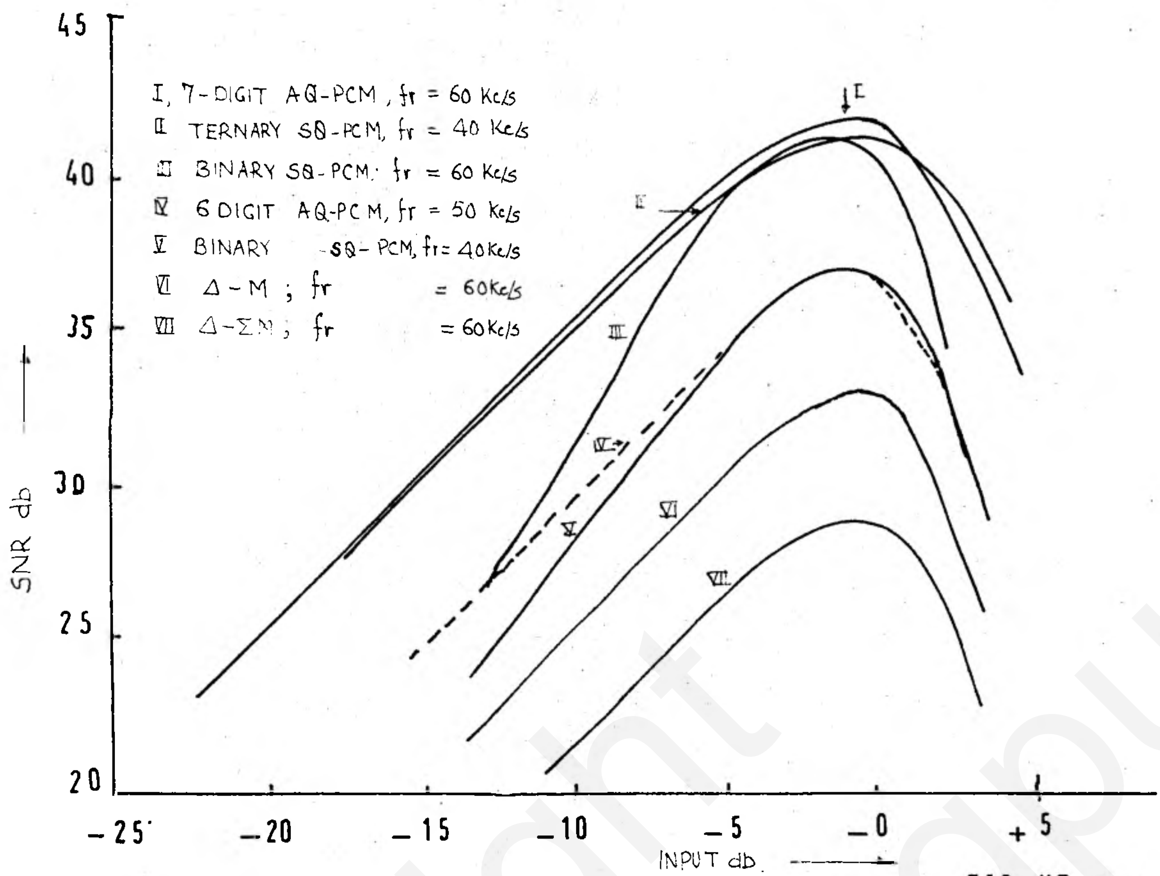
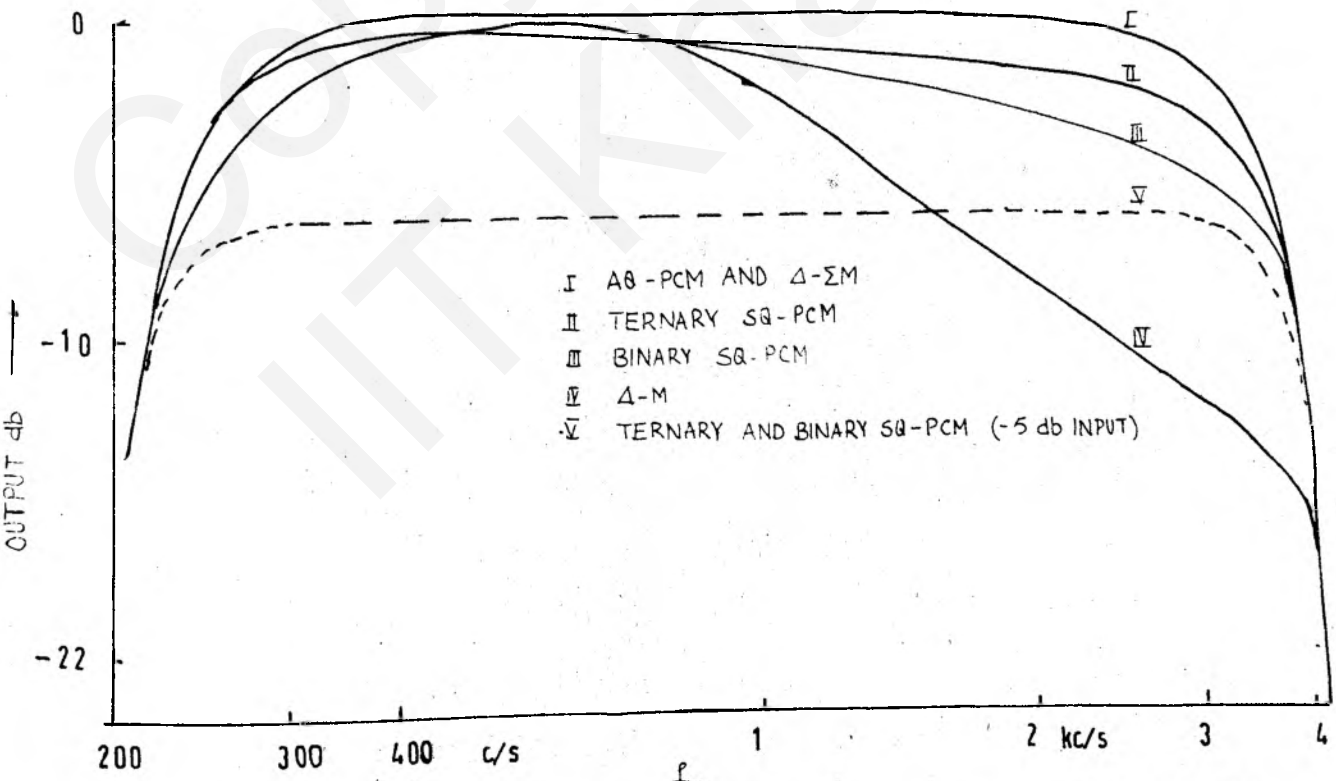


FIG. NO. 6·1

VARIATION OF SNR WITH INPUT LEVEL FOR DIFFERENT SYSTEMS



GAIN CHARACTERSTICS VS. fm FOR DIFFERENT SYSTEMS

FIG. NO. 6·2

The variation of the SNR(Q) with signal frequency in all the systems is within 3 db at -5 db input level.

The gain characteristics vs. signal freauency for the different systems is shown in Fig.6.2. The output voltage vs. signal frequency curve is approximately flat for the AQ-PCM, the Ternary SQ-PCM and the Δ-ΣM systems. For the Binary SQ-PCM system, the output falls by about 5 db at 3 Kc/s signal frequency at an input level of 0 db. At an input level of about -5 db, however, the frequency distortion is negligible. In the Δ-M system, as reported by Zetterberg[11], the output at 3 Kc/s falls by about 9 db as compared to 1 Kc/s signal. Actually the Δ-ΣM system achieves a flatter gain vs. frequency characteristics of the system at the cost of a lower SNR. The frequency distortion in the Δ-M system is not a serious drawback for processing speech signals, as will be discussed later.

The variation of SNR(Q) with PRF for the different systems is shown in Fig.6.3. The improvement in SNR(Q) in the SQ-PCM system is about 9 db/octave change of PRF, as in other Uni-digit systems of Δ-M and Δ-ΣM . In all these systems, the SNR(Q) is proportional to $(f_r)^{3/2}$ (refer Appendix B and Section 2.4). In the AQ-PCM system the improvement in SNR(Q) is exponential with PRF, and for a n digit system, the SNR(Q) is (6n + 3) db (refer Appendix A). This is one major advantage of the AQ-PCM system, as larger increase in SNR can be obtained with a proportionately smaller increase in the PRF, although the circuitry for n = 8, or more, is very complex. On the other hand, all the Uni-digit systems are very simple from the circuit point of view. The results

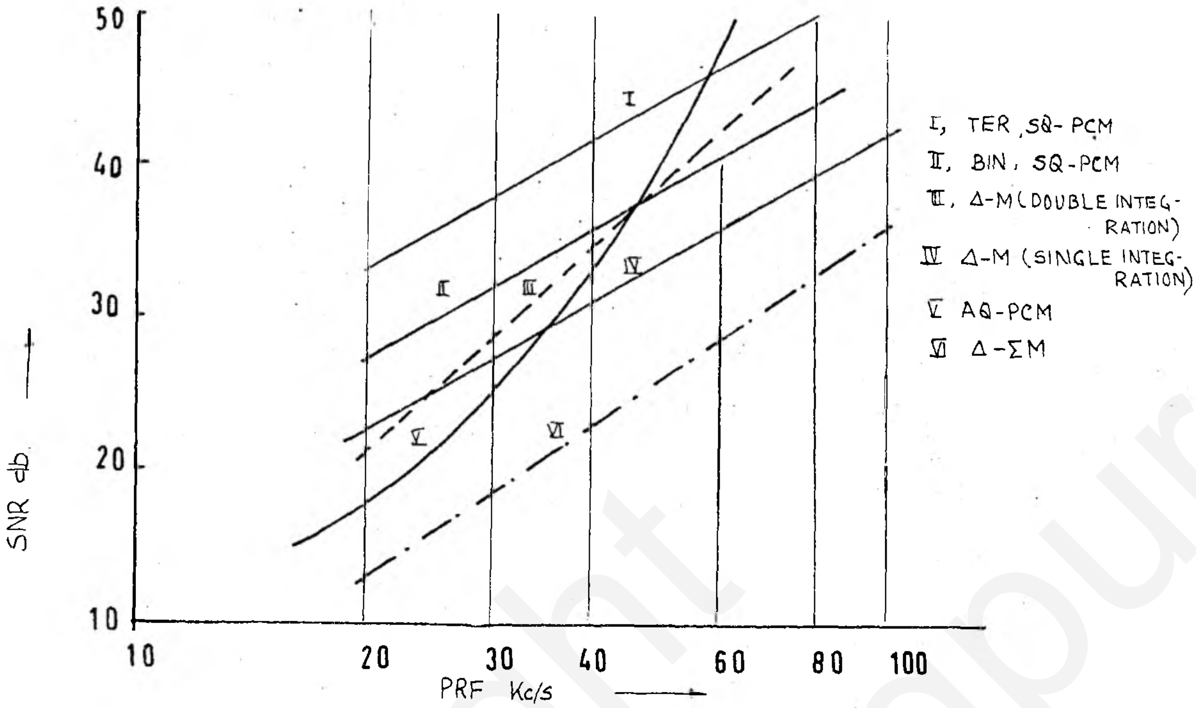


FIG. NO. 6·3

VARIATION OF SNR WITH PRF FOR DIFFERENT SYSTEMS

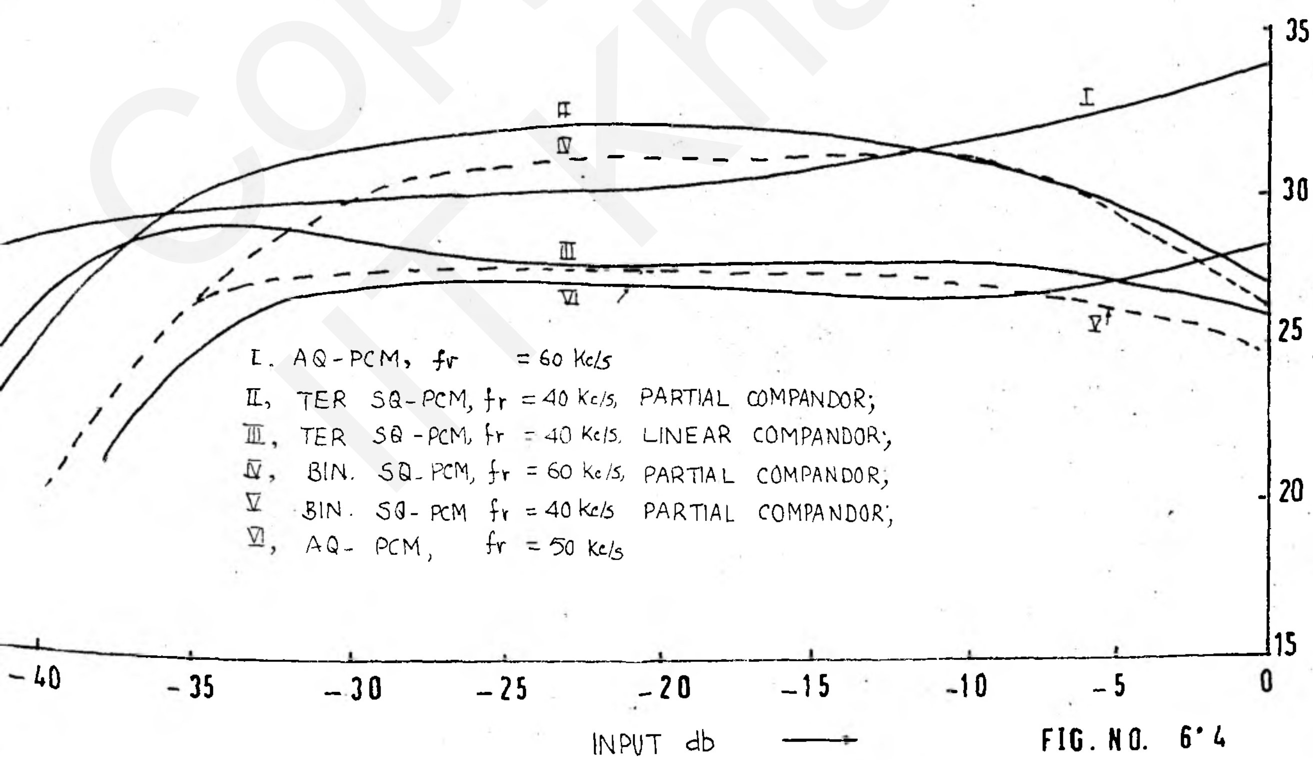


FIG. NO. 6·4

VARIATION OF SNR WITH INPUT LEVEL FOR COMPANDORED SYSTEMS for a complex signal input, like the FM signals and the band-limited noise signals, have a similar character, except that the optimum SNR(Q) is about 8 - 10 db below the full load sinusoidal signal. Except the Δ-M system, the input-output voltage linearity is quite good in all the systems under consideration. The Ternary SQ-PCM system (without feedback) has not been included in the above comparison, because there the output voltage is not proportional to the input (Section 3.2), and the SNR is poorer. The system, however, is quite adequate for processing of some special input signals like FM and narrow band signals, as has been discussed in Section 3.2.0.

6.1.1. THE DYNAMIC RANGE.

It has been shown that the dynamic range is small in all systems, and a compandor, either logarithmic or syllabic, should be used to extend this range. The syllabic compandor (Section 2.5) is suitable only when few channels are required and each channel uses one compandor. When a large number of channels are to be multiplexed, a common instantaneous compressor and a common instantaneous expandor will result in economy in equipment. The SQ-PCM systems, both Ternary and the Binary, have been experimented with using an instantaneous compandor (Section 2.5), and the results are compared with the AQ-PCM system (Appendix A) in Fig.6.4. The dynamic range of input (with the partial compandor) is 40 db and 35 db in the Ternary and the Binary SQ-PCM, respectively. The best SNR(Q) is 32 db in the Ternary case and about 27.50 db in the Binary case. This compares fairly well with a dynamic range of 44 obtained for the AQ-PCM (60 Kc/s), and 34 db obtained for the AQ-PCM (50 Kc/s). A linear compandor in the Ternary SQ-PCM system at the 40 Kc/s PRF gives a dynamic range of 41 db, and the SNR in the middle of the range is about 27 db as compared to the 30 db for the AQ-PCM (60 Kc/s) system. So far, no results have been reported for companding in Δ-M and Δ-ΣM systems. The gain characteristics with the signal frequency in Δ-M is almost similar to the spectral density of speech and therefore, it has been felt that for processing of speech signals, no special efforts for reducing the frequency distorting are necessary[4]. Either an over-modulation, or a syllabic compandor, has then to be used to handle the dynamic range of the input speech.

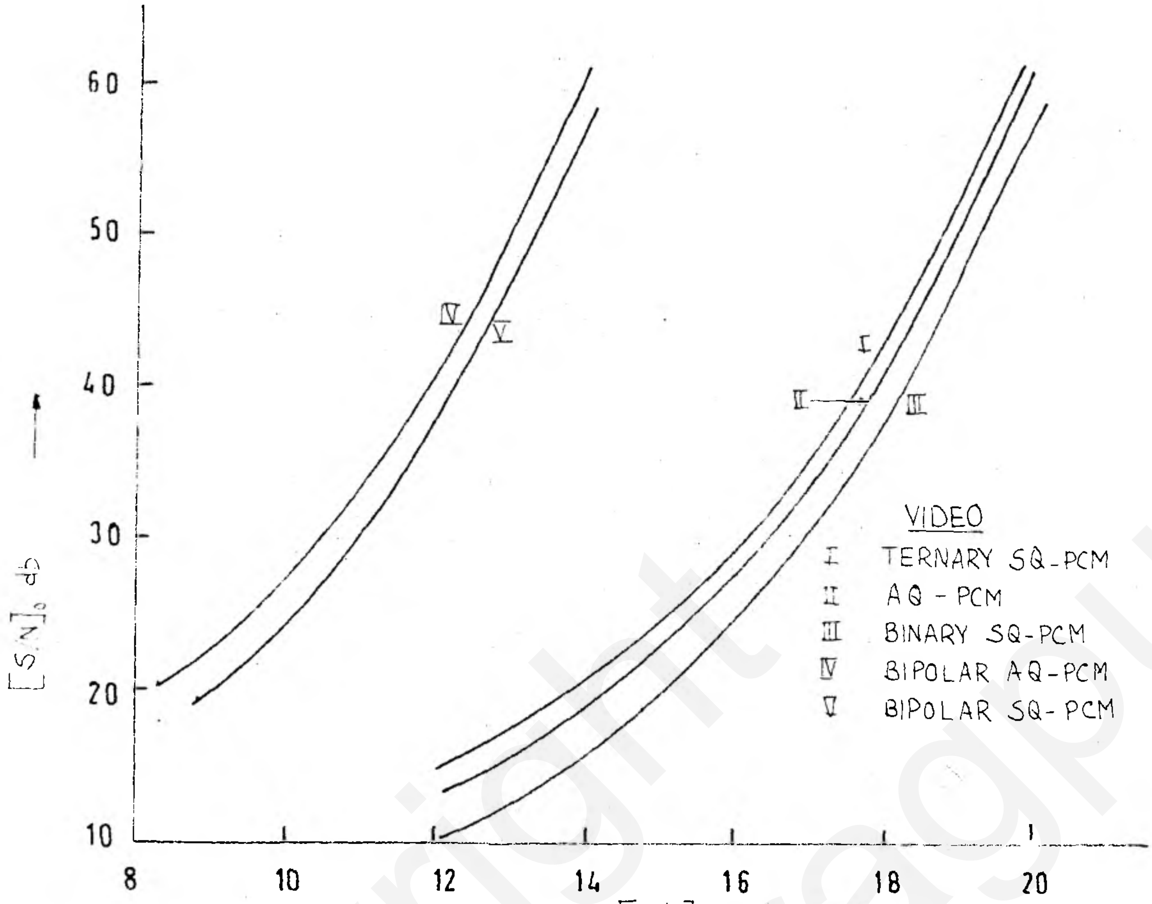
The Binary SQ-PCM system can use an instantaneous compandor to advantage, with of course a partial compression still left in the output of the system. The output of the system at the higher end of the signal frequency is compressed to about 8 db for an input change of 20 db (Fig.4.16). This may not be a serious difficulty for a commercial quality of speech signal. The attempts to get a linear compandor for the Binary SQ-PCM were not very successful in general, probably because of the limited bandwidth of the whole system. Speech tests with the partial compandor give good results, as expected, in both the Binary and the Ternary SQ-PCM systems.

Basically, the use of an instantaneous compandor in the AQ-PCM system gives a nonlinear quantization characteristic and the weak signals are processed with a good SNR(Q). The quantizer in the Ternary SQ-PCM is just a three level quantizer with abrupt nonlinearities, but as the sampling and coding is done at a much higher frequency, the compandor will allow the processing of weak signals at a higher SNR(Q), in a manner similar to the AQ-PCM (Section 2.5). The compandor in any of these systems is a necessary adjunct because, inspite of giving a much poorer peak SNR than a noncompanded system, it allows for a large variation of speech volumes at a constant SNR, as may be necessary in the commercial applications (where the talking volume of persons differs).

6.2.0 TRANSMISSION CHARACTERISTICS.

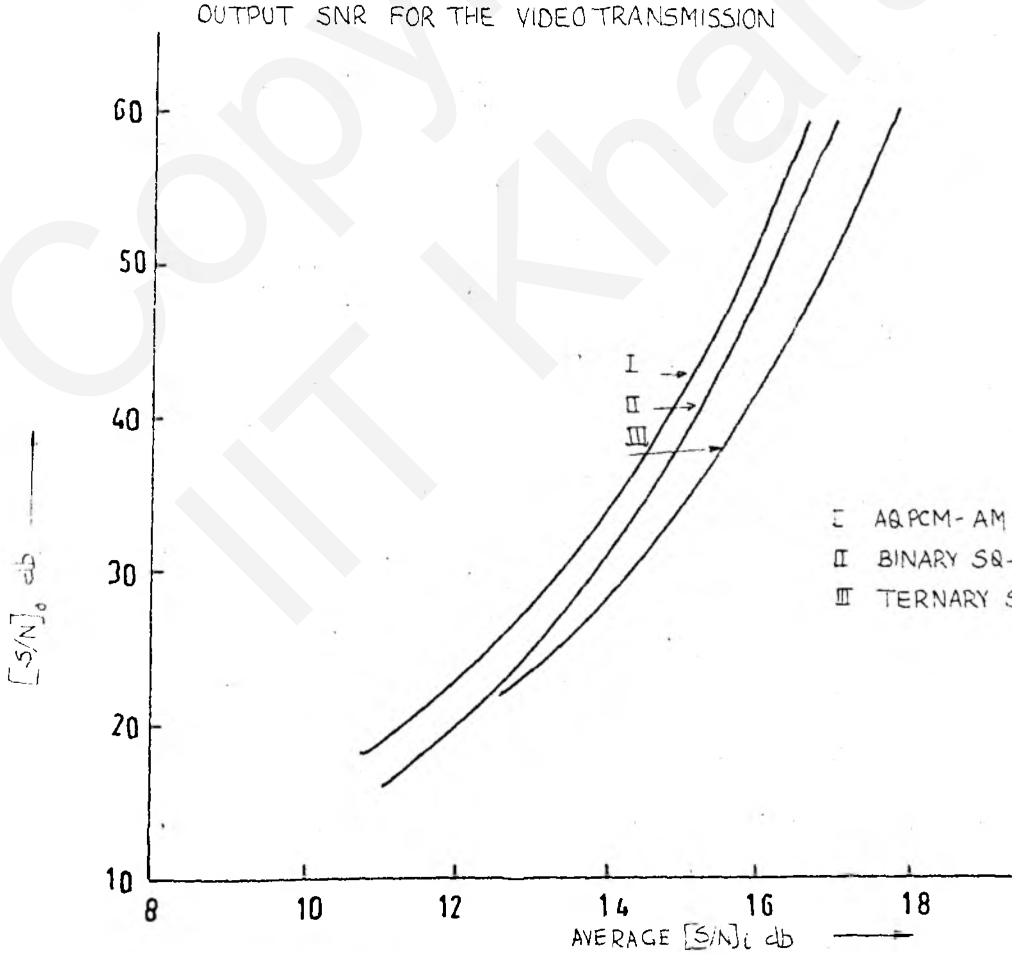
The transmission of the pulses of the PCM and Δ-M systems could be effected by Video transmission or RF transmission, employing, among others, Amplitude Modulation, Frequency modulation and Phase modulation etc. The point of interest is the transmitted power necessary, with a given channel capacity, for a desired output SNR. As the output quality attainable is limited by the original quantization noise introduced in the processing of the signal itself, SNR(C) required to produce this particular output SNR will depend upon the system under consideration and the modulation scheme used for transmission. The quality of transmission of the Δ-M and Δ-ΣM systems will be similar, though poorer, as compared to the SQ-PCM system and therefore the results and the comparison for these systems could be obtained by extrapolating from those for the Binary SQ-PCM system.

Considering the Video transmission first, the SNR(C) required at the input of the receiver for any desired SNR at the output of the receiver is shown in Fig.6.5. The peak power of the transmitter is generally the limiting factor, and the figure of 45 db is chosen as the maximum SNR(Q) possible in the Ternary SQ-PCM at 40 Kc/s PRF. The SNR(C) is seen from the figure to be 18.2 db. The AQ-PCM system (refer Appendix A) requires an SNR(C) of 18.4 db for the S_o/N_o of 45 db, and the Binary SQ-PCM at 40 Kc/s PRF requires an SNR(C) of 18.2 db for an output SNR of 40 db. However, the 1/0 pulses of AQ-PCM and Binary SQ-PCM systems could be converted to +1/-1 pulses, and the peak power reduced to one fourth for the same noise advantage. The peak and average power are the same now and are 6 db below the previous figure. For the Bipolar pulses input SNR of 12.4 and 12.2 db, respectively, is obtained for the required quality at the output. The average power in the Ternary system is, however, about 1.78 db below the peak power, and therefore, on the average power basis, the SNR(C) is 16.42 db for a 45 db output SNR. It is to be seen that the average power in the Ternary SQ-PCM system cannot be reduced further without lowering the two thresholds of slicing (Section 5.1.1). Therefore, for the same slicing levels, or in other words, for the same channel noise, the Ternary SQ-PCM system requires larger average signal power (4.0 db approx.), as compared to the Binary systems for a similar output SNR. This will mean that the channel capacity (per cycle of bandwidth) required for the transmission of the Ternary SQ-PCM system pulses will be more than that for the AQ-PCM and Binary SQ-PCM system pulses. The required channel capacity in bits/second, however, will be lower for the Ternary SQ-PCM than for the AQ-PCM, because of the lesser bandwidth involved (20 Kc/s vs. 30 Kc/s).



PEAK INPUT SNR VS. OUTPUT SNR FOR THE VIDEO TRANSMISSION

FIG. NO. 6·5(a)



... OUTPUT SNR FOR AM TRANSMISSION

FIG. NO. 6·5(b)

The video signalling by 1/0 pulses on cables has also the problem of low frequency and d.c. transmission, and Aaron[27] has shown that, in AQ-PCM systems, a conversion to Pseudo-Ternary pulses shifts the spectral energy at the very low frequencies to slightly higher frequencies (Appendix A), and thus avoids the use of costly transformers etc. The Ternary SQ-PCM here partly overcomes this difficulty and it is seen (Section 3.1.3) that the sideband energy contained around the fundamental and harmonics of the PRF is quite large, so that the audio band spectrum can be easily suppressed. (It is, however, not known definitely whether this technique will succeed because no experimentation has been done with SQ-PCM transmission over any length of cable). Since statistically the number of positive and negative pulses are equal, the system is more or less balanced and therefore intuitively it can be said that the d.c. and low frequency transmission requirements of unbalanced 1/0 pulses will not particularly apply here. The Binary SQ-PCM system can be converted to a Bipolar a Pseudo-ternary system and all the relevant technique employed about AQ-PCM can be used there.

6.2.1. RF MODULATION.

The pulses of Binary SQ-PCM and the AQ-PCM systems could be used to amplitude-modulate a carrier satisfactorily. No satisfactory scheme could be proposed for the amplitude modulation of a carrier by the pulses of the Ternary SQ-PCM system. A three level amplitude modulation though theoretically possible is not very attractive in practice, because the advantage of the on-off method of transmission is lost and the required noise threshold will be small. Fig.6.5(b) shows the SNR(C) required at the input of the receiver of PCM-AM system for a desired output SNR. The SNR(C) in the Binary SQ-PCM-AM system is 15.2 db (at 40 Kc/s PRF) and in the AQ-PCM-AM (50 Kc/s) system, the SNR(C) required is 14.9 db, for an output SNR of 40 db in both the cases. The bandwidth required for the transmissions is twice the video bandwidth in all the cases.

The Frequency modulation, Frequency Shift Keying, and Phase modulation of a carrier by the pulses of these systems can be employed with advantage. The PCM-FM with a deviation ratio of unity has been chosen here for comparison and, if the receiver detector is a simple discriminator, it is known that the SNR(C) advantage (above the threshold) over AM is 4.8 db only, and the bandwidth is twice the bandwidth of the AM. A wider deviation ratio will give a further improvement in the SNR(C) i.e. a lesser SNR(C) is required for the same S_o/N_o, but at the cost of a bandwidth larger than twice the AM bandwidth.

Fig.6.6 shows the input SNR vs. output SNR in the PCM-FM systems. It is seen that the 40 Kc/s Ternary SQ-PCM-FM system will require an SNR(C) of 15.85 db for an S_o/N_o of 45 db, and the 60 Kc/s AQ-PCM system will require an SNR(C) of 14.8 db for the same S_o/N_o. The 40 Kc/s Binary SQ-PCM-FM, and 50 Kc/s AQ-PCM-FM system will require an SNR(C) of 14.6 db and 14.2 db, respectively, for the output SNR of 40 db. As shown in Section 5.2.3, the FM (FSK), using a decision circuit in the receiver requires a much

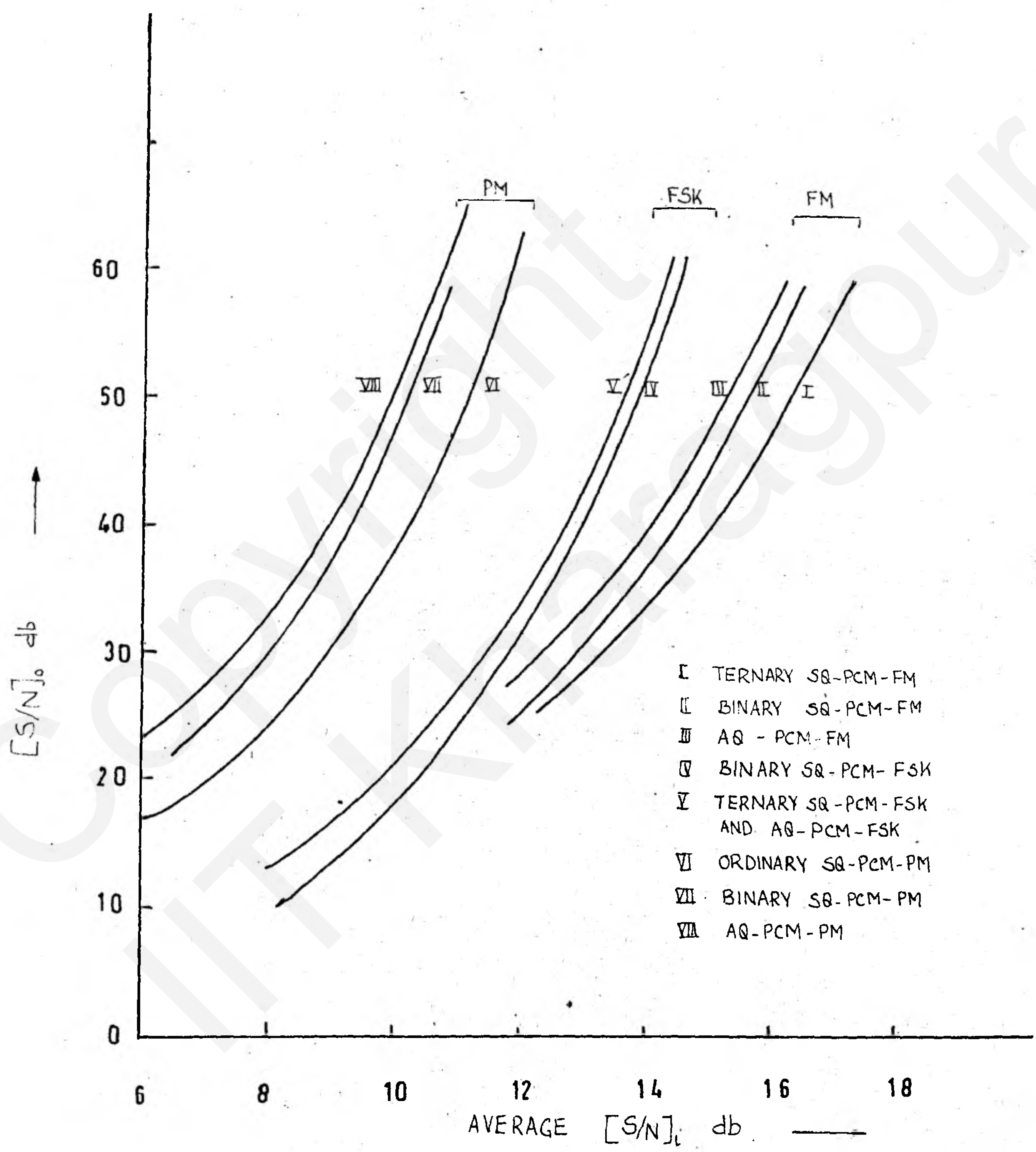


FIG. NO. 6·6

AVERAGE INPUT SNR VS. OUTPUT SNR
FOR FM, FSK, AND PM TRANSMISSION SCHEMES

lower SNR(C) for a particular S_o/N_o. The two-tone AQ-PCM-FSK at 50 ~~60~~ Kc/s PRF has an input SNR(C) of 12.2 db for an output SNR of 45 db, and a three-tone Ternary SQ-PCM-FSK at 40 Kc/s PRF also has an input SNR(C) of 12.2 db for the same S_o/N_o, as shown in Fig.6.6. A two-tone Ternary SQ-PCM-FSK has, on the other hand, an input SNR(C) of 16.4 db for the same S_o/N_o. In the Binary SQ-PCM-FSK at 40 Kc/s PRF, and in the AQ-PCM-FSK at 40 Kc/s PRF, the input SNR(C) is 13 db and 12.7 db for the required S_o/N_o of 40 db, as shown in the Fig.6.6.

A phase modulation of the carrier by the PCM pulses will give even better performance in terms of the transmitted power necessary to get a particular S_o/N_o at the output of the receiver. The Ternary SQ-PCM-PM system at 40 Kc/s PRF requires an input SNR(C) of 10.6 db only for an S_o/N_o of 45 db, as seen in Fig.6.6, and the AQ-PCM-PM requires an input SNR(C) of 9.5 db for the same S_o/N_o. The Binary SQ-PCM-PM at 40 Kc/s PRF requires an input SNR(C) of 9.2 db as seen from Fig.6.6, as compared to the AQ-PCM-PM at 50 Kc/s PRF which, requires an input SNR(C) of 8.9 db for the same S_o/N_o of 40 db.

Fig.6.7 shows the channel SNR required at the input to the receiver with the desired output SNR, for the PCM-RF modulated systems. The curves for SQ-PCM-RF Modulation and AQ-PCM-RF modulation have been levelled off at the limiting values of the quantizing-noise already present in the system. As has been explained before, there is no need for increasing the signal power further. The greatest advantage of these methods of signal approximations lies, therefore, in the undisturbed and noise free transmission

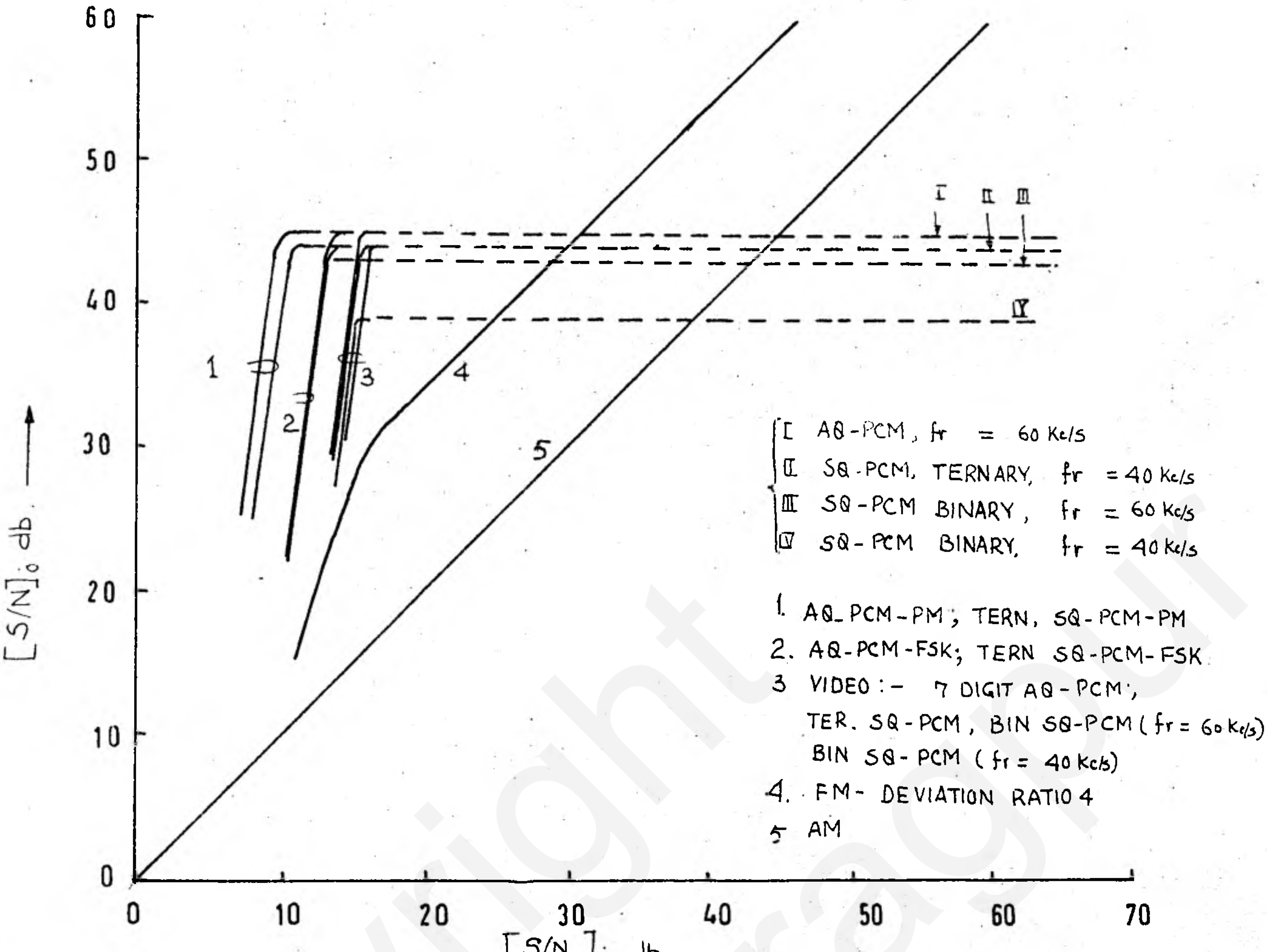


FIG. NO. 6·7

COMPARISON OF RF MODULATED SCHEMES, QUANTIZING NOISE IS THE LIMITING FACTOR IN CURVE I, II, III & IV

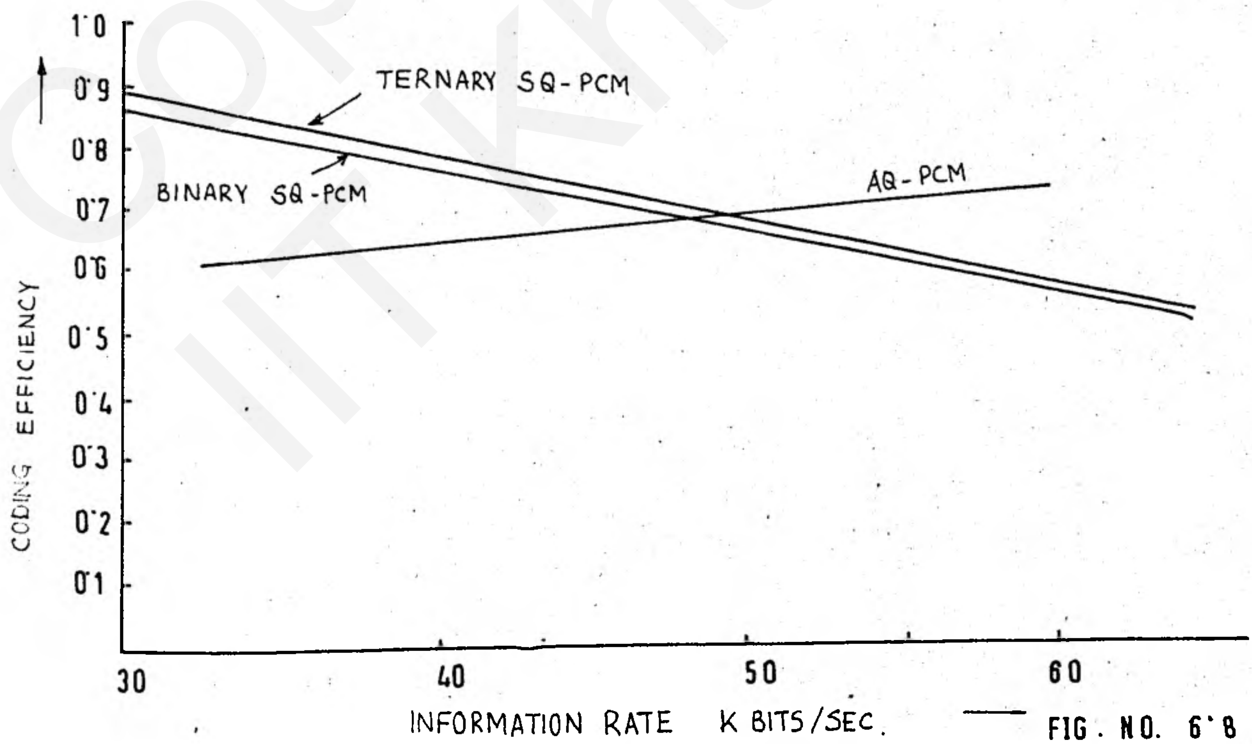


FIG. NO. 6·8

CODING EFFICIENCY OF THE SYSTEMS VS INFORMATION RATE over the medium once the threshold SNR(C) is exceeded. For the message signal - RF Modulation Scheme resulting in FM and AM, the input SNR(C) for satisfactory signalling in the presence of noise is also shown in the figure (curve 4). It is seen that the wide band FM, with a deviation ratio of four or more, will require about 15 db more power than the PCM-RF modulated systems. Similarly, the message signal - AM modulation (curve 5) will require about 30 db more power than the PCM systems for a similar performance.

It is seen that Ternary and Binary SQ-PCM systems require a slightly larger amount of power, as compared to the AQ-PCM systems, for a similar quality of transmission, but the bandwidth required for SQ-PCM transmission is less in most of the cases, and therefore, another comparison of these systems is based on the efficiencies as discussed below.

6.3.0 EFFICIENCIES AND BANDWIDTHS.

The coding efficiency, the power efficiency, and the communication efficiency have been defined in Sections 5.4.1, 5.4.2 and 5.4.3, respectively. A comparison of the coding efficiencies of the Ternary SQ-PCM, the Binary SQ-PCM, and the AQ-PCM is shown in Fig.6.8. At information rates of below 50 K bits/sec. the SQ-PCM systems are more efficient. The reason for this improvement with H is that, in the AQ-PCM systems, the quantizing SNR improves very rapidly with an increase in the number of quantizing levels. This is also a reason why the coding efficiency for the SQ-PCM system falls with higher information rate because, in the SQ-PCM systems the improvement in SNR(Q) with PRF is very slow. The coding efficiency of the Ternary SQ-PCM system is better than the Binary SQ-PCM system at a particular information rate.

6.3.1. POWER EFFICIENCY.

The power efficiencies of the Ternary SQ-PCM-VIDEO, -AM, -FM, -FSK and -PM at 40 Kc/s PRF, have been compared with the corresponding AQ-PCM systems at 60 Kc/s PRF (Appendix A) in Table 1, (for the channel noise = quantizing noise). The lowest figure of β (a lower β indicates a more efficient system) is obtained for SQ-PCM-PM, and is 7.26 compared to 8.9 for AQ-PCM-PM. The unity-deviation-ratio-FM has the highest β of 60.4 for the AQ-PCM- system. The power efficiency of the Ternary SQ-PCM systems is better than those of the AQ-PCM systems, mostly because of lower bandwidths required for transmission (also indicated in Table 1). For the ternary SQ-PCM, the bandwidth required per bit (= α) is much lower compared to that required by the AQ-PCM system.

The power efficiencies of the Binary SQ-PCM-RF-modulated systems at 40 Kc/s PRF are compared with the AQ-PCM-modulated systems at 50 Kc/s PRF in Table 2. The β of the SQ-PCM is poorer than that of the AQ-PCM system because, the bandwidth required per bit being the same, the SQ-PCM requires more transmitter power for the same communication function. The bandwidths required for different modulation schemes are also indicated in the Table 2.

TABLE 1.

S_o/N_o = 45 db.

Mode of Transmission.	Ternary SQ-PCM PRF = 40 Kc/s.		AQ-PCM - 7 digit. PRF = 60 Kc/s.	
	Bandwidth in Kc/s.	β	Bandwidth in Kc/s.	β
VIDEO.	20	13.8	30	17.34
AM.	40	28	60	34.67
FM.	80	48.7	120	60.4
Two-tone FSK.	80	55.4	120	41.30
Three-tone FSK.	120	39.2	-	-
PM.	40	7.26	60	8.91

TABLE 2.

S_o/N_o = 40 db.

Mode of Transmission.	Binary SQ-PCM PRF = 40 Kc/s.		AQ-PCM - 6 digit. PRF = 50 Kc/s.	
	Bandwidth in Kc/s.	β	Bandwidth in Kc/s.	β
VIDEO.	20	15.81	25	14.92
AM.	40	31.62	50	29.85
FM.	80	55.72	100	50.24
FSK.	80	39.0	100	35.98
PM.	40	8.13	50	7.59

As an example, the β efficiency of Ternary SQ-PCM - Video being 13.8, is better than the Binary SQ-PCM-Video efficiency of 15.81. Similarly for other modulation methods and in general, therefore, the Ternary SQ-PCM is more efficient than the Binary SQ-PCM and the equivalent AQ-PCM.

6.3.2. COMMUNICATION EFFICIENCY.

The communication efficiency of the different modulation schemes for the Ternary SQ-PCM at 40 Kc/s PRF and for the 60 Kc/s PRF AQ-PCM systems (Appendix A) is shown in Fig.6.9. The η of video transmission of the pulses is -2 db for the Ternary SQ-PCM, with a channel SNR of 16.4 db, and -3.8 db for the AQ-PCM system, with the same channel SNR, but the output SNR in the AQ-PCM is about 55 db against only 45 db in the Ternary system. The communication efficiency is best for the phase modulation scheme and poorest for the frequency modulation scheme. It is seen that from the efficiency view point, the frequency modulation of the Ternary SQ-PCM pulses is much better than that of the AQ-PCM-FM system. The two-tone FSK of the AQ-PCM and three-tone FSK of the Ternary SQ-PCM are equivalent in terms of communication efficiency.

In Fig.6.10, the communication efficiency of the Binary SQ-PCM-RF-modulated systems at 40 Kc/s PRF is compared with that of the AQ-PCM-modulated systems at 50 Kc/s PRF. Because of the smaller ratio of RF bandwidth to the message bandwidth, the η of the Binary system is better than that for a similar AQ-PCM

COMMUNICATION EFFICIENCY VS INPUT SNR FOR TERNARY SQ-PCM AND 7 DIGIT AQ-PCM MODULATED SCHEMES

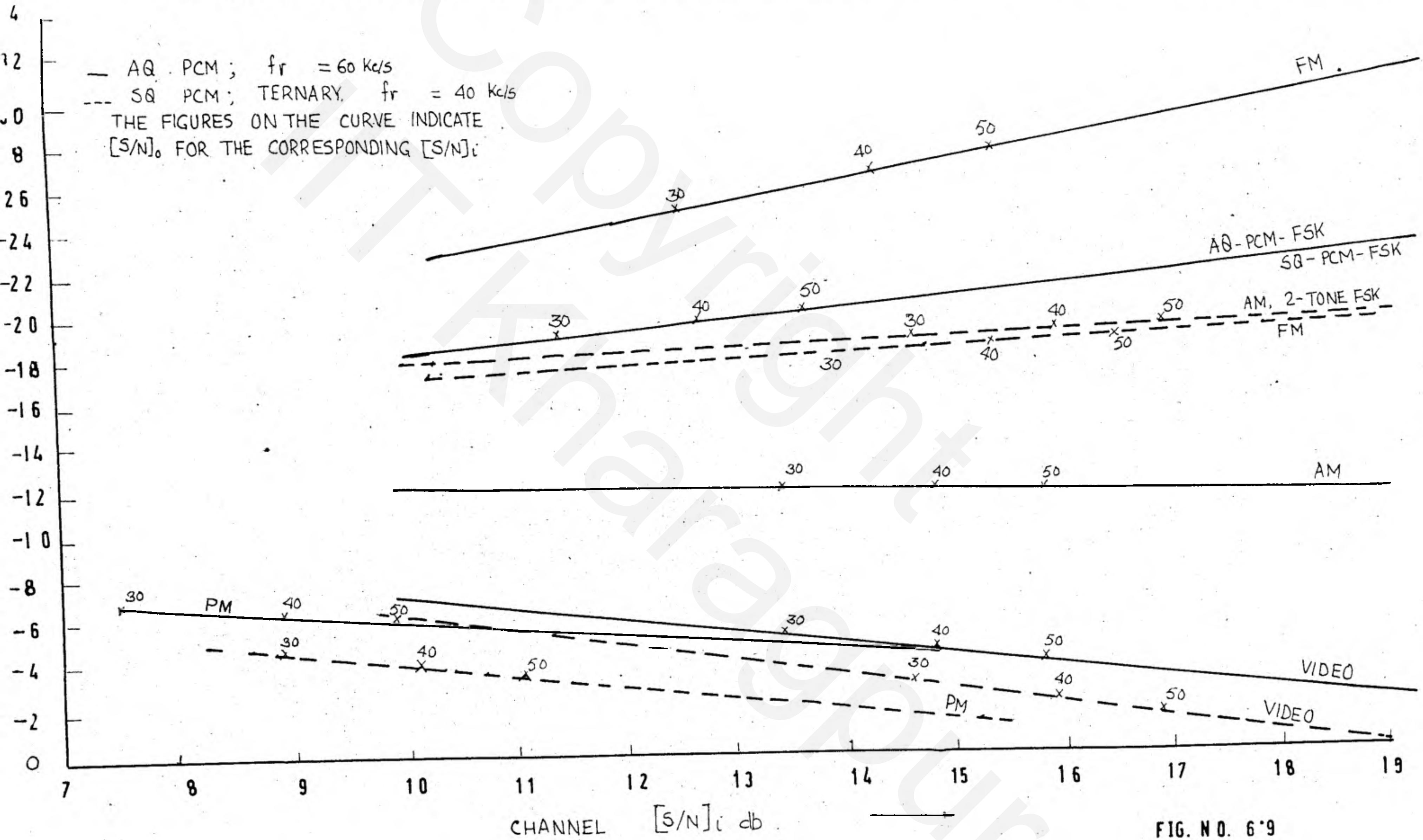


FIG. NO. 6·9

COMMUNICATION EFFICIENCY VS INPUT SNR FOR BINARY SQ-PCM AND 6 DIGIT AQ-PCM MODULATED SCHEMES

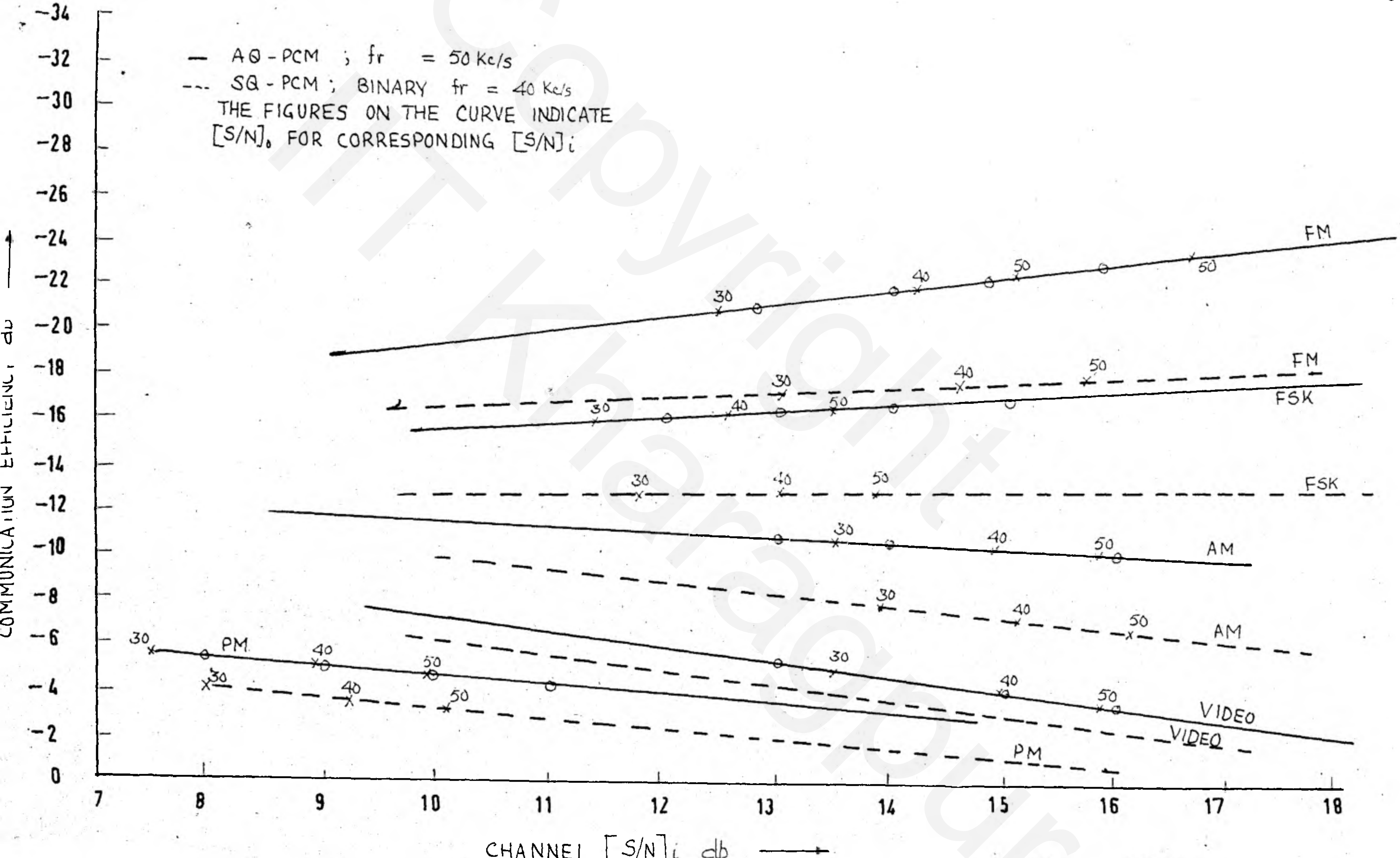


FIG. NO. 6'10

system. As an example, for video transmission, the η of the Binary SQ-PCM is -3.1 db for an input SNR of 15.1 db, and is -4 db for the AQ-PCM system with the same input SNR. The communication efficiency of the Ternary SQ-PCM is better than that of the Binary SQ-PCM system e.g. -2.3 db for the Ternary, and -3.1 db for the Binary video transmission (of course, for the same output SNR of about 40 db).

6.4. SUMMARY.

In summarising the comparisons of the different coded pulse modulation systems, it can be said that the Ternary SQ-PCM at 40 Kc/s PRF has an equivalent performance to AQ-PCM system at 60 Kc/s PRF, in almost all respects, e.g., quantizing noise, flatness of the gain vs. frequency curve, linearity of input-output voltage, the dynamic range etc. The improvement in the dynamic range due to companding is less in the Ternary SQ-PCM, and the multiplexing of many channels is as complex, if not more. The transmitted power required for transmission over either a video channel, or an RF channel is more in the Ternary SQ-PCM system, but this is more than off set by the advantage of using a lesser bandwidth. The net result being that power efficiency and communication efficiency of SQ-PCM are superior to that of the AQ-PCM system. From another point of view, perhaps the most important one, the Ternary SQ-PCM has the great advantage of being very simple in circuitry. The coding and quantizing circuit, which can be made common for multiplexing, employs very few circuits and components, and is very stable against voltage variations in the quantizer and sampler circuits. It has been shown that even in the extreme case when one of the quantizers is not working, the circuit only degenerates into a system similar to the Binary SQ-PCM. Small variations like 1% change of the reference bias voltage, has a negligible effect. In the AQ-PCM coder, however, a noise uncertainty equal to one quantizing step results in a SNR deterioration of 3 db.

The Binary SQ-PCM system at 40 Kc/s PRF is similarly equivalent to the AQ-PCM system at 50 Kc/s PRF, and to the Δ-M system at 60 Kc/s PRF. The advantage of the Binary SQ-PCM system over Δ-M is the comparative flatness of the gain vs. frequency characteristics and of course, a better SNR at much lower PRF. The Δ-ΣM achieves an even better frequency response than the Binary SQ-PCM, but unfortunately the quantizing SNR is much poorer. The partial companding of the Binary SQ-PCM has been successfully tried, but no results are available regarding the companding of Δ-M and Δ-ΣM for comparison. The power efficiency of the Binary SQ-PCM-RF transmission is poorer but the communication efficiency is better than those of the similar AQ-PCM system. The stability of the Binary SQ-PCM system is not as good as that of the Ternary system, but the small variations of bias in the quantizer circuit do not effect the performance to any appreciable degree.

CHAPTER VII.

C O N C L U S I O N S.

7.0. GENERAL.

In view of the limitations of the existing methods of analogue-to-digital conversions and digital signal transmissions, the author has tried to develop some simple methods of signal coding using Ternary and Binary pulses, resulting in Ternary and Binary SQ-PCM systems. The coding and transmission characteristics of the systems have been examined in detail, and the superiority of the ternary coding over the binary has been established. The Ternary SQ-PCM system shows great promise, not only because it is more efficient, but also because it is very simple. Application of a similar technique results in an improved Binary system also. The discussion and the results presented in the foregoing chapters show that the performances of the Binary SQ-PCM is better than or equal to those of AQ-PCM, Δ-M and Δ-ΣM for an useful range of PRF. The SQ-PCM systems are suitable for applications in speech communication, digital computers, hybrid computers, digitalised feedback systems, telemetry, electronic exchanges, and such other digital systems. In the present chapter the salient features of the SQ-PCM systems, along with their possible applications and the scope of future work are briefly discussed.

7.1. SALIENT FEATURES OF THE SQ-PCM SYSTEMS.

The coding of Ternary and Binary SQ-PCM has been developed by utilising the quantization of signal slopes, a method similar to the equal-slope approximation used in Network synthesis. The straight-forward three-level quantization of signal slopes results in a compressed output at the receiver, but for a particular input level, the output SNR is found to be quite acceptable. The quality of this approximation improves considerably, giving higher SNR and dynamic range, if a suitable feedback from the output to the input is used and the difference signal is quantized. The two thresholds used for the Ternary coding, together with the negative feedback, minimise the errors in the approximation. The circuit developed is very simple, high overload tolerance and is sufficiently stable against drifts of the reference and bias voltages. The two-level quantization of signal slopes also gives a fairly improved performance as compared to other similar digital systems. The improvement is attributed to (i) the particular exponentially decaying impulse response of the feedback network, and (ii) the use of threshold in the quantizer-comparator circuit. Instantaneous companding has been successfully used to extend the dynamic range of the input for both the Ternary and the Binary systems. As sufficiently high SNR's are obtained at lower PRF's, smaller bandwidths will be required for RF transmission. Since the bandwidths are small, the power and communication efficiencies are high.

A quantitative critical evaluation of the SQ-PCM systems with reference to other digital systems is given in Table I and II. Table I gives a broad summary of the coding characteristics of the Ternary SQ-PCM (with and without feedback at a PRF of 40 Kc/s),

TABLE I

CODING CHARACTERSTICS

SYSTEMS → / SYSTEM PARAMETERS ↓	TERNARY SQ-PCM WITHOUT FEED BACK PRF 40 Kc/s	TERNARY SQ-PCM WITH FEEDBACK PRF 40 Kc/s	BINARY AQ-PCM 7-DIGIT PRF 60 Kc/s	BINARY SQ-PCM PRF-40 Kc/s	BINARY AQ-PCM 6-DIGIT PRF 50 Kc/s	BINARY Δ-M PRF 60 Kc/s	BINARY Δ-ΣM PRF 60 Kc/s
OPTIMUM SNR	‡ 32 db	41 db	42 db	37 db	37 db	33 db	29 db
DYNAMIC RANGE ABOVE 25 db.	‡ 21 db	29 db	29 db	17·5 db	19 db	13 db	9 db
COMPANDORED DYNAMIC RANGE	–	40 db	44 db	35 db	34 db	–	–
INPUT-OUTPUT VOLTAGE LINEARITY	NOT LINEAR	LINEAR	LINEAR	LINEAR	LINEAR	NOT LINEAR AT HIGH fm	LINEAR
FALL IN OUTPUT AT MAX fm INPUT 0 db	+ 5 db	* 3 db	0 db	* 5 db	0 db	** 16 db	0 db
FALL IN SNR AT MAX fm INPUT 0 db	+ 10 db	* 6 db	0 db	* 8 db	0 db	16 db	0 db
INCREASE IN SNR WITH INCREASE IN PRF	7·5 db PER OCTAVE	8·0 db PER OCTAVE	6 db PER DIGIT	8·6 db PER OCTAVE	6 db PER DIGIT	** 9 db PER OCTAVE	9 db PER OCTAVE
SNR WITH ½ BAND FM SIGNAL	+ 30 db	33 db	$ 34 db	28 db	$ 28 db	⊗ 25 db	⊗ 20 db
SNR WITH NARROWBAND FM SIGNAL	+ 32 db	34 db	$ 34 db	30 db	$ 28 db	⊗ 26 db	⊗ 21 db
SNR WITH ½ BAND NOISE SIGNAL	‡ 24 db	32 db	33-34 db	27 db	28 db	⊗ 24 db	⊗ 20 db
QUALITY OF SPEECH SIGNAL	FAIR	EXCELLENT	EXCELLENT	GOOD	GOOD	FAIR	FAIR

+ ORDINARY INTEGRATOR ‡ NON-LINEAR INTEGRATOR IN THE RECEIVER

* AT AN INPUT LEVEL OF −5 db, THE OUTPUT AND SNR ARE CONSTANT WITH fm

$ IN AQ-PCM, THE SNR FOR THE COMPLEX SIGNALS IS THE SAME.

** THE OUTPUT AND SNR FALLS BY 6 db / OCTAVE CHANGE OF fm

⊗ EXTRAPOLATED FROM SINUSOIDAL SIGNAL INPUT RESULTS

Binary SQ-PCM (PRF 40 Kc/s), AQ-PCM (7-digit and 6-digit), Δ-M (PRF 60 Kc/s) and Δ-ΣM (PRF 60 Kc/s) systems. It is seen that the optimum SNR of the Ternary with feedback is comparable to the 7-digit AQ-PCM and higher than that of all the other systems. The Binary SQ-PCM has a higher optimum SNR than those of the Δ-M and Δ-ΣM systems, and is equivalent to the 6-digit AQ-PCM system. The dynamic range and the extension of the dynamic range by the instantaneous compandor follow the same general pattern. The Ternary system (without feedback) has a compressed output but has a good optimum SNR and dynamic range which will be useful in some particular applications. The input-output voltage relation is linear in all cases, except in Δ-M . The optimum SNR falls at the higher f_m in Ternary and Binary SQ-PCM and Δ-M systems. However, in the SQ-PCM system the SNR is constant with f_m at a slightly lower input level of -5 db. The output is almost constant at all f_m in the SQ-PCM, AQ-PCM and Δ-ΣM systems but falls in Δ-M system at the rate of 6 db/octave change of f_m.

In fact, instead of making point by point comparison of these systems for different f_m, a very good picture of the performance of a system can be obtained by comparing the results for a most general input signal e.g., noise signal. The noise signal test on the Ternary SQ-PCM at 40 Kc/s PRF, gives an SNR of 32 db, as against the SNR of 33 - 34 db in the 7-digit AQ-PCM system. The Binary SQ-PCM, at 40 Kc/s PRF, has an SNR of 27 db, as compared to the SNR's of 28, 24 and 20 db, obtained in the case of 6-digit AQ-PCM, Δ-M and Δ-ΣM , respectively. The judgement of the speech quality has also been made on the same basis, as a system with Noise-Signal-SNR of above 30 db will give excellent results in speech reproduction.

Table II deals with the transmission characteristics of the SQ-PCM and AQ-PCM systems. In general, the results of the coding characteristics of the Δ-M and Δ-ΣM (at 60 Kc/s PRF) are similar, though poorer than those of the Binary SQ-PCM (PRF 40 Kc/s) system, and therefore, their transmission characteristics will be similar, but poorer. In view of this, the transmission characteristics of the Δ-M and Δ-ΣM are not discussed here. In the SQ-PCM systems, the information is coded into occurrence and nonoccurrence of pulses and a number of channels can be multiplexed on the time-division basis. The TDM pulses can be transmitted by lines or coaxial cables, reconstructed completely at each repeater, and the final signal at the receiver is almost unaffected by channel noise, if an SNR(C) of about 20 db at the input of the receiver is maintained. The transmission over radio medium has to be necessarily in U.H.F. and Microwave region as the bandwidth of the SQ-PCM output will be fairly large. An wideband modulation scheme, such as SQ-PCM-FM or SQ-PCM-PM will have an additional noise suppressing property.

As the Ternary transmission consists of 3 levels, the receiver will have to have two threshold circuits to distinguish between +1 and 0, and -1 and 0 levels. The channel noise will have an adverse effect, and compared to the Binary system, this system will be more disturbed by channel noise bursts. However,

TABLE II

TRANSMISSION CHARACTERSTICS

SYSTEM PARAMETERS ↓ / TRANSMISSION SCHEMES →		TERNARY SQ-PCM WITH FEEDBACK PRF 40 kc/s	BINARY AQ-PCM 7 DIGIT PRF 60 Kc/s	BINARY SQ-PCM PRF 40 Kc/s	BINARY AQ-PCM 6 DIGIT PRF 50 kc/s
CODING EFFICIENCY (ζ)		0·59	0·74	0·76	0·70
INPUT SNR REQUIRED AT THE RECEIVER FOR AN OUTPUT SNR OF 45 db IN THE CASE OF TERNARY SQ-PCM/7-DIGIT AQ-PCM, AND OUTPUT SNR OF 40 db IN THE CASE OF BINARY SQ-PCM/6 DIGIT AQ-PCM SYSTEM, THE FIGURES ARE IN db	VIDEO PEAK POWER	18·2	18·4	18·2	17·8
	-AM *	16·4	15·4	15·2	14·8
	-FM	15·8	14·8	14·5	14·2
	-FSK	13·2	13·2	13·0	12·7
	-PM	10·6	9·5	9·2	8·9
POWER EFFICIENCY (β)	VIDEO	13·8	14·92	15·81	17·34
	-AM	28	29·85	31·62	34·67
	-FM	48·7	50·24	55·74	60·4
	-FSK	39·2	36·0	39·0	41·30
	-PM	7·26	7·58	8·13	8·9
COMMUNICATION EFFICIENCY (η) FOR OUTPUT SNR OF 45 db IN TERNARY SQ-PCM/7 DIGIT AQ-PCM; AND 40 db IN BINARY SQ-PCM/6 DIGIT AQ-PCM SYSTEM THE FIGURES ARE IN db.	VIDEO	-2	-4	-3·1	-4·2
	-AM	-19·5	-12	-7·2	-10·5
	-FM	-19	-27·8	-17·8	-22·2
	-FSK	-20	-20	-13	-16·3
	-PM	-3·8	-6	-3·8	-5

* — AM INDICATES THE MODULATION SCHEME e.g. AQ-PCM-AM OR SQ-PCM-FSK

the effect on the signal built up, by the erroneous decoding due to the noise burst, is much less here than that in the Binary AQ-PCM. On a proper evaluation, therefore, the required input SNR here would be similar to that of the 7-digit AQ-PCM system. The Ternary sys-tem is better adapted to an FM or PM wideband modulation scheme, and either a 3-tone FSK or a PM will require the least power for satisfactory transmission.

The power efficiency of the Ternary SQ-PCM (Video or RF modulation) is higher than that of the 7-digit Binary AQ-PCM system, because of a smaller bandwidth necessary for signalling. The power efficiency of the SQ-PCM-PM is highest, again because of the lower SNR(C) and smaller bandwidth than those of the other systems. The communication efficiencies of the Ternary and the Binary SQ-PCM-Video or RF modulation schemes are higher than those of the 7-digit and 6-digit AQ-PCM systems respectively. The communication efficiency of the Ternary SQ-PCM-Video transmission is the highest.

The SQ-PCM systems have some important limitations also. First of all, the improvement in SNR with an increase of PRF is very slow, being at best about 9 db/octave. By using a double integrator in the feedback loop, it is possible to increase the amount of improvement in SNR to 15 db per octave change of PRF. However, the systems' performance with double integration has been found to be inferior to that of the system using only a single integration (for the PRF's of 40 - 60 Kc/s). Debart[34] has shown that a system with multiple integration does not improve the information capacity; it offers, however, advantages in reconstruction of the signal.

Secondly, the coding efficiency of the Ternary SQ-PCM system is lower than that of the Binary systems. This is mostly because the PRF being the same, the actual information rate, H, in a Ternary system is much higher than that in the Binary system. However, as the coding efficiency of SQ-PCM is higher than that of AQ-PCM at lower PRF's, a 2-digit coding, instead of the Uni-digit coding, would improve the overall efficiency, and the improvement per octave change of PRF would also be better. The coding technique for Ternary, developed so far, has proved to be superior to those of the Binary systems in many respects but there is still scope for further improvements in obtaining higher coding efficiency.

Thirdly, more transmitter power is required for the transmission of Ternary signals. Fortunately, this is more than offset by the smaller bandwidth necessary for signalling and hence, the power and communication efficiencies are high. The ease of generating the ternary code, combined with satisfactory results for the coding and transmission characteristics, make the Ternary SQ-PCM system very suitable for all digital applications. The Binary SQ-PCM also will be more useful than such other Binary systems.

7.2. SOME APPLICATIONS OF SQ-PCM.

The SQ-PCM system may be employed with profit either, as a means of communication or, in analogue to digital conversion schemes. Since the basic circuits are simple, and the coding and transmission characteristics are comparable to other digital systems, the applications of the Ternary SQ-PCM can be foreseen in all situations where digital signal processing is advantageous. The first and foremost application of these systems is in speech communication either, through coaxial cables or, by wideband modulation schemes. An useful dynamic range of approximately 40 db and an SNR of about 30 db is normally quite adequate for speech transmission and the SQ-PCM has been shown to provide these with reasonable bandwidth requirements.

As video signal transmission over ordinary telephone wires is possible, the SQ-PCM could be used in integrated electronic exchanges, where it is advantageous to use the same digital method for both the speech transmission and control switching. Thus, the advantages of using coded signals as well as of introducing electronics into telephone switching would be obtained in the form of reduction in the volume of equipment, greater reliability, lesser distortion in signals and increased flexibility of operation. Also, existing junction routes between exchanges can use digital techniques because a TDM SQ-PCM will extend the transmission capacity of the conventional paths. The repeaters would be simple and can be conveniently located. Most of the schemes proposed for the integrated electronic exchanges[35], have concentrators for every 150-200 subscribers and the output of these concentrators is coded with SQ-PCM. No change is made in the subscribers set, and all the processing to the digital form is confined to the concentrators. However, a transistorised version of the SQ-PCM coder can be made small enough to be placed in the subscriber's telephone equipment so that the signals emanating from the subscriber will be in the digital form. The advantage of any such scheme will be the homogeneity of the techniques used in modulators, demodulators, control and supervision circuits, regenerators and channel dropping equipment etc., as all signals will be in the digital form.

Speech information appearing as a series of ternary or binary pulses can take advantage of all facilities offered by digital information handling techniques, e.g., analysis, memorizing, ciphering and correlation etc. As a result, this digitalised speech information could be sent in a lesser bandwidth by using a technique known as Coding downwards. A 95% intelligible speech may be obtained by an SQ-PCM system working at, say, a PRF of 10 Kc/s only, and if the groups of 5 (or more) pulses are converted with a ternary code to PAM pulses of much lower PRF of, say, 2 Kc/s, then the filtered wave will have a bandwidth of 1 Kc/s only for the original 3 Kc/s signal bandwidth. In the receiver, if the complementary functions are performed, then the message signal would be of a much better quality than that obtained by other band-compressing methods for the same compression ratio. Alternately, the multiplexed SQ-PCM pulses may be grouped together to form the PAM pulses and transmitted in a reduced video bandwidth. The penalty for this compression by coding downwards would be such that, an SNR of about 40 db in the transmission medium would be required for efficient transmission of the 100 level PAM pulses.

The digitalised speech information may also be transmitted with lesser power, than that required in the usual systems, by using a technique similar to the Datec[44]. In such a digital adaptive technique, the average pulse rate is reduced to 1/5 of the equivalent Δ-M and efficient voice communication may be obtained with an average of 3500 pulses/sec. only. Using SQ-PCM, this compound coding would be more efficient, and only about 2000 pulses/sec. need be transmitted.

For the processing of wideband signals, e.g., Television signals, the equivalent PRF required by Ternary SQ-PCM will be quite low as compared to that of all other systems. Inose[5] has shown that for a TV of 3.5 Mc/s video-signal bandwidth, a PRF of 30 Mc/s is sufficient for a fairly good reproduction using Δ-ΣM coding, where the effective SNR is about 20 db only. The Ternary SQ-PCM system has an SNR of 25 - 30 db for 3.5 Mc/s message band at a PRF of 20 Kc/s only, and by extrapolating these results, it can be said that a PRF of 20 Mc/s would be more than adequate for reproducing fairly good TV pictures. In fact, it is expected that a PRF of 15 Mc/s only will give quite meaningful results. The Binary SQ-PCM system will process the same quality of TV signals at a PRF of 20 Mc/s only.

SQ-PCM systems will have wide applications in telemetry and other analogue-to-digital conversion applications, particularly so in the industrial digital feedback-control systems and hybrid computers. Information about the state of a process is, ordinarily, in the form of electrical signals from transducers measuring pressure, temperature, displacement, acceleration, and so on. For digital automation, data handling, switching, computing, and other operations that are most readily carried out in a pulse or digital form, it is necessary to convert these analogue informations into digital informations. The SQ-PCM coding can be used for such conversions with ease and lesser error. It can be said that wherever AQ-PCM has been used in such applications, the SQ-PCM systems are also applicable and to a better advantage because of their simple signal processing technique. In fact, even in cases where coded and digitalised systems are advantageous but AQ-PCM is not used because of its complexity, the Ternary and Binary SQ-PCM systems could be readily used. The FDM telemetry signals could also be transmitted in the form of FDM-SQ-PCM, as these systems could easily be adapted for wideband applications. In the field of computation, the Analogue and Digital Computers have so far found distinctive applications, but recently it has been felt that a combination of analogue and digital techniques would be best suited for solution of the special problems generated by the increasingly complex technology. As a result, a growing number of hybrid computers are in use, and it can be readily seen that SQ-PCM may easily be adapted for such applications.

Finally TDM-SQ-PCM systems combined with some RF modulation schemes can be used in Radio Relays and Satellite communications

Due to the Doppler shift in satellite communication, the requirements of accurate synchronisation are critical in synchronous type of modulation scheme, like AQ-PCM and SQ-PCM. However, compound modulation schemes such as SQ-PCM-FM-FB (frequency modulation using negative feedback) are likely to overcome or at least reduce these difficulties.

7.3. SCOPE OF FUTURE WORK.

One of the major drawbacks of the Ternary or Binary SQ-PCM system is that the SNR does not increase rapidly with an increase in the PRF. It may be possible to process the signals in such a way that a cumulative advantage in the SNR will be obtained if the signal is approximated successively, and $\Delta f(t) = \left[f(t) - \widetilde{f(t)}\right]$ is again processed to give $\widetilde{\Delta f(t)}$. Then the combined signal $\left[\widetilde{f(t)} + \widetilde{\Delta f(t)}\right]$ will approximate f(t) with an SNR almost twice of that obtained by a single approximation. A 2-digit Ternary SQ-PCM may, thus, give an SNR of about 60 db, if the sampling rate is 20 Kc/s, giving an effective PRF of 40 Kc/s. But, in actual practice, the SNR of a 2-digit system will not be twice that of the Uni-digit case, because the improvement in SNR with feedback depends on the strong correlation between the signal samples, and the difference signal $\Delta f(t)$ will have larger bandwidth and less correlation at a given PRF. The samples of the $\Delta f(t)$ will be more random than the samples of the original f(t), and the SNR for the second approximation will generally be poorer. However, it is expected that a 2-digit Ternary SQ-PCM system will have approximately an SNR of 50 db at an equivalent PRF of 40 Kc/s.

The processing of $\Delta f(t)$ will improve with higher PRF's and it may be possible to get an SNR of 75 db with an equivalent PRF of 60 - 65 Kc/s. The investigations, along the line suggested, have already been started and the 2-digit Ternary system is in the experimental stage. The Binary SQ-PCM system also can be processed in a similar manner, and a good improvement in SNR may be expected.

With a view to eliminate some of the defects in SQ-PCM coding, certain modifications in the process of approximation using feedback have been suggested by other workers in the laboratory. The field of their work has been mostly in the binary systems and it has been found that the sampler of the coder could be taken outside the feedback loop without introducing any deterioration in the performance. The residual frequency distortion in the Binary SQ-PCM system may be corrected by a modification where two feedback loops are used. One feedback loop is around the quantizer and the other around the complete coder including the sampler and the quantizer. Such a system has an optimum SNR of the same order as the present Binary SQ-PCM, but has the additional advantage of having constant output and SNR for the whole of the message band. The Ternary SQ-PCM system could also be improved in a similar manner.

The Δ-ΣM system has a poorer SNR than the Δ-M because the equalisation of SNR with f_m in Δ-ΣM has been done at the SNR value obtained for the highest f_m. It is felt that the large aliasing distortion in the sampler has been responsible for the poor SNR in Δ-ΣM , as the quantizer is placed after the sampler. If the sampler precedes the quantizer, with other circuits remaining the same, the optimum SNR has been found to improve by about 6 db. The other properties of Δ-ΣM remain as they are reported by Inose. Such a modified Δ-ΣM circuit can also be converted to a ternary-coder. The optimum SNR now will be slightly poorer (expected value is 36 db at 40 Kc/s PRF) but the system will have the advantage of giving a very wide message bandwidth along with constant output and constant SNR throughout the whole band.

The problem of designing a suitable device for direct 3-level quantization, instead of using two binary circuits in parallel, is yet to be solved. Some preliminary experiments with various circuits and elements have been made but the stability of these devices is poor and the two thresholds are not symmetrical. One possibility would be to use a matched pair of PNP-NPN transistors along with Zener diodes to establish the +ve and -ve thresholds, symmetrically and accurately. The use of a separate compressor for nonlinear coding is also a problem which should be eased by designing coders with compressor as an integral part, so that the overall circuit is further simplified. For TDM operation, generation and transmission of synchronising signals have not been properly investigated as yet and more work is called for to find out a stable synchronising signal to be transmitted along with the signal codes.

The complexity in the circuits of a Ternary coded AQ-PCM, although shown to be more efficient than Binary AQ-PCM, has been a major hindrance for its general applications. It would, however, be possible to simplify this ternary coder by using delay lines and suitable feedback around the coder. But for useful range of PRF's, a two-digit Ternary SQ-PCM would have superior characteristics with simpler circuitry as compared to an equivalent Ternary AQ-PCM. It is, thus, seen that further investigations should be carried out with other promising Ternary-coded systems, specially of the type using only Uni-digit or two-digit codes, similar to the SQ-PCM. With the proper solutions of some of the outstanding problems mentioned above, it is expected that future wideband communication and other digital systems would be more efficient, but much simplified.

APPENDIX A.

PULSE CODE MODULATION (AQ - PCM).

A.0. GENERAL.

The aim of any communication system is to transmit information from one place (source) to another (receiver) through some medium with as much efficiency as possible. The medium may be a cable or a radio frequency link, and the efficiency of interest normally is the amount of power required for a satisfactory signal to noise ratio at the receiver output. It has been shown by Shannon[8] that a communication system using a wideband coding (modulation) will give a better signal to noise ratio at the receiver output as compared to a narrow band coding for the same transmitter power. The Pulse Code Modulation system (AQ-PCM) is one of the most efficient systems, where by using binary on-off pulses for transmission, a high quality of transmission can be obtained under such adverse noise conditions that it is just possible to recognise the presence of a pulse. A brief review of the AQ-PCM system - its principle of operation, quality of transmission, bandwidths, and efficiencies is now given.

A.1.0. CODING CHARACTERISTICS.

A continuous signal f(t), of duration T seconds, and limited in bandwidth to W_m c/s, could be represented by $2W_mt$ independent samples, taken at a regular rate of $2W_m$ samples/second. The message signal could be represented as,

$$f(t) = \sum_{n} f(n/2W_m) \cdot \frac{\sin \pi(2W_m t - n)}{\pi(2W_m t - n)} \qquad \ldots \text{(A.1).}$$

that is, the f(t) is a sum of a series of simple functions of the Sin x/x type, centred at the sampling instants, $t = n/2W_m$

The peak value of the Sin x/x function is equal to the value of the sample at the corresponding sampling point. At the receiver, the signal can be reconstructed from the sample values representing the signal by generating a proportional impulse from each sample and then passing the series of impulses through an ideal low pass filter of cut-off frequency W_m c/s. The response of an ideal low filter to an impulse is a Sin x/x pulse and therefore the principle of equation (A.1), applies.

The method of sampling and sending only samples at regular intervals of $1/2W_m$ can be used for a fairly perfect transmission of information. The technique, however, suffers in one very important respect, and that is, that it is impossible to transmit the exact amplitude of the sample under practical and physical conditions. The amplitude of the sample is sent as an amplitude of a pulse, a position of a pulse, or some other property of a pulse. On the line, noise, distortion, and crosstalk between pulses will disturb the amplitude or position of the pulse and therefore, after recovery, the size of sample will not be the same as the original. If the transmission is over longer distance where repeaters are necessary, the error is cumulative, and inspite of sufficient amplifications along the way, the final amount of error may become excessive, and therefore, limit the distance of transmission.

An excellent solution of this problem is to send only discrete values of the amplitudes or position of the pulses instead of the continuous range of values. The discrete amplitude transmitted is nearest the true sampled value of the signal. At the receiver, due to the noise introduced on the way, the amplitude received will be different from all expected discrete amplitudes. However, if the noise and distortion introduced is small, the erroneous amplitude can be identified as the nearest correct amplitude. Quantizing the signal amplitudes into some discrete levels only, therefore, introduces some error initially in the transmitted amplitude itself. This error, generally called quantizing noise, can however, be made very small by dividing the total range of the signal values into a large number of discrete levels; that is, the separation between the levels can be made small. On the other hand, if the separation between the levels becomes too small, the difficulty of recognising the quantized levels in presence of noise increases. However, once the amplitude has been quantized, it can be sent over long distances using repeaters, subject to the condition that at each repeater the noise added is small enough to allow for a correct recognition of the quantized amplitude level. Since the correct level at the repeater is recognised, at the output of the repeater the signal is as good as the original and there is no cumulative effect of the noise on the line.

From a theoretical point of view, the sampling and quantizing of the amplitudes are the two basic things in AQ-PCM, but unfortunately, with the practical circuits, it is not possible to distinguish all the discrete amplitudes which the single sample pulse at the instant may be representing. However, the quantized levels could be coded into a binary number, where any particular amplitude may be represented by a set of on-off pulses. The receiver is only required to distinguish between the presence, or absence, of a pulse. Thus, several pulses as a code group are used to represent one quantized amplitude. From the law of binary numbers, it is known that a group of 6 pulses can represent numbers upto 64, and a group of 7 pulses can represent a number upto 128. The signal amplitude may be quantized into, say, 128 levels and each level could be represented by this code group of 7 on-off pulses. For decoding at the receiver, each pulse of the code group is multiplied by a weighting factor (the position of the pulse in the group), and then linearly added to form a pulse which has the exact amplitude of the original quantized signal. The coding of the quantized levels is necessary for the successful implementation of the idea of sampling and quantizing.

The AQ-PCM system consists essentially of these three elements, - sampling, quantizing, and coding[15]. The message signal is sampled at a rate slightly greater than $2W_m$ samples per second, quantized into ℓ different levels and finally, coded into a group of n binary pulses, where

$$\ell = 2^n$$

The sampling is really a quantization in time and 'quantizing' here is an amplitude quantization. As has been mentioned previously, the error (quantizing noise) depends on the size of the step of the amplitude quantizer. To determine the relation between the distortion or error, and the size of the quantizing step, the signal is impressed on a "staircase transducer", having an input-output relation as shown in Fig.A-1 (a).

The staircase transducer sorts the input signal voltage into small compartments forming the tread of the staircase and all signals within plus or minus half a step of the midvalue are replaced at the output by the midvalue. The effect of such quantization on a smoothly varying function of time is illustrated in Fig.A.1(b), and it is seen that the output remains constant, so long as the signal is confined to a tread, and jumps by one full step when the input increases beyond the tread. The signal therefore is approximated with such a staircase, and the difference between the original wave and this approximated wave will be the error. The instant by instant error curve is also shown in the same figure. The maximum value of the error is clearly a half of one step and varies between the limits of plus half a step to minus half a step. This error waveform can be approximated, for all practical purposes, by a triangular waveform as shown in Fig.A.2 and the mean square value of the error could be calculated. The slope of the triangular waveform will be quite arbitrary, but the amplitude of the triangle will vary between plus and minus half a step. The calculation is the same if the mean square value of a straight line of arbitrary slope but amplitude confined to plus and minus half a step is calculated. If E_0 is the voltage corresponding to one step and s is the slope of the line, then

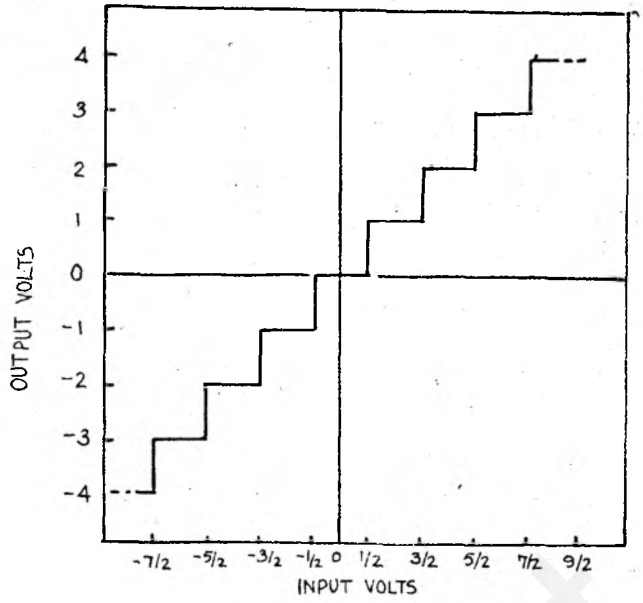


FIG. NO. A·1 (a)

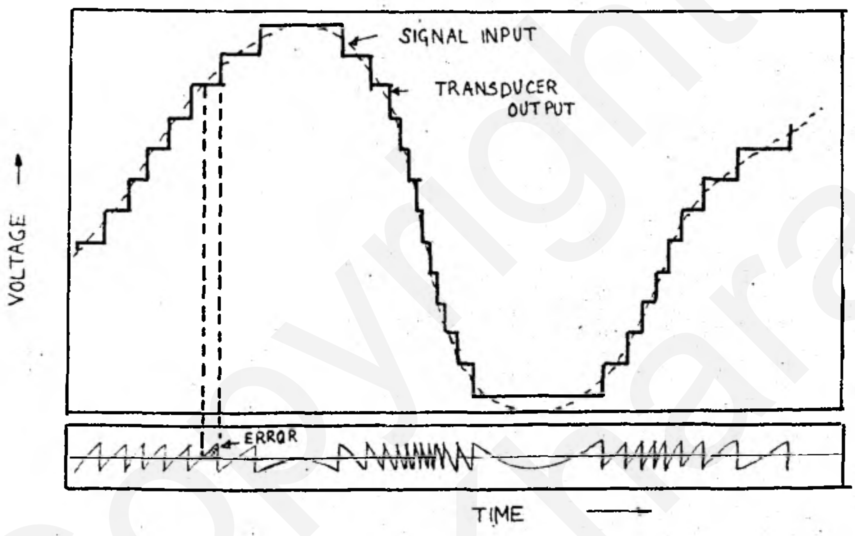


FIG. NO. A·1 (b)

(a) STAIRCASE TRANSDUCER

(b) A QUANTIZED SIGNAL WAVE AND THE CORRESPONDING ERROR WAVE

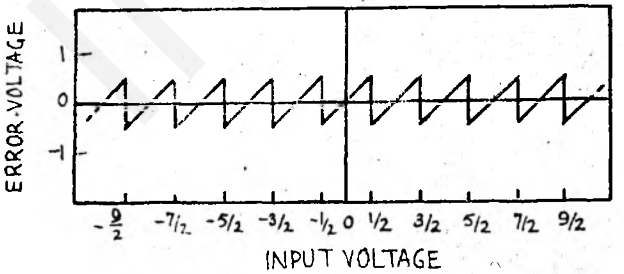


FIG. NO. A·2

CHARACTERSTICS OF THE ERROR WAVEFORM

the equation of a line is

$$\epsilon = st \quad , \quad -\frac{E_o}{2S} < t < \frac{E_o}{2S}$$

where ϵ is the error voltage, and t is the time with reference to the midpoint as the origin.

Therefore,

$$\overline{\epsilon^2} = \frac{S}{E_o} \int_{-E_o/2S}^{E_o/2S} \epsilon^2 \, dt \qquad \text{... (A.2).}$$

$$\text{or} \quad \overline{\epsilon^2} = E_o^2/12$$

The mean square value of the error is therefore 1/12th of the size of the square of the quantum step. This indicates that there will be large errors on quantizing, but fortunately, all the error power does not fall in the signal band. In fact, the application of the signal to the nonlinear staircase characteristics could be thought of as a modulation process where the error is also distributed in the higher order modulation products which could be removed by a low pass filter. The error spectrum for a general signal, like band limited noise, has been determined by Bennett[17] and shown here in Fig.A.3. The error spectrum for different number of quantized amplitude levels like 16, 32, 64, and 128 i.e. for number of binary digits n like 4, 5, 6 and 7 respectively, has been shown in the figure. The error spectrum is fairly wide spread and the area under each curve represents the error power density. As the number of digits is increased, the error power spectrum becomes flatter and more widespread, but the maximum density is reduced. The error power falling in the signal band is just the area under the curve from zero to unit

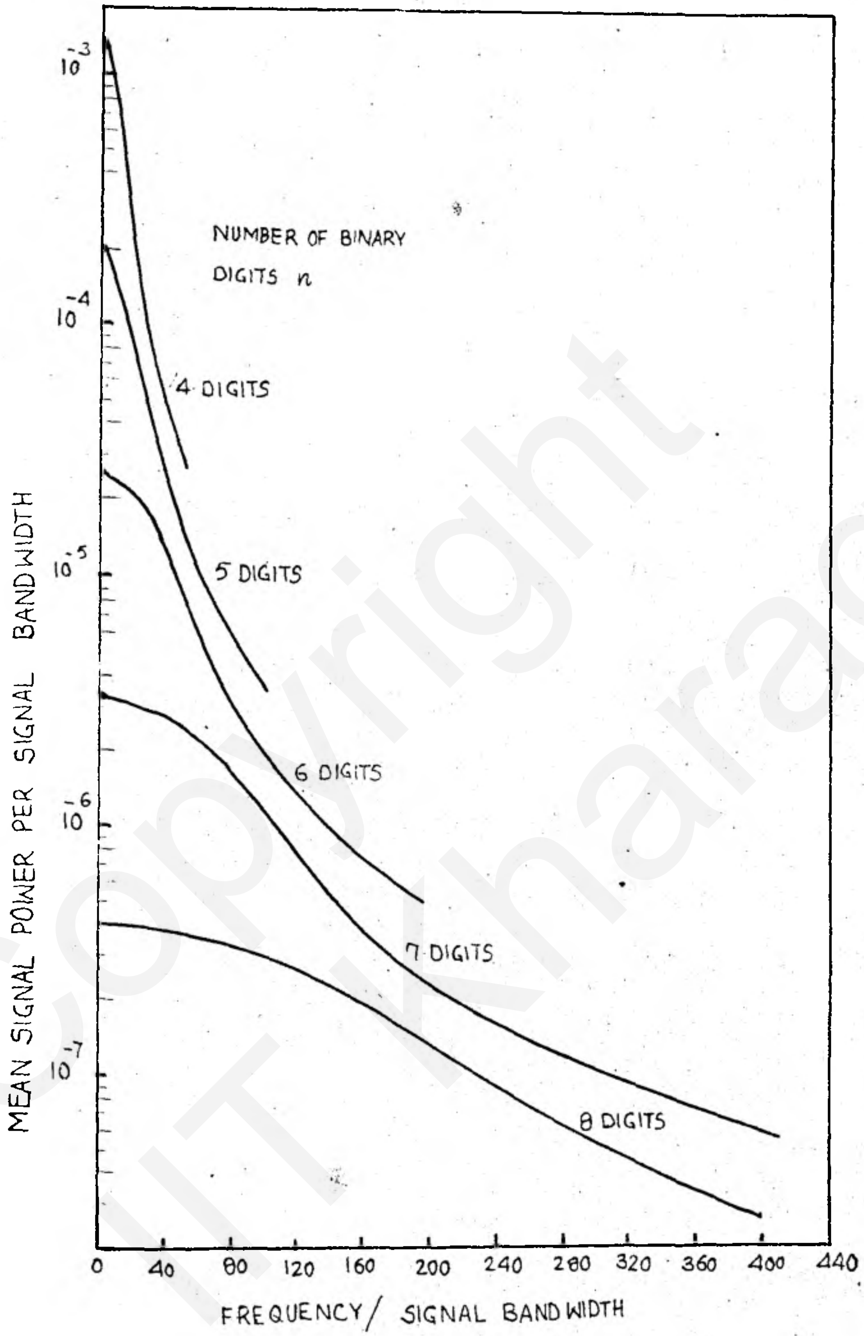


FIG. NO. A·3

ERROR SPECTRUM FOR A RANDOM NOISE SIGNAL abscissa, and thus, it is seen that with the increase in number of quantizing levels the error reduces.

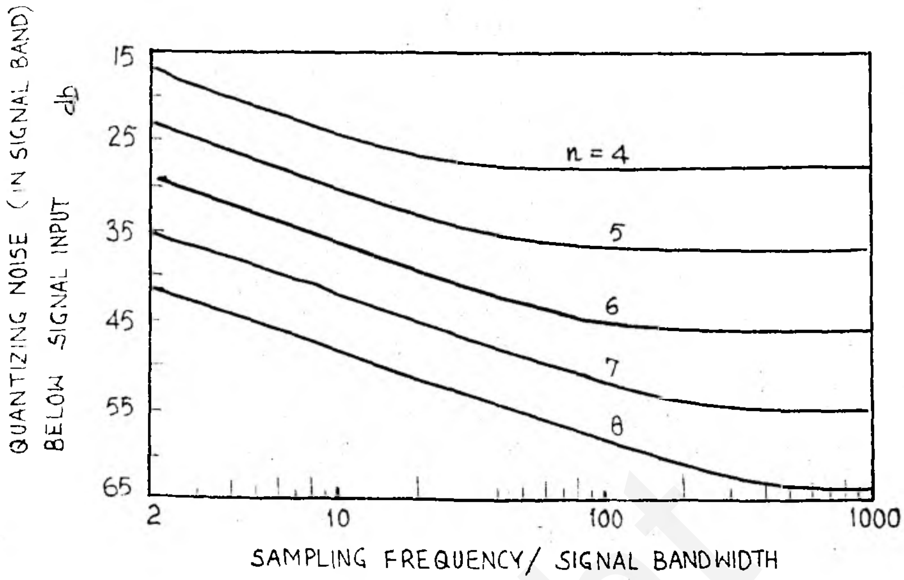
As was seen earlier, quantization and transmission does not simplify the problem, because physically it is impossible to recognise the different quantum levels. Also, the bandwidth required to transmit this signal, such that the discrete quantum steps are preserved, is very large. The quantization in amplitude could be combined with quantization in time. The sampling frequency would be equal to or slightly in excess of $2W_m$ c/s. The sampling could be thought of as a multiplication of the signal by the switching function. The switching function has a finite value during the very short period (approaching zero) of switch closure, and zero value otherwise; the frequency of the switching is the same as the sampling frequency. The switching function can be expanded by Fourier Series to give a constant term, the repetition frequency and the harmonics of the repetition frequency. The effect of the multiplication of the signal with the switching function will therefore, give a constant term proportional to the signal itself, upper and lower sidebands on the repetition frequency, and upper and lower sidebands on the harmonics of the repetition frequency. These may be applied to the staircase transducer now. At the output of the transducer will appear the signal, as well as the error due to quantization. If the sampling frequency is slightly more than the highest signal frequency, the original signal could be recovered from the output by passing it through a low pass filter.

As the error was spread over a much wider band than the original signal, the filter output will not consist of all the error voltages. To calculate the error actually present in the output signal band, the error spectrum can be multiplied by the switching function, and all the contributions to the original signal band made by the harmonics of the switching function summed up. The contributions by the harmonics are due to the beating of each harmonic with the band of error spectrum near it, and this gives a beat frequency in the signal band. Hence, the sampling introduces some more error in the signal band, which can be reduced by increasing the sampling frequency. This price is easily paid because the quantized sample can be directly encoded into a set of binary pulses.

The calculated values of the error power in the signal band are shown in Fig.A.4, after Bennett[17]. The abscissa is the ratio of the sampling frequency to signal bandwidth, and the input signal is band-limited noise. The sloping nature of the curves indicates a reduction of noise with the increase of sampling frequency, because by doing so, the harmonics of the sampling frequency are pushed up into the less dense regions of the error spectrum. By increasing the sampling frequency to a large value, the effect of sampling becomes almost negligible, and the system returns to a case of quantization alone. The flat portion of the curves in Fig.A.4 then represents the quantization noise alone, without the sampling.

A.1.1.QUANTIZING NOISE.

The signal to noise ratio for full load sinusoidal signal



ERROR POWER IN THE SIGNAL BAND

FIG. NO. A' 4

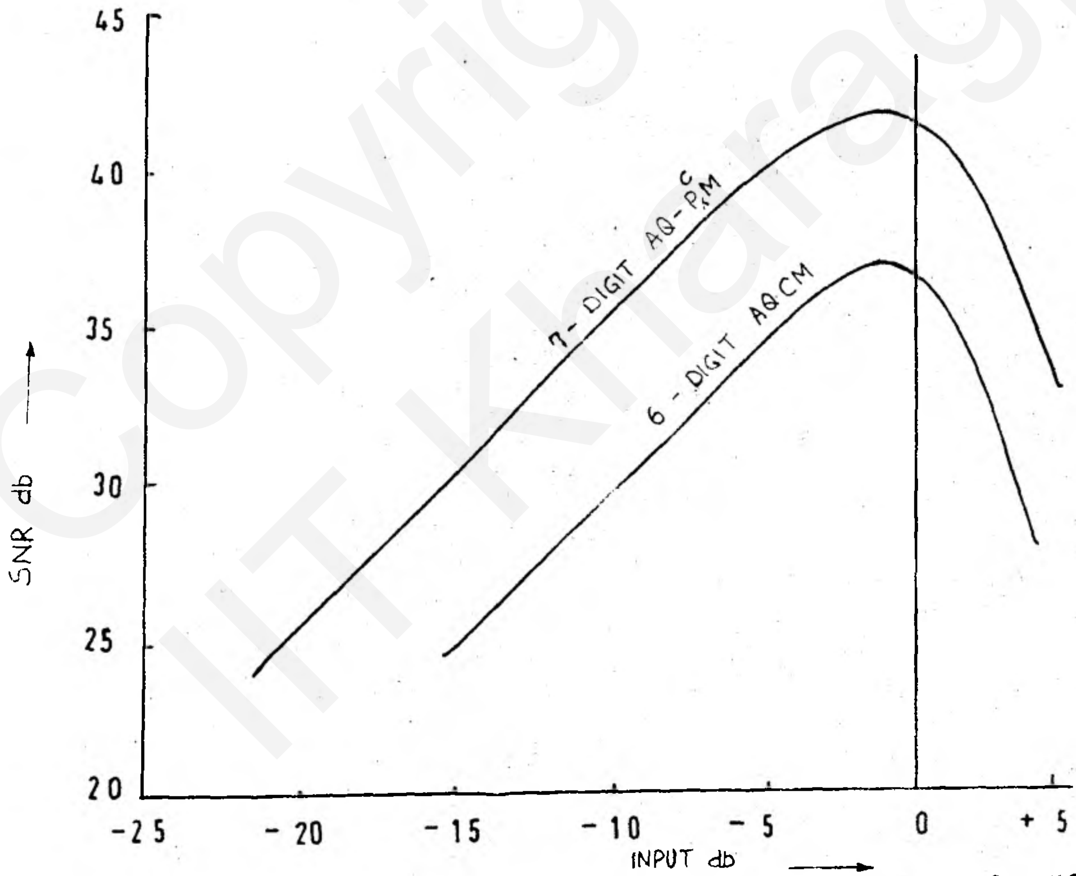


FIG. NO. A'5

VARIATION OF SNR WITH INPUT LEVEL

in an AQ-PCM system can be calculated with the help of equation (A.2). If the full load sinusoidal signal is E volts peak, then its mean square values is $E^2/2$, and the total range of the quantizer is -E to +E, that is 2E. The height of one step has been assumed to be E_0 and therefore, the total number of steps = $2E/E_0 = r$. The quantizing noise, which can be assumed to be something like an independent source of noise, is given by eq.(A.2) as, $E_0^2/12$. Hence, the ratio of mean square signal to mean square noise is

$$\frac{E^2/2}{E_0^2/12} = \frac{6E^2}{4E^2/r^2} = \frac{3r^2}{2} \qquad \text{... (A.3).}$$

Because the signal band is slightly less than half the sampling frequency, the equivalent rectangular band in which the noise acts is about 3/4 of the signal band, and is therefore, less than that given in equation (A.3), by the same factor. The signal to quantizing noise ratio is,

$$S/N = 10 \log_{10} \frac{3r^2}{2 \times 3/4}$$

$$= 20 \log_{10} r + 3 \text{db} \qquad \text{... (A.4).}$$

As the value of r is equal to 2^n, the S/N is given as

$$S/N = 20 \log_{10} 2^n + 3 \text{ db}$$

$$\text{or} \quad S/N = 6n + 3 \text{ db} \qquad \text{... (A.5).}$$

The SNR for various number of binary digits is shown below:

n	SNR
3	= 21 db
4	= 27 db
5	= 33 db
6	= 39 db
7	= 45 db.

The SNR increases by 6 db for every increase in the number of digits.

In general, the signal is speech, or some other complex signal. The estimate of the SNR, which would be obtained for these signals, can be made from the results of a band limited noise signal. It has been shown that the SNR results for such a noise signal is about 10 - 11 db below the full load sinusoidal signal. Since the distribution of the amplitudes in a noise signal is assumed to be Gaussian, the probability of the amplitude exceeding 4 times the mean square value is extremely small. The input level of the noise signal is kept about 12 db below the full load sinusoidal signal.

The variation of input level below that for the full load sinusoid produces a proportionate decrease in the SNR. As the quantizing noise is substantially uncorrelated and uniform over the signal band, it remains constant. Fig.A.5 shows the variation of SNR with input level for a 6 digit and a 7 digit AQ-PCM systems. The SNR for noise type signal can be read from Fig.A.4. For a ratio of sampling frequency to signal bandwidth of 2, it is seen to be 35 db and 29 db for a 7-digit and a 6-digit system, respectively. The frequency response, that is, variation output with frequency is constant in the message band. The effect of the quantizing noise on speech will be similar to that of a thermal noise with the same mean power, and a noise level of approximately 25 db below the signal is acceptable and quite comfortable for listening.

A.1.2. NON-LINEAR QUANTIZATION.

It is seen that the signal to noise ratio falls linearly with a decrease in the input, and the range of input, for which the SNR (in a 7-digit AQ-PCM system) is above 25 db, is only about 20 db (Fig.A.5), below the full load sinusoid. This will mean that signals with low amplitudes will be reproduced with a very poor SNR. This can be looked at from another point of view. Assuming that, to the staircase transducer with uniform steps a large and a small amplitude signal is applied. The transducer will produce an equal quantizing error, but the error will be a small percentage of the large amplitude signal, whereas, the error will be comparable to, or form a large percentage, of the low amplitude signal. Thus the low amplitude signals will be adversely effected by the quantizing noise. It may then be advantageous to use a quantizer with non-uniform steps, tapered signals, or some other device, so that the size of the quantum steps is small for weaker signals.

Of course, increasing the total number of steps will reduce the quantizing error, but it will require more coder decisions in less time and an increased bandwidth for transmission in the line. Another alternative is to taper the size of the quantum steps over the signal range in such a way that weak signals traverse more number of steps. This poses a serious problem in the coding accuracy of the smallest steps. A third method is to taper the signal in the manner shown in Fig.A.6, where the weaker signals are spread over a considerable number of quantum steps. This technique is certainly feasible and almost invariably used in practical circuits. The signal is first compressed, which allows for a large amplification of lower amplitude signals and less amplification or saturation of higher amplitudes, and then encoded with circuits of moderate speed and accuracy. At the receiver, accordingly, if a linear decoder is used, it must be followed by an expandor which puts back the signal amplitudes in their correct perspective. It is important that transmission between the compressor and expandor be linear. Since the compressor has to act on short duration pulses, an instantaneous compressor and an instantaneous expandor is used.

For speech signals, a modified logarithmic characteristic is used for the compandor, and thereby, the quantizing noise for weak signals is reduced to an acceptable level (accompanied by an acceptable deterioration for strong signals). Fig.A.7 illustrates one such desirable companding characteristics. In the case of the compressor, x represents the input and y the output, while for an expandor, y represents the input and x the output, and the relation between the input and output is

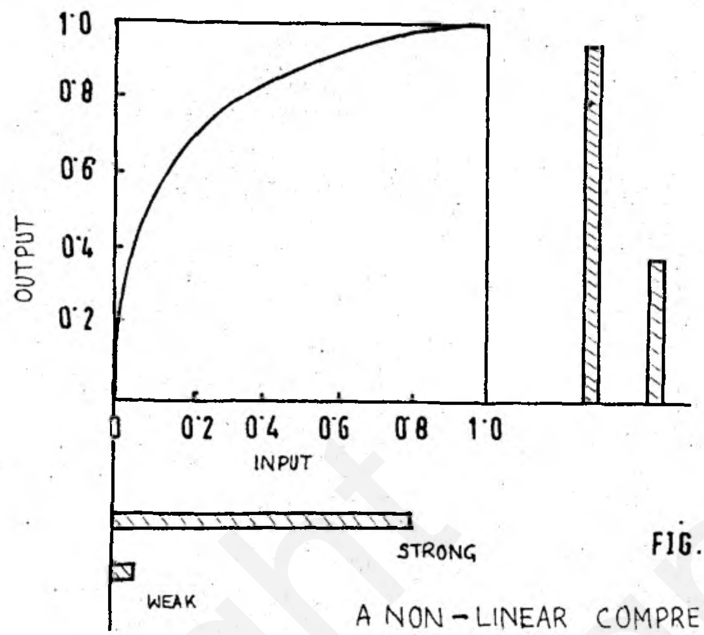


FIG. NO. A· 6

A NON-LINEAR COMPRESSOR

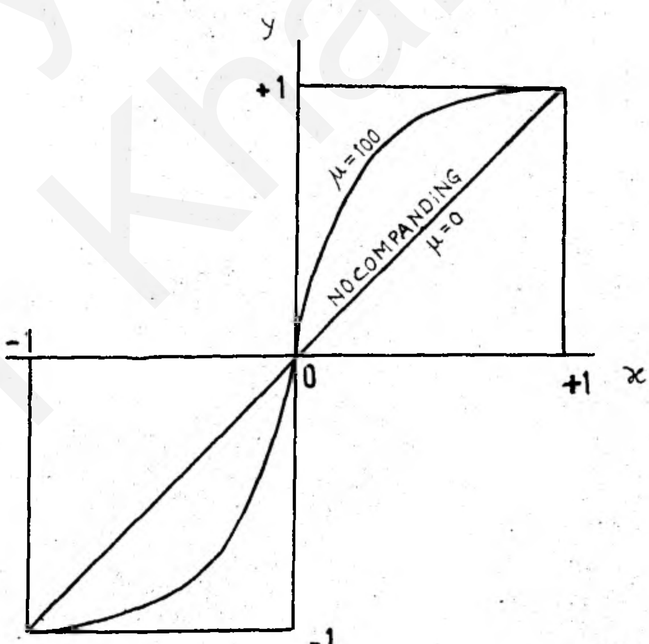


FIG. NO. A·7

DESIREABLE COMPANDING CHARACTERSTICS

$$y = \left[\frac{1}{\log_e (1+\mu)} \right] \cdot \log_e (1+\mu x)$$

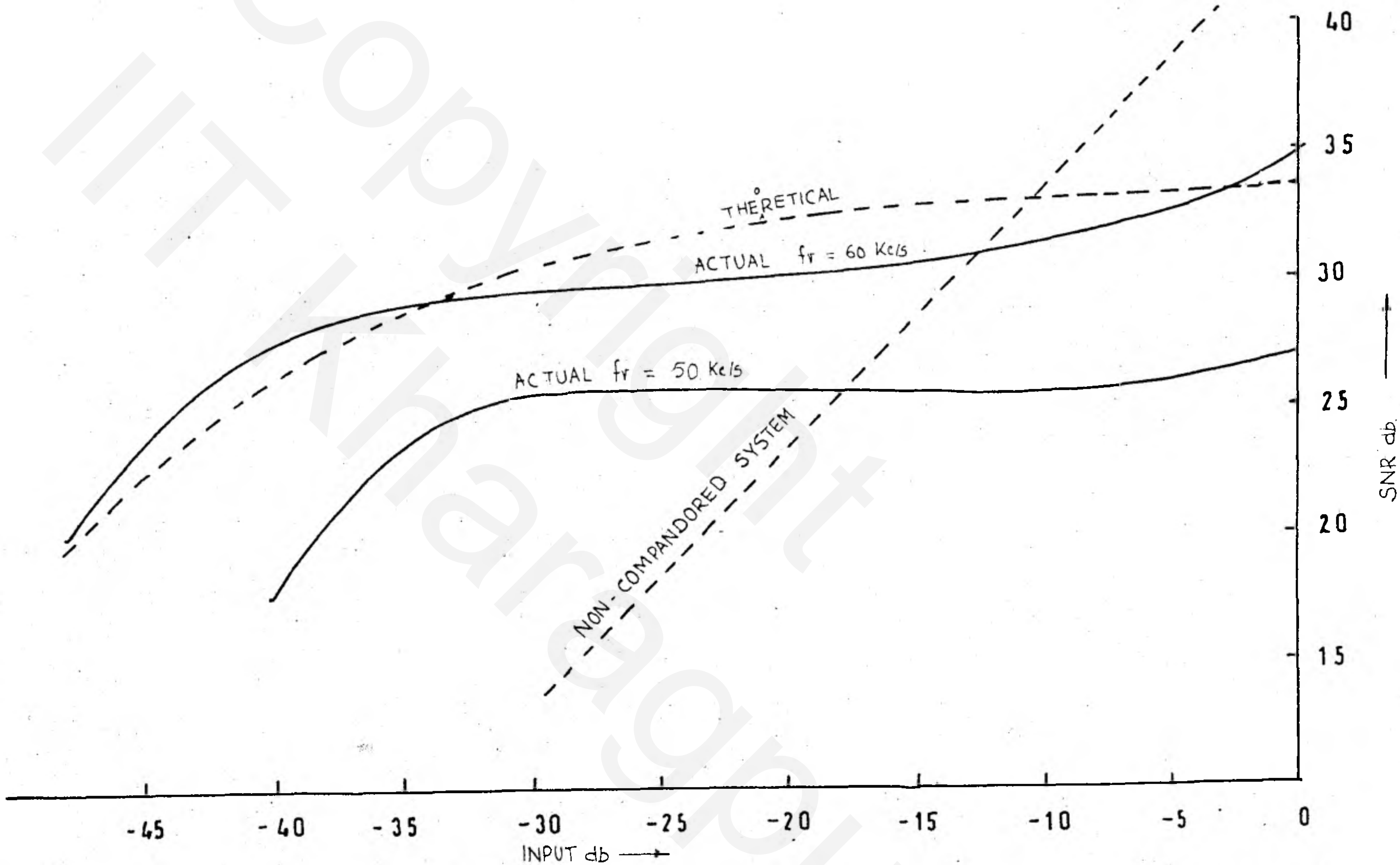
The μ is a parameter which determines the degree of compression (or expansion) and therefore, the companding improvement for weak signals. The companding improvement is the change in signal to noise power ratio relative to the non-companded quantizer. Mann et al[36] have discussed the choice of the optimum value of μ and have concluded that, a μ of 100, which gives a companding improvement of 26 db, is most satisfactory. Fig.A.8 shows the quantizing noise performance of the theoretical logarithmic companding characteristics for μ = 100, and 7-digit encoding. The figure also shows the practical results obtained for a 24-channel, 7-digit PCM link given by Gray and Shennum.[37] and a 6-digit. It is seen that the SNR is almost constant at approximately 30 db for an input voltage range of about 40 db and the total dynamic range is 44 db.

A.2. TRANSMISSION CHARACTERISTICS.

It can be easily shown that a channel of bandwidth W c/s can transmit 2W independent pulses per second. The pulses occur (or do not occur) at $t = 0, \tau, 2\tau \ldots m\tau$, where $\tau = 1/2W$ seconds and the received pulse is of the form

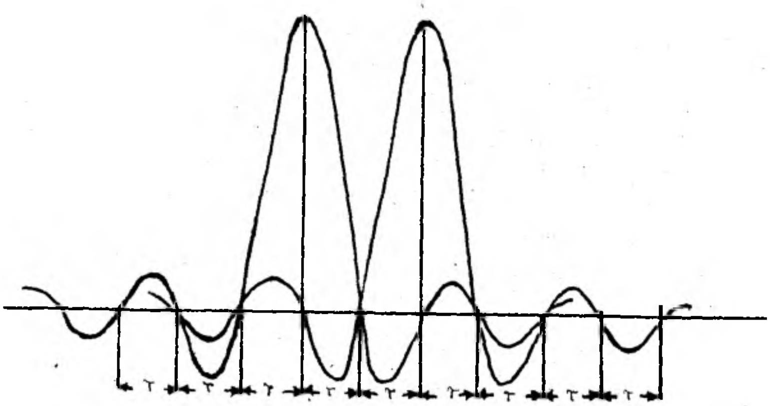
$$V = V_0 \frac{\sin \frac{\pi}{\tau}(t - m\tau)}{\frac{\pi}{\tau}(t - m\tau)} \qquad \ldots (A.6).$$

and is shown in Fig.A.9.



VARIATION OF SNR WITH INPUT LEVEL FOR THE COMPANDERED SYSTEM

FIG. NO. A'8



Sin x/x TYPE OF PULSES AT THE RECEIVER FIG. NO. A·9

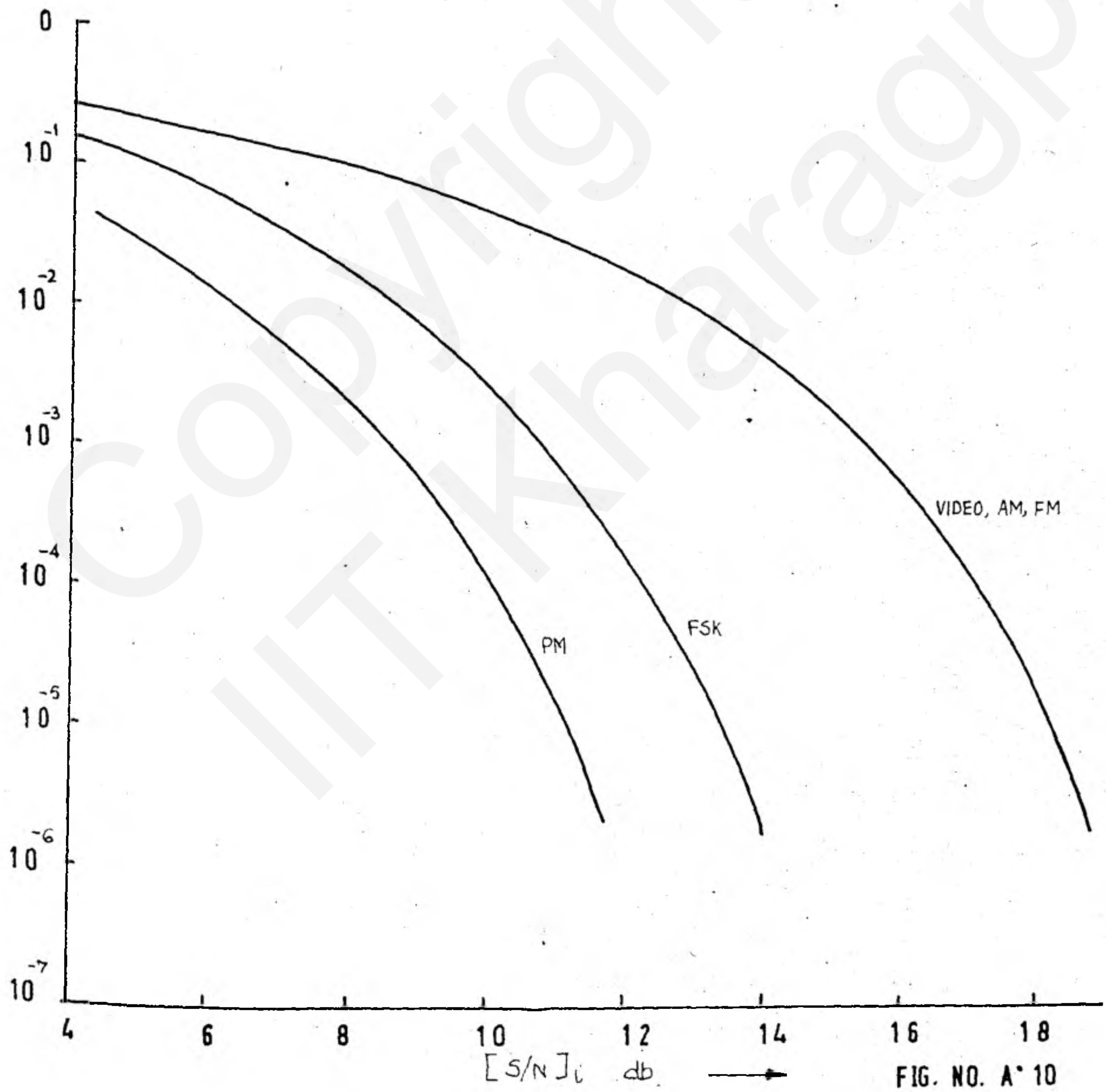


FIG. NO. A·10

A PLOT OF Pe WITH INPUT SNR FOR
SCHEMES

It is seen that the amplitude τ of the Sin x/x pulse centred at $m\tau$ is zero at $t = K\tau$ where $K \neq m$. Since the transmission is synchronized, the pulse train is sampled at $\tau = m\tau$. At the sampling instant, only the pulse centred at that point will be seen and none of the others. The Sin x/x pulse given by eq.A.6 is limited to a band of W c/s, because it is the output obtained after applying a very short pulse to an ideal low pass filter of W c/s bandwidth. In AQ-PCM, a message signal of bandwidth W_m c/s is sampled at the rate of $2W_m$ c/s and each sample is coded into a group of 'n' pulses. Therefore, $2n\ W_m$ number of pulses per second are required to be transmitted and the bandwidth of the video channel will be nW_m c/s. If the $2nW_m$ on-off pulses are transmitted by modulating the amplitude of a carrier, the bandwidth required will be $2nW_m$ c/s. In the case of transmitting the pulses through FM or FSK, the bandwidth required will be larger (as shown in Section 5.3) and will be $4nW_m$ c/s for a unity deviation - ratio FM, and for a two tone FSK. For a AQ-PCM-PM transmission, similarly the bandwidth required is equal to that in the double sideband AQ-PCM-AM case.

Considering the video transmission of the 1/0 pulses, the presence, or absence, of a pulse at the receiver can be reliably detected only if the signal is larger than the noise by a certain amount. The noise is assumed to have a uniform power spectrum and a Gaussian distribution of the amplitudes. To achieve ideal detection, the signal is passed through an ideal low pass filter of bandwidth W (= nW_m), and sampled at times $K\tau$. Suppose the height of the pulse is 2V when it is correctly received. The sampled signal is passed through a slicer which gives an output pulse if the signal is greater than V, and no pulse if it is less than V. If a positive noise burst occurs at the particular instant when a pulse was supposed to be present, there will be no error, but if it occurs at an instant when no pulse is sent (with a magnitude greater than V) there will be an error. Similarly, a negative noise burst of magnitude greater than V will obliterate the presence of a pulse, but will have no effect if a 'zero' has been sent. The probability of error will therefore depend upon the input SNR, and as has been shown in Section 5.1, the P_e is given as

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{1}{2}\sqrt{S_i/N_i}} e^{-x^2/2}\, dx$$

The curve for P_e, for different values of input SNR, is shown in Fig.A.10. The probability of error in a "word" of n pulses is

$$\left[1 - (1 - P_e)^n\right]$$

Now in AQ-PCM, the occurrence of a noise pulse in the n different positions of the "word" pulses produces different amounts of change in the final detected output of the signal. For instance, an error pulse at the first position of the 7-digit "word" will produce an error of about 50%, but an error pulse at the last digit place will have less than 1% error finally. The mean square change is

$$\sum_{i=1}^{n} \frac{2^{2i-2}}{n} = \frac{4^n - 1}{3n}$$

Therefore, the mean noise power in the output is N_o, and is given by,

$$N_o = \frac{A^2}{(2^n-1)^2}\left[1-(1-P_e)^n\right] . \frac{4^n-1}{3n}$$

where A^2 is the signal level. The mean-signal power is

$$A^2/8 = (A/2\sqrt{2})^2$$

and hence, the $(SNR)_o$ is,

$$\left[S_o/N_o\right] = \frac{(2^n-1)^2}{8} \cdot \frac{3n}{4^n-1} \cdot \frac{1}{\left[1-(1-P_e)^n\right]} \quad \text{... (A.7).}$$

If P_e is very small and n is more than 4, the equation (A.7) can be reduced to

$$\left[S_o/N_o\right] = \frac{3}{8} \times \frac{1}{P_e} \quad \text{... (A.8).}$$

The output SNR depends upon P_e which in turn, is based on the input SNR. A curve of input SNR vs. output SNR is drawn in Fig.A.11. It is seen that if the input SNR is large enough to make the signal intelligible, a small increase will make the transmission perfect. An idea of the rapid improvement in the output can be had from Table I, given below, for a pulse rate of 100 Kc/s.

TABLE I.

Input (SNR) peak.	Probability of Error.	One error every.
13.3 db.	10^{-2}	10^{-3} sec.
17.4 db.	10^{-4}	10^{-1} sec.
19.6 db.	10^{-6}	10 sec.
21.0 db.	10^{-8}	20 min.
22.0 db.	10^{-10}	1 day.
23.0 db.	10^{-12}	3 months.

Evidently, there is a threshold at about 20 db, below which there is a serious interference, and above which the interference is negligible.

The above derivation has been based on the peak power limitation in the transmission link. The average power is 3 db below the peak power because, on an average, the pulses are present half the time. On the other hand, the transmission could be bipolar, where a 'one' pulse is sent as +V and a 'zero' pulse is sent as -V. The peak to peak signal swing is the same 2V as in the case of 1/0 transmission for the same noise margin. The peak power is now 1/4th the peak power of 1/0 pulses, and since a pulse, either positive or negative, is always present, the average power is the same as the peak power. Therefore, for a bipolar or balanced transmission, the power is reduced by 6 db for the same error rate P_e at the output.

A.2.1. RF MODULATION.

In the AQ-PCM-AM system, the error probability is calculated in the fashion given in Section 5.2, and the P_e is given by,

$$[P_e]_{AM} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{1}{2}\sqrt{S_i/N_i}} . \; e^{-x^2/2} . dx$$

A plot of P_e vs. S_i/N_i is given in Fig.A.10. The mean noise power is then obtained from equation A.7 and the (SNR) output is

$$[S_o/N_o]_{AM} = \frac{3}{8.[P_e]_{AM}} \qquad \text{... (A.9).}$$

Fig.A.11 shows the input SNR (Average basis) vs. the output SNR. The unity deviation-ratio AQ-PCM-FM has an improvement of about 4.8 db, but in all other respects, it is similar to AM. The input SNR (average) vs. output SNR in this case is also shown in Fig.A.11, where to every value of S_o/N_o, this improvement of 4.8 db has been added.

The error probability in the AQ-PCM-FSK is found in a manner similar to the one given in eq.5.24 and P_e is

$$[P_e]_{FSK} = \frac{1}{2} e^{-S/2N}$$

The variation of P_e with S/N is plotted in Fig.A.10. The output SNR from eq.(A.8) is

$$[S_o/N_o]_{FSK} = \frac{3}{8} \cdot \frac{1}{[P_e]_{FSK}} \qquad \text{... (A.10).}$$

and is shown in Fig.A.11.

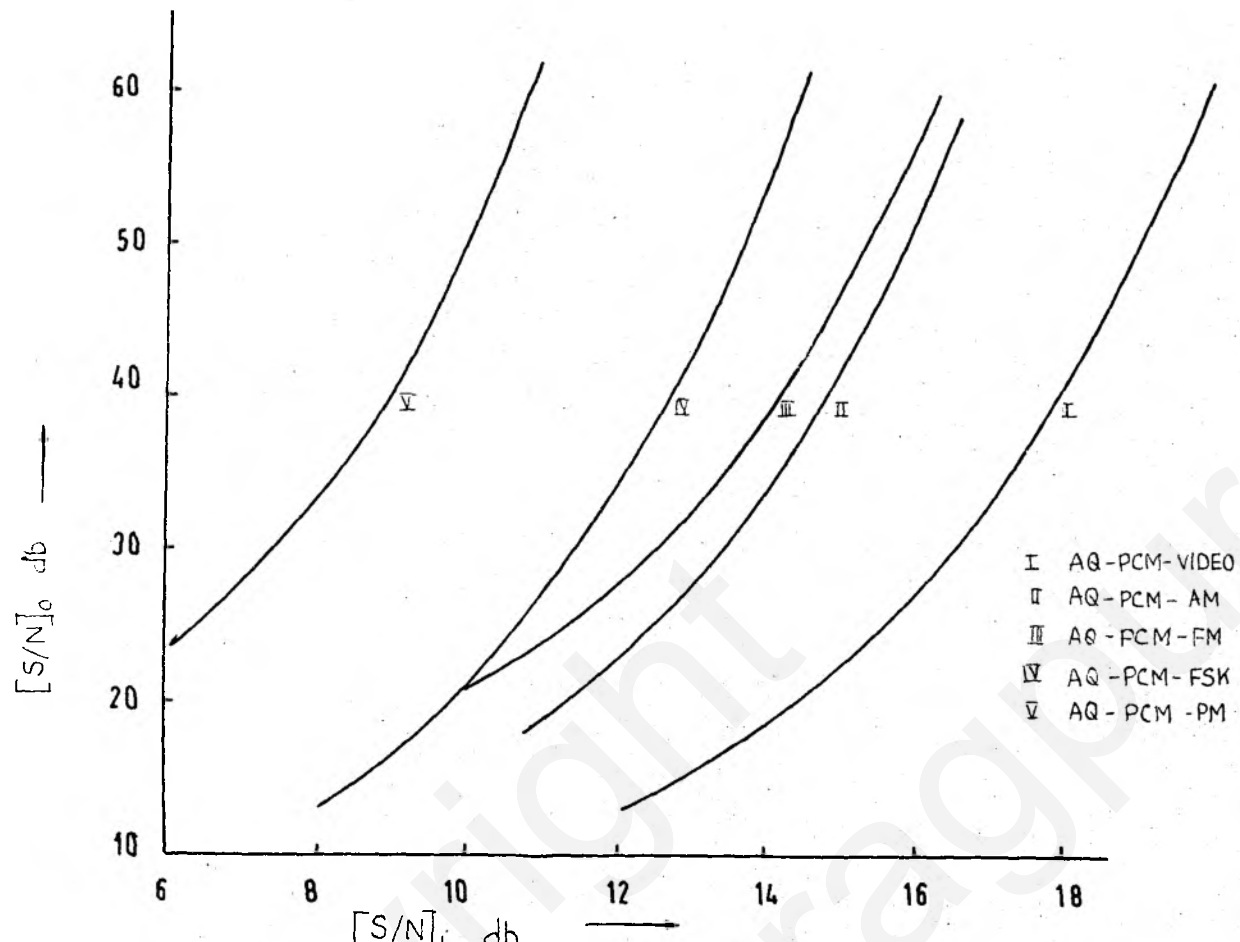


FIG. NO. A· 11

VARIATION OF OUTPUT SNR WITH INPUT SNR FOR DIFFERENT MODULATION SCHEMES

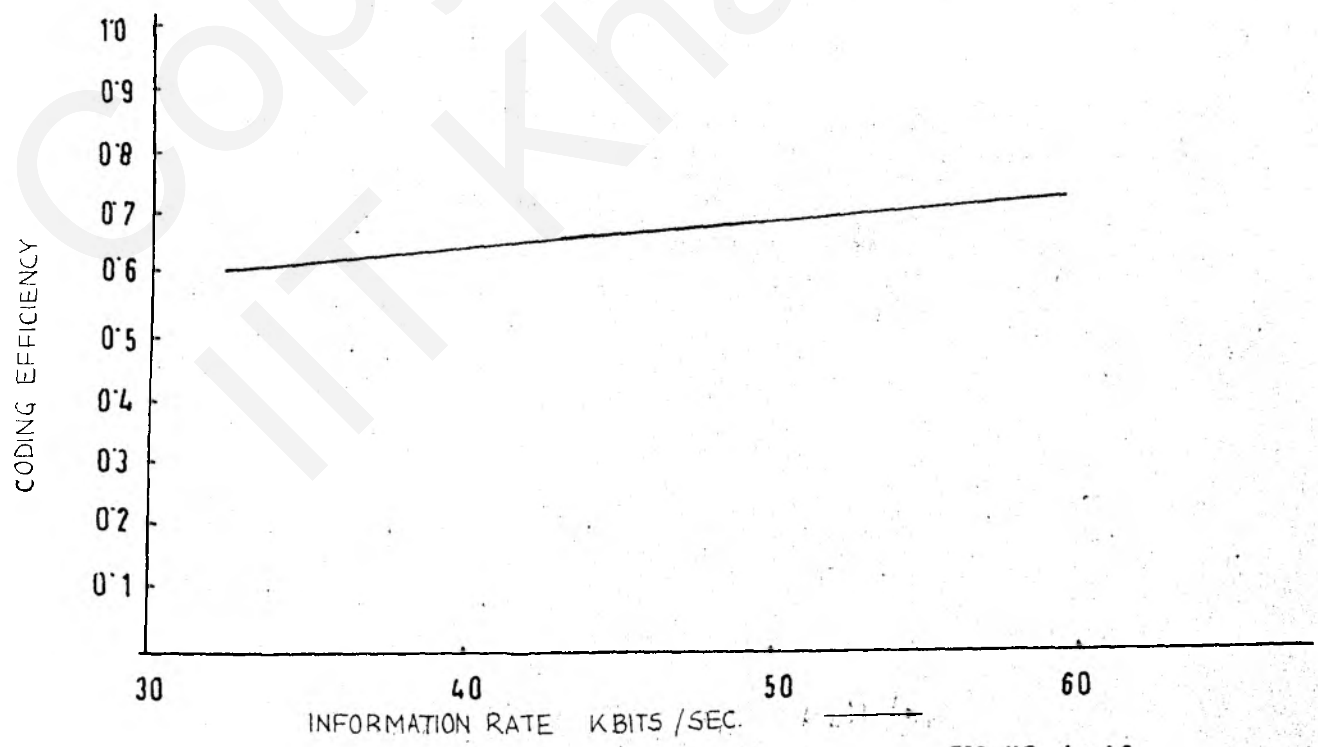


FIG. NO. A· 12

CODING EFFICIENCY VS INFORMATION IN AQ-PCM SYSTEM

For the AQ-PCM-PM, the probability of error (refer eq.5.29) is

$$P_e = 1 - \int_{-\pi/2}^{+\pi/2} P(\theta)\, d(\theta)$$

where

$$P_\theta = \frac{1}{2\pi} e^{-S/N} \left[1 + \sqrt{\frac{4\pi S}{N}}\; e^{\left(\frac{S}{N}\right)\cos^2\theta} \cdot \cos\theta \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{\frac{2S}{N}}} e^{-y^2/2}\, dy \right]$$

The variation of P_e with input SNR is plotted in Fig.A.10. The output SNR is given by

$$[S_o/N_o]_{PM} = \frac{3}{8} \cdot \frac{1}{[P_e]_{PM}} \qquad \ldots (A.11).$$

and is plotted in Fig.A.11. A comparison of all these RF modulation schemes shows that AQ-PCM-PM will require the least transmitter power for a given error rate at the output.

In most transmission systems the noise and interference are cumulative. Normally, the overall transmission takes place through several repeater stations, and at each repeater, when the signal is amplified the noise also gets amplified by the same amount, and there is no way of separating the noise from the signal. If there are 100 repeaters, the quality of transmission has to be 100 times better than for a single link transmission. In AQ-PCM system, however, the signal can be regenerated as often as necessary without impairing the quality provided, it is ensured that the noise and interference are below the threshold in each link. Actually, if the noise in the single link causes a certain fraction p of the pulses to be regenerated incorrectly, then after m links, if $p \ll 1$, the fraction incorrect will be approximately mp. However, to reduce the error p to a value p' = p/m requires only a slight increase of power in each link, as has been seen in connection with the probability of error vs. input SNR. Thus, it can be said that in AQ-PCM, the transmission requirements are almost independent of the total length of the transmission circuit.

A.3. EFFICIENCY.

Relative evaluation of the different systems has been done in Section 6.3 on the basis of coding, power, and communication efficiencies defined earlier in Section 5.4. The three efficiencies for the AQ-PCM system are given below.

Coding Efficiency.

The coding efficiency has been defined in Section 5.4.1 as

$$\zeta = W_m/H \cdot \log_2 (1 + S/N)$$

Fig.A.12 shows the ζ with the information rate H for the AQ-PCM system. The coding efficiency increases with the information rate, indicating increasingly better coding with the increase in number of quantizing levels.

Power Efficiency.

Power efficiency has been defined in Section 5.4.2, as

$$\beta = \frac{P_{min}}{\epsilon^2 H} = \frac{S_i}{N_i} \cdot \alpha$$

β for a 6-digit and a 7-digit AQ-PCM has been calculated and given in Table II. The information H, and the bandwidth B, have also been indicated in the same table. The values of S_i/N_i have been taken from Fig.A.11.

TABLE II.

Mode of transmission.	AQ-PCM 7-digit; SNR 45 db PRF 60 Kc/s.			AQ-PCM 6-digit; SNR = 40 db PRF 50 Kc/s.		
	H Kbits/sec	B Kc/s	β	H Kbits/sec	B Kc/s	β
VIDEO.	60	30	17.34	50	25	14.92
AM.	50	60	34.67	"	50	29.85
FM.	60	120	60.4	"	100	50.24
FSK.	50	120	41.30	"	100	35.98
PM.	60	60	8.9	"	50	7.58

Communication Efficiency.

The communication efficiency has been defined in Section 5.4.3. as,

$$\eta = \frac{\text{Received power in the ideal system.}}{\text{Received power required in the actual system for the same information rate/cycle.}}$$

$$= \frac{(S_i/N_i)_{ideal}}{(S_i/N_i)_{actual}}, \quad \text{for the same information rate.}$$

The method of calculation of η has also been indicated there. The communication efficiency for a 6 and 7-digit AQ-PCM-modulated systems is given in Fig.A.13(a) and (b), respectively. The output SNR obtained is indicated on the curves. It is seen that η of

COMMUNICATION EFFICIENCY WITH INPUT SNR DIFFERENT MODULATION SCHEMES FOR 7-DIGIT AQ-PCM SYSTEM

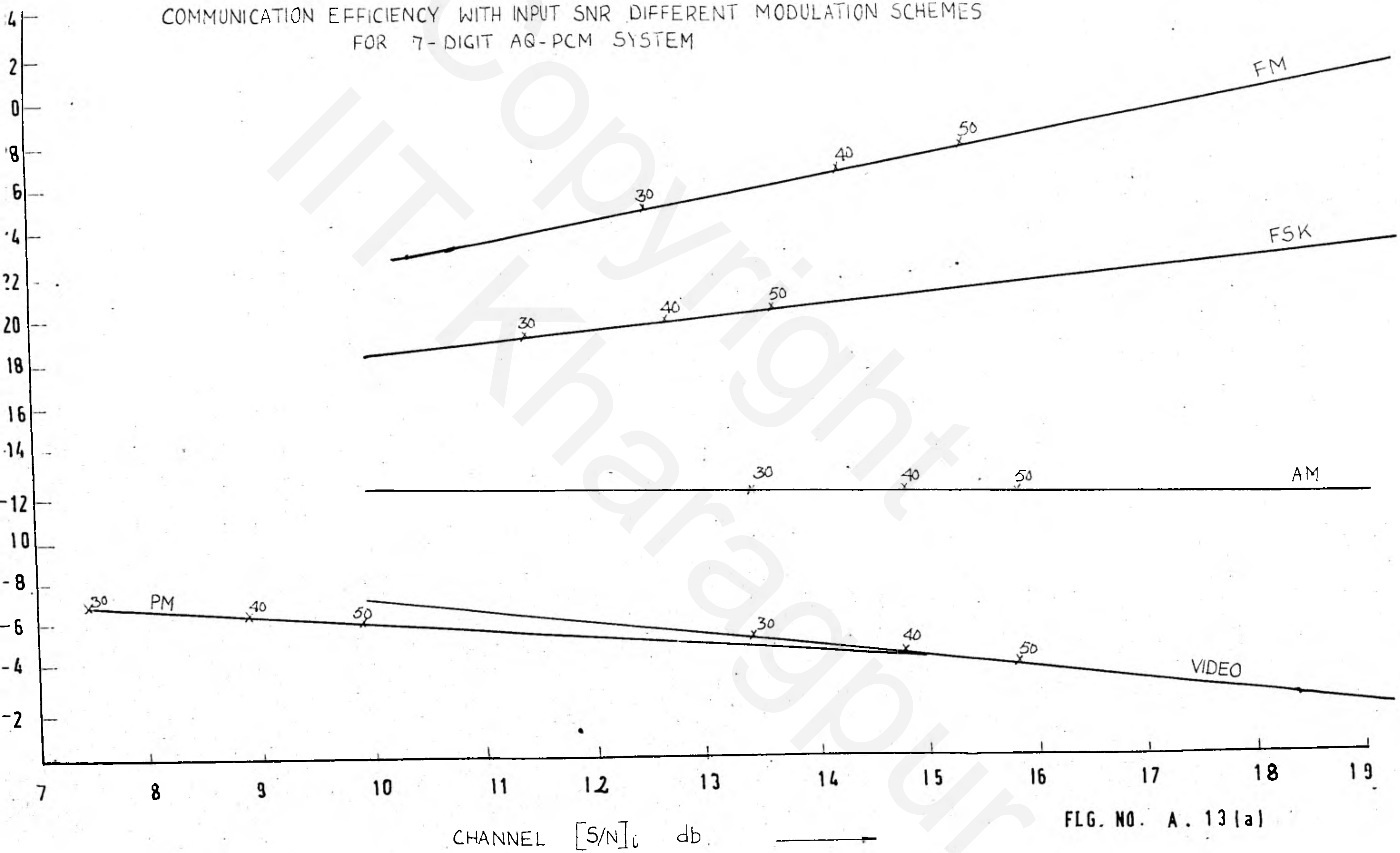


FIG. NO. A. 13 (a)

COMMUNICATION EFFICIENCY WITH INPUT SNR FOR DIFFERENT MODULATION SCHEMES FOR THE 6-DIGIT AQ-PCM SYSTEM

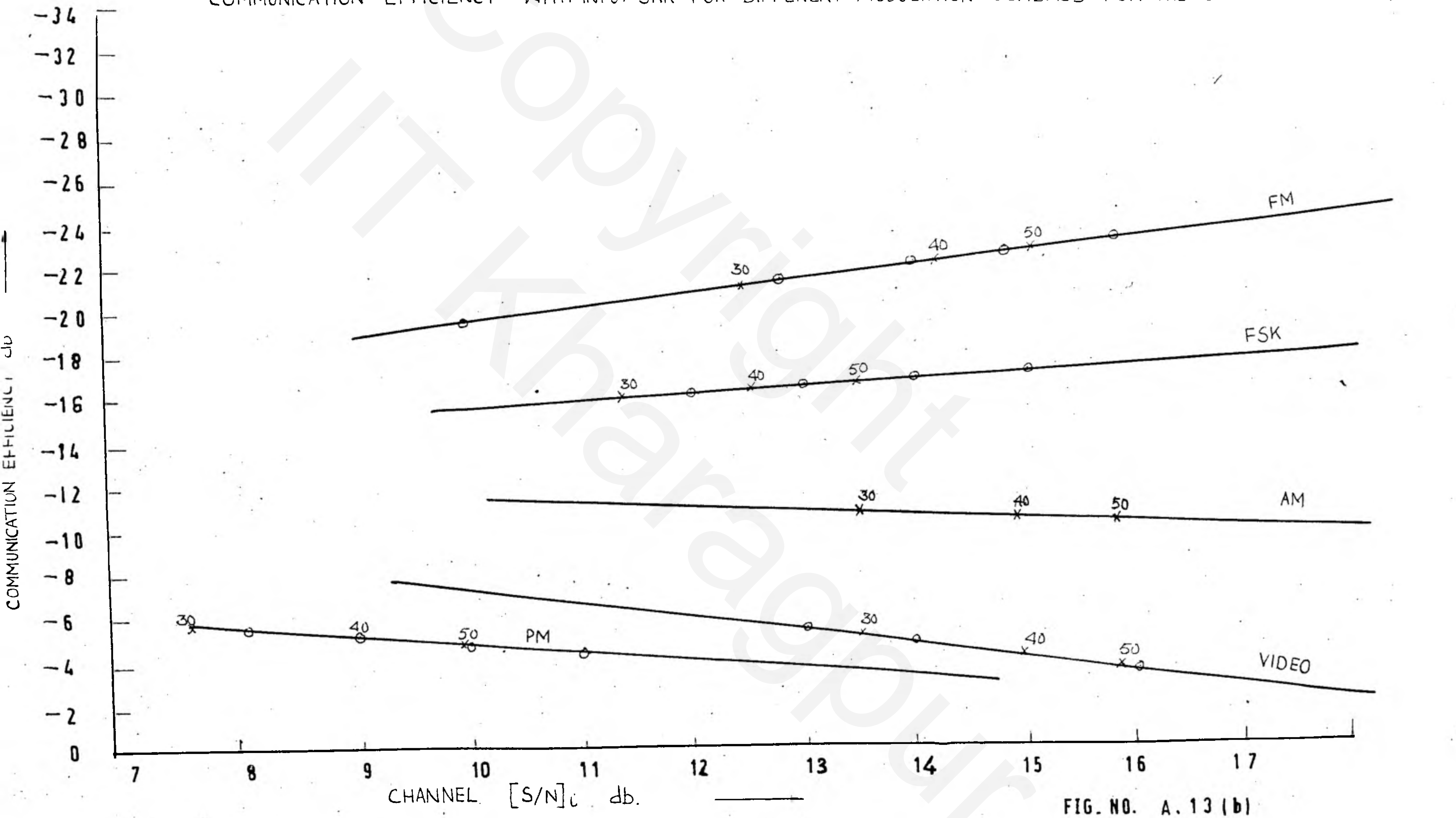


FIG. NO. A.13 (b)

the AQ-PCM-PM is the highest and of the AQ-PCM-FM is the lowest.

A.4. CHANNEL CAPACITY.

The information capacity of a system is defined as the number of independent symbols or characters which can be transmitted by it in unit time. The information capacity is normally expressed in terms of number of binary digits per second, C, which can be handled by the channel. The units of binary digits are used because they are simplest in character and all systems are expressed in these units. Shannon and others have shown that the capacity of an ideal system is

$$C = W \log_2 (1 + P/N) \qquad \ldots \text{(A.12)}.$$

where W = bandwidth,

and P/N = the ratio of average signal power to mean noise power.

Actually all the systems have a much smaller channel capacity than the one given by eq.(A.12). To calculate the channel capacity of the AQ-PCM system one can proceed as follows. Assuming that the system is working above the threshold so that the errors are negligible,

$$C = 2\,W_m\, m.$$

where $2\,W_m$ = sampling frequency.

m = equivalent number of binary digits/code group.

Now, there are ℓ quantizing levels and the number of binary digits per code group is $\ell = 2^n$, while the actual number of digits n to the base 'b' will be $l \cdot b^n$. Therefore,

$$2^m = b^n$$

$$\text{or} \quad m = n \log_2 b$$

$$\text{and} \quad C = 2 W_m \cdot n \log_2 b.$$

$$= 2 W \log_2 b \qquad (\text{As } 2W_m \cdot n = 2 W).$$

$$= W \log_2 b^2 \qquad \ldots (A.13).$$

If pulses have 'b' different amplitude levels and σ is the rms value of noise, then an amplitude separation of $K\sigma$ must exist between the levels to provide enough safeguard against noise, where K is a const. At the threshold in AQ-PCM system the value of K = 10 approximately. The total amplitude range is $K \cdot \sigma \cdot (b - 1)$, and the average signal power S, assuming all levels to be equally likely, is

$$S = K^2 N \cdot (b^2 - 1)/12 \qquad \ldots (A.14).$$

Substituting the value of b^2 from eq. (A.14) in eq.(A.13).

$$C = W \log_2 (1 + 12S/K^2 N) \qquad \ldots (A.15).$$

Eq.(A.15) gives the channel capacity of the actual AQ-PCM system and this will be equal to the ideal channel capacity in eq.(A.12) if the signal power is increased so that

$$S = (K^2/12) \cdot P$$

This is equivalent to an increase in power of approximately 9 db, required in AQ-PCM, to get a given channel capacity for a given bandwidth. Equation (A.15) shows that the power and bandwidth are exchanged on a logarithmic basis and the channel capacity is proportional to the bandwidth.

A.5. T.D. MULTIPLEXING IN AQ-PCM.

One of the greatest advantages of the AQ-PCM system is that it lends itself to a time division multiplex. Many message channels can be multiplexed and a standard quality and high reliability can be obtained[38]. The instantaneous compressor and coder is common to all channels in the transmitter, and similarly the instantaneous expandor and decoder are common equipment in the receiver.

APPENDIX B.

DELTA-MODULATION AND DELTA-SIGMA MODULATION.

B.0. GENERAL.

Among the existing coded pulse modulation system, the AQ-PCM system discussed earlier in Appendix A, gives the best performance with moderately low PRFs. The complexity of the AQ-PCM circuits has led other workers in the field to try some other methods of approximation. Noticeable among these have been the Δ-M and Δ-ΣM systems.

In the Δ-M system developed by De Jager[4], a step approximation of the message signal waveform, by +1/-1 pulses, is obtained by a simple circuit. The Δ-M system has been found suitable for some special class of signals like speech. As a differentiation of the message waveform is involved in the coding process of the Δ-M system, it is incapable of transmitting d.c. and very low frequency signals. The Δ-ΣM system has been proposed by Inose et al[5], as a modification of the Δ-M system. It overcomes the shortcomings of the Δ-M system in many respects, while still retaining the advantage of simple circuitry. The quantizing noise, however, is more in the Δ-ΣM, and a larger PRF is required for a similar SNR at the receiver output. The advantage of d.c. transmission, and of uniform frequency response of the message signals, are decidedly in favour of Δ-ΣM.

The present Appendix is intended to give a review of the

Δ-M and Δ-ΣM systems with emphasis on their coding characteristics. The systems are similar in principle to Binary SQ-PCM system, but inferior in coding characteristics (for the same PRF). Their transmission characteristic will also be similar to Binary SQ-PCM system and, therefore, it will not discussed here. Section B.1 reviews the coding characteristics of the Δ-M system in detail and Section B.2 reviews the same for the Δ-ΣM system. In Section B.3, a discussion of the two systems is given.

B.1.0. Δ-M SYSTEM.

A simple block diagram of the Δ-M system is shown in Fig.B.1. The pulse modulator allows a positive, or a negative pulse, from the pulse generator to pass through to the feedback network C, depending upon the positive or negative polarity, respectively, of the error E(t). The feedback network C is normally an integrator and builds up the signal with positive and negative steps depending on the polarity of the pulses at its input and the time constant of the integration. The output B(t) of the feedback network is compared with the original signal f(t) in the difference circuit D to give the error signal E(t). The approximating signal B(t), therefore, has the form of a step curve oscillating around the message signal f(t). The negative feedback from the output to the input and the high PRF used helps in the reduction of the error to a small value. Fig.B.2 shows the approximated curve and the series of positive and negative pulses produced at the output of the Δ-M system. The approximated curve has been drawn for a single integrator having a large time constant in the feedback

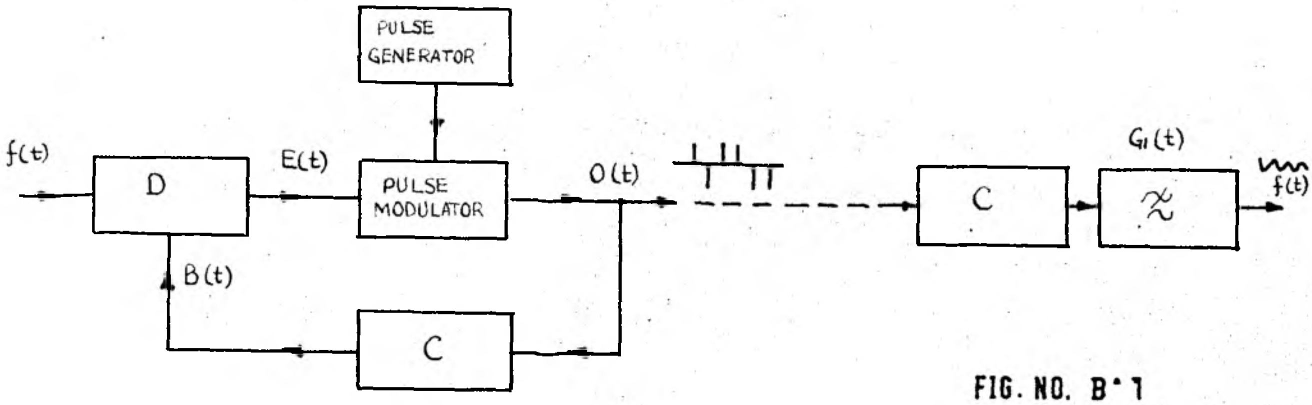


FIG. NO. B.1

A BLOCK DIAGRAM OF Δ-M SYSTEM

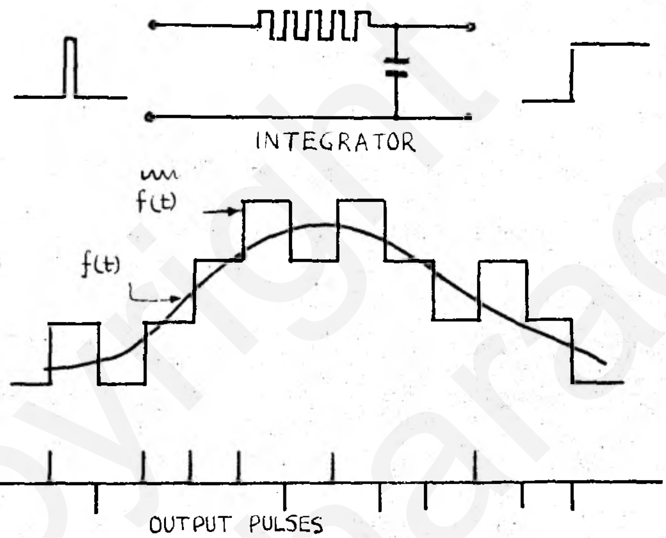


FIG. NO. B.2

AN APPROXIMATED CURVE USING A SINGLE INTEGRATOR

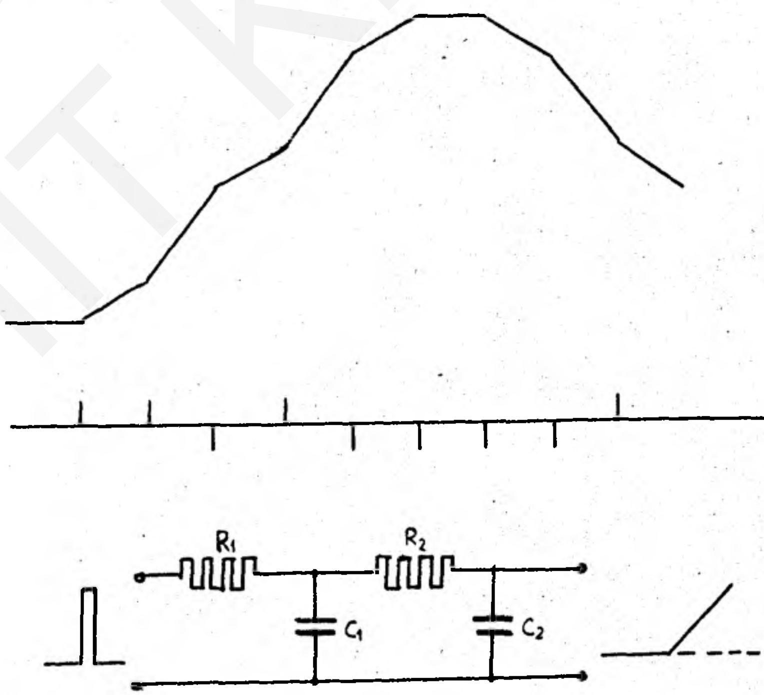


FIG. NO. B.3

loop. By applying the corresponding series of pulses to another integrating network at the receiving end, the same approximation of the original is obtained. The approximated curve can be further smoothed by passing it through a low-pass filter.

As the PRF is limited, there is always some difference between the input signal f(t) and the approximated signal $\tilde{f}(t)$. In processing of speech signals, the difference between f(t) and $\tilde{f}(t)$ (called quantizing noise), is audible as a granular noise. This quantizing noise can, however, be diminished by increasing the PRF, when a better approximation is obtained. The zero-level is transmitted by an alternating pulse series, which when integrated and filtered, will give a zero output.

The information contained in the transmitted pulses is mainly correlated to changes of input signal, and not to its amplitude. An overloading of the system will occur when the slope of the signal exceeds a certain limit. De Jaeger has shown that for a sinusoidal signal input the maximum amplitude that can be transmitted is,

$$A = \frac{f_r}{2\pi f_m} \cdot V_e \qquad \text{... (B.1).}$$

where A = peak amplitude of the sine wave $A \sin W_m t$

f_r = PRF

f_m = $W_m/2$ = signal frequency.

V_e = height of one step in the approximating function.

In other words, the height of one step in the approximating function V_e is

$$V_e \gg \frac{\omega_m A}{f_r}$$

where $\omega_m A$ is the slope of the sinusoidal signal. It is seen that the maximum amplitude and the number of distinguishable levels decreases with increase in f_m.

The negative pulses at the transmitter output may be omitted without decreasing the SNR at the receiving end. In fact, the pulse modulator at the transmitter end need not give positive and negative pulses, but may give only positive and zero pulses, depending upon the polarity of the error signal E(t). Lender et al[39] have shown that with a proper adjustment of the time constant of the feedback network, it is possible to simplify the circuit considerably (using 1/0 code only).

It is also possible to use a modified network in the feedback loop with a view to reduce the quantizing noise and thus, improve the system. If double integration is used in the network C, then, every pulse at the input has an effect of changing the slope of the reconstructed signal, instead of the amplitude, as is the case with the single integrator network. This is similar to the approximation of a waveform by linear segments used in networks. The distant receiver is a replica of the local receiver (network C) and, therefore, the reconstructed signal is related to the received pulse pattern, as shown in Fig.B.3. The signal in this figure is built up from a series of pulses where every pulse changes the slope of the output signal by the same amount in the positive or negative direction. If this approximated signal is now passed through a low-pass filter, a better SNR will be obtained at the output.

In practice, however, the double integration, in the feedback loop of the system above will lead to serious difficulties of instability and self oscillation. As the frequency response of the feedback network will now be falling at the rate of 12 db/octave, the stability conditions will be critical. The oscillations can be avoided if the straight line approximation in the double integration scheme is combined with some sort of prediction. The value which the straight line will have after a time interval τ can always be predicted if no further changes in the derivative occur. Thus, use can be made of an extrapolation of the approximating curve and comparison of these values with the original input signal. Fig.B.4(a) shows one such approximating curve, where it is shown that if the extrapolated value (shown dotted) is lower than the wanted signal, one unit is added to the derivative of the approximating curve. This will tend to reduce the error between the original and the approximated signal. The feedback network is shown in Fig.B.4(b), where the resistance in the second integrator is split into two parts of R_2 - r and r ohms. The output of the double integrator is taken from the junction of these two resistors. The value of r is adjusted such that

$$\tau = r C_2$$

In practice, however, the receiver uses only a single integrator and the break point of the second integrator in the feedback loop is generally placed at a frequency higher than the

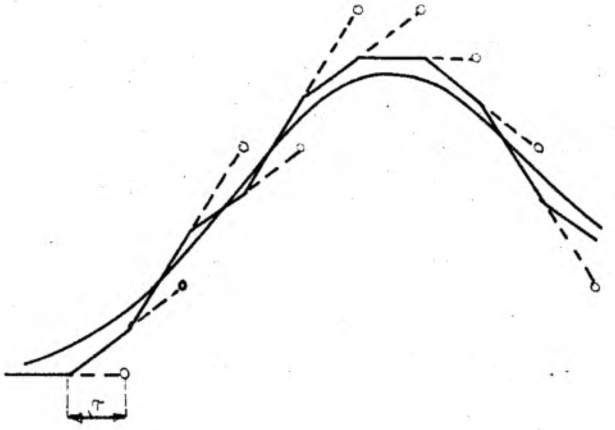


FIG. NO. B·4 (a)

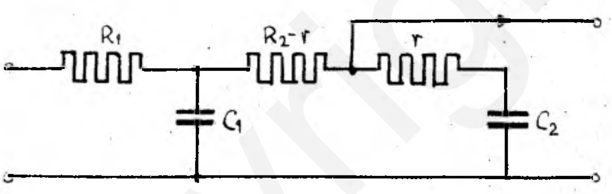


FIG. NO. B·4 (b)

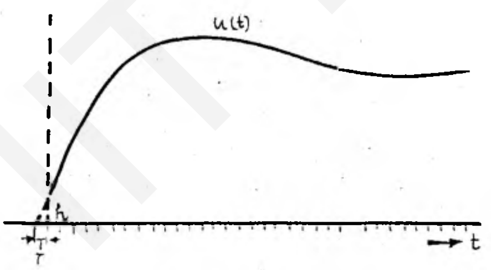


FIG. NO. B·4 (c)

APPROXIMATION USING DOUBLE INTEGRATION WITH PREDICTION,

(a) APPROXIMATING CURVE

(b) FEEDBACK NETWORK, AND

(c) IMPULSE RESPONSE OF THE FEEDBACK NETWORK highest signal frequency. The response of this feedback network to an impulse is shown in Fig.B.4(c). If the value of τ is kept slightly less than T (the time interval between two pulses), the quantizing noise is minimum.

B.1.1. CODING CHARACTERISTICS OF Δ-M.

The SNR(Q) in the Δ-M can be found by estimating the error produced in quantization and the signal built up by the pulses at the receiver. The error signal is assumed to be somewhat random and uncorrelated for time intervals which are large compared to T. The total quantizing error is quite large and spread out over a large band of frequencies. Fortunately, the error falling in the pass band of the low pass filter is within reasonable limits. As the quantizing noise is random, (irregular and nonperiodic) it has a continuous frequency spectrum. The amount of quantizing noise in the pass band will depend on the cut-off frequency, f_o, of the low pass filter. For simplicity of analysis, a sinusoidal signal is assumed as f(t), and the consequent error waveform is assumed as approximate rectangular pulses of random amplitudes. The feedback network is assumed to be a single integrator. To calculate the SNR characteristics of the system (Fig.B.1), in a way similar to that given in Section 2.4, the frequency domain characteristics of quantities like f(t), E(t), O(t), B(t), $G_1(t)$ and $\tilde{f}(t)$ are taken as f(w), E(w), O(w), B(w), $G_1(w)$ and $\tilde{f}(w)$, respectively. The equations governing the system in the frequency domain are,

$$E(w) = f(w) - B(w)$$
$$B(w) = O(w) \times 1/jw.$$
$$N(w) = f(w) - \widetilde{f(w)}$$

where N(w) is the noise power density in the message band. Now,

$$E(w) = f(w) - O(w).\ 1/jw.$$

or $E(w).\ G_1(w) = f(w).\ G_1(w) - O(w).\ 1/jw.\ G_1(w).$ ... (B.2).

If the receiver filter is assumed to have a brickwall characteristics with a cut-off frequency at f_o, then

$$G_1(w) = \begin{cases} 1, & \text{for } |\omega| \leq \omega_o \\ 0, & \text{for } |\omega| > \omega_o \end{cases}$$

and $f(w),\ G_1(w) = f(w)$ in the pass band.

Equation (B.2) is modified as

$$E(w).\ G_1(w) = f(w) - \widetilde{f(w)}$$
$$= N(w).$$

because $O(w).\ 1/jw.\ G_1(w) = \widetilde{f(w)}$ at the receiver.

The modulator has an abrupt nonlinearity, and the assumed error waveform is rectangular in nature. The power spectral density of this random error waveform, varying between the limits $\pm V_e$, can be found by resolving it into a product of two components. One component is a regular rectangular waveform which is multiplied by the other component of random amplitudes (with a mean zero value and peak limits of $\pm V_e$). The amplitude component varies randomly around zero with a probability defined by p(y). The correlation function of this oscillating function is $\overline{v^2}$, where $\overline{v^2}$ is the mean square amplitude of the pulses for the probability distribution assumed. The power density spectrum of the error waveform is, therefore, given as,

$$N_E(\omega) = \frac{\overline{v^2}}{\tau} \cdot \left| E_T(j\omega) \right|^2$$

when $E_T(jw)$ = Fourier transform of the regular rectangular pulse

$\tau = 1/f_r$, where f_r is the PRF.

As,

$$E_T(j\omega) = \tau^2 \cdot \left[\sin \frac{\omega\tau}{2} \Big/ \frac{\omega\tau}{2} \right]^2$$

for a rectangular pulse of unit height, the power density spectrum is,

$$N_E(\omega) = \overline{v^2}\,\tau \cdot \left[\sin \frac{\omega\tau}{2} \Big/ \frac{\omega\tau}{2} \right]^2 \qquad \ldots \text{(B.3)}.$$

Therefore the noise power N in the message band is

$$N = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} N_E(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} \overline{v^2} \cdot \tau \cdot \left[\sin \frac{\omega\tau}{2} \Big/ \frac{\omega\tau}{2} \right]^2 d\omega.$$

$$\cong \frac{\overline{v^2}}{2\pi} \cdot \int_{-\omega_0}^{\omega_0} \tau \cdot d\omega \qquad \left[\begin{array}{l} \text{since } \sin x/x = 1 \text{ for} \\ \omega \ll \omega_0 \text{ and } \omega_0\tau \ll 1 \end{array}\right]$$

$$\therefore \quad N = 2\,\overline{v^2} \cdot (f_0/f_r). \qquad \ldots \text{(B.4)}.$$

and the rms noise voltage is

$$N_v = (2\overline{v^2})^{1/2} \cdot (f_0/f_r)^{1/2} \qquad \text{... (B.5).}$$

If the frequency of the message signal is f_m, then f_r/f_m number of pulses occur during the full period of the input signal, and ideally only half of these i.e. $f_r/2f_m$ pulses are responsible for building up the signal from negative peak to the positive peak. The rms value of the signal voltage, then, is

$$S_v = K \cdot \frac{V_e}{2\sqrt{2}} \cdot (f_r/2f_m) \qquad \text{... (B.6).}$$

where K is a factor of proportionality depending on the input signal amplitude. The value of K is unity for the maximum input signal amplitude that the system can handle without overloading.

Therefore, the rms signal to noise ratio for the Δ-M system

$$[S_v/N_v] = K \cdot \frac{V_e}{8\sqrt{\overline{v^2}}} \cdot \frac{f_r^{3/2}}{f_0^{1/2} \cdot f_m} \qquad \text{... (B.7).}$$

If it is assumed that all error amplitudes are equally probable between the limits $\pm V_e$, the mean square amplitude $\overline{v^2}$ (for the probability distribution of amplitudes p(y)) is found to be $V_e^{2}/3$. Substituting the value of $\overline{v^2}$ in the equation B.7,

$$[S_v/N_v] = 0.22\, K \cdot \frac{f_r^{3/2}}{f_0^{1/2} \cdot f_m} \qquad \text{... (B.8).}$$

This result agrees well with the SNR obtained by De Jager and others. The SNR, eq.(B.8), is directly proportional to $(f_r)^{3/2}$ and inversely proportional to f_m.

Considering the system with double integrator the SNR evaluation proceeds in a similar fashion, except that the signal build up is now different. Each pulse (positive or negative) builds up a ramp voltage at the output of the integrator, and the final height reached is the sum of all ramps produced. Hence, the peak signal built up by the q pulses is

$$[S]_{PEAK} = V_e \cdot \frac{q(q+1)}{2} \qquad \ldots \text{(B.9)}.$$

where $q = f_r/4f_m$

Equation(B.9) can now rewritten as,

$$[S]_{PEAK} = \frac{V_e}{32} \cdot (f_r/f_m)^2 \qquad \text{since } f_r/4f_m \gg 1$$

$$\text{and } S_{rms} = \frac{V_e}{32} \cdot (f_r/f_m)^2 \qquad \ldots \text{(B.10)}.$$

The noise waveform, instead of being rectangular, is now assumed to triangular, and the noise voltage is calculated according to equation 2.26(b). The SNR, then, becomes,

$$S_v / N_v = 0.05 \frac{f_r^{5/2}}{f_m^2 \cdot f_o^{1/2}} \qquad \ldots \text{(B.11)}.$$

If a single integrator is used at the receiver, eq.(B.11) will be modified. As the SNR will be proportional to $1/f_m$ and not to $1/f_m{}^2$, the SNR, according to De Jager, comes out to be

$$S_o/N_o = 0.026 \frac{f_r{}^{5/2}}{f_m \cdot f_o{}^{3/2}} \quad \ldots (B.12).$$

The SNR is proportional to $(f_r)^{5/2}$, and the improvement in SNR with the increase of PRF will be 15 db/octave. Whereas, in the single integrator system, the improvement in SNR is only 9 db/octave (Eq.B.7). At about 100 Kc/s PRF, the double integrator has 10 db higher SNR than the single integrator.

Use of a double integrator network in the feedback loop and the receiver (either with or without prediction), presents some serious difficulties. The output at the higher frequencies of the message band is severely reduced. This will result in an overload at lower input levels of the higher frequency input signals. The SNR will be very poor because there is no matching between the output and the input, and the feedback does not function. No doubt the lower frequencies will be reproduced fairly well, but little advantage can be taken of this fact. The input has to be decreased to a level where the highest speech frequency does not overload the system. At such input levels, the SNR at the lower input frequencies is also quite poor.

A 12 db/octave change of slope in the feedback network will also lead to instability and oscillations, as mentioned earlier. A modification of the system will consist of using only a single integrator at the receiver, and retaining the double integrator (with prediction) in the feedback loop. This does not interfere with the advantage obtained in approximation of the signal at the transmitter. The received output will have the same frequency characteristics, i.e., a fall of 6 db octave, as the mean power spectrum of speech signals[40]. This is desirable, and quite satisfactory, if only the speech signals are being transmitted.

B.1.2. EXPERIMENTAL RESULTS.

The Δ-M system has been extensively tested for speech type of signals,[41,42,43,44] and a PRF of about 60 Kc/s has generally been found adequate for a fairly good reproduction. The variation of SNR with input signal level for a 1 Kc/s sinusoidal signal is shown in Fig.B.5, for a PRF of 60 and 40 Kc/s. The best SNR at the optimum input is 32 and 28 db, respectively. The SNR deteriorates for lower input levels and the dynamic range of input for an output SNR of 25 db is 13 and 9 db, at the PRFs of 60 and 40 Kc/s, respectively. The variation of output with the input signal frequency is shown in Fig.B.6, where it is seen that the output decreases by 6 db for an octave change of f_m. These results have been taken from Lender etc. and a filter at a cut-off frequency of 250 c/s used. The variation of SNR with f_m is also shown in the same figure and it is seen that the SNR falls by 8 db at the higher frequency compared to the SNR at the 1 Kc/s signal. Variation of SNR with PRF is shown in Fig.B.7, for both the single and the double integrator. The theoretical increase in the SNR with PRF is shown dotted in the figure. The experimental results have

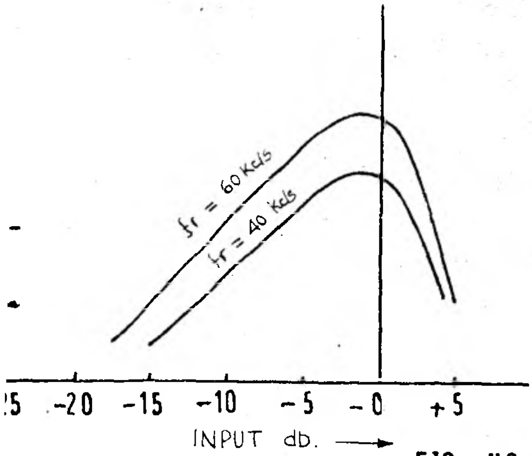


FIG. NO. B·5

VARIATION OF SNR WITH INPUT LEVEL

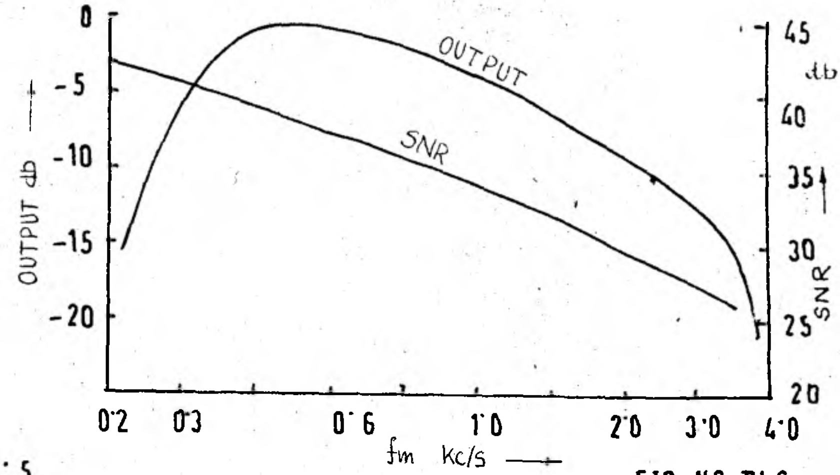
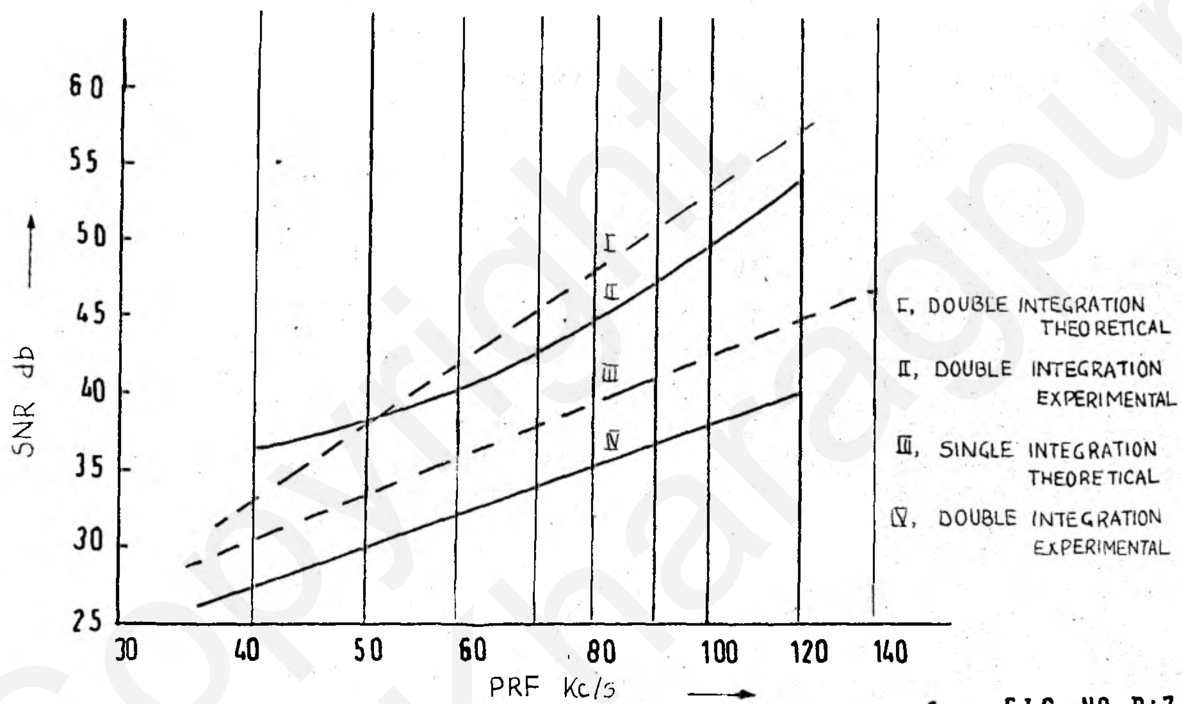


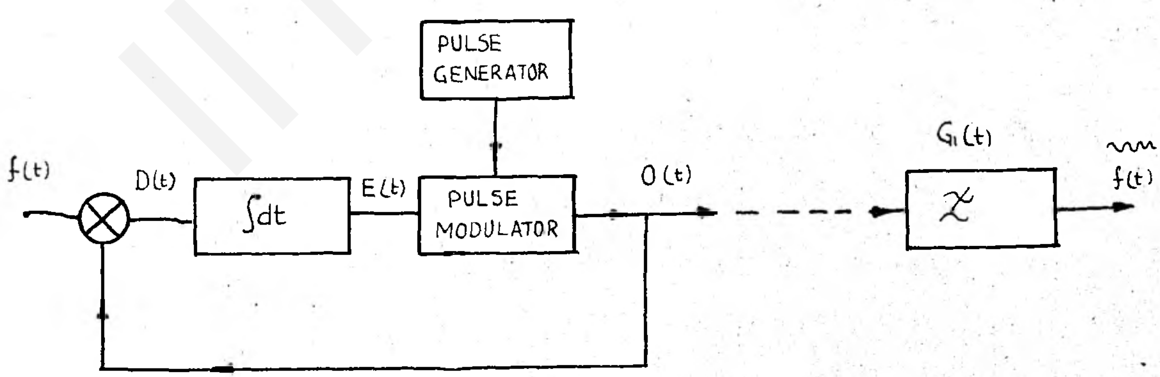
FIG. NO. B·6

VARIATION OF OUTPUT AND SNR WITH fm



VARIATION OF SNR WITH PRF

FIG. NO. B·7



A BLOCK DIAGRAM OF Δ-ΣM SYSTEMS

FIG.NO. B·8

approximately the expected slope of 9 db/octave for the single integrator, and 15 db/octave for the double integrator.

The SNR for the FM and the band limited noise signals will, in general, be less by about 8 - 10 db, as compared to the optimum SNR for the sinusoidal frequency.

B.2.0. Δ-ΣM SYSTEM.

As shown above, the Δ-M system is not always suitable for the transmission of the signals (specially the low frequencies) in digital form. To compensate for the unavoidable differentiation of the input signal in Δ-M, the Δ-ΣM system has a signal integration process added at the input to the Δ-M. A rearrangement of the blocks in this modified system (mainly to make the process of integration realizable by ordinary networks), gives the Δ-ΣM system as proposed by Inose et al. A block diagram of the Δ-ΣM system is shown in Fig.B.8, where the input to the integrator is the difference of the input signal f(t) and the output pulse signal O(t). The error signal E(t) is the integrated output of the difference circuit, that is,

$$E(t) = \int \left\{ f(t) - O(t) \right\} dt$$

The pulse modulator compares the amplitude of the E(t) with a predetermined reference level, and opens the gate to pass a pulse from the pulse generator only when the error signal is larger than the reference level. For the amplitude of E(t) less than that of the reference level, the gate closes and there is no pulse at the output. Through the negative feedback the integrated difference signal is always kept close to the reference level of the modulator, provided, there is no overload. As the amplitude of the input signal becomes larger, the output pulses appear more frequently. In other words, the output pulses carry the information corresponding to the input signal amplitude.

At the receiver, the pulses are reshaped and passed through a low pass filter to recover the signal. As no integration process is involved in the receiver, no accumulative error due to transmission disturbances results at the output. Unlike the Δ-M system, where the allowable input signal amplitude that does not overload the system is inversely proportional to the signal frequency, the maximum allowable amplitude in Δ-ΣM is independent of the signal frequency. The maximum signal amplitude allowed is proportional to the product of the amplitude and the number of the output pulses. The integrator in the forward path of the transmitter could be either a single or a double integrator, or any other signal processing network favourable to the system of transmission.

B.2.1. CODING CHARACTERISTICS OF Δ-ΣM SYSTEM.

Proceeding in a manner similar to that given for the Sη-PCM and Δ-M systems, the analysis of Δ-ΣM can be simply accomplished as follows. Denoting the frequency domain characteristics of f(t), D(t), E(t), O(t), G_1(t), and $\tilde{f}(t)$, by f(w), D(w), E(w), O(w), G_1(w) and $\tilde{f}(w)$, respectively, the frequency domain equations for the system with single integration can be written as

$$D(w) = f(w) - O(w)$$
$$E(w) = D(w) . \quad 1/jw$$
$$N(w) = f(w) - \tilde{f}(w)$$

where N(w) is the noise power density in the message band. Now

$$E(w) = D(w). \quad 1/jw = \left[f(w) \times 1/jw - O(w). \ 1/jw \right]$$

or $\quad jw. \ E(w) = f(w) - O(w)$

or $\quad jw. \ E(w). \ G_1(w) = f(w). \ G_1(w) - O(w) \ G_1(w)$

... (B.13)

If the receiver filter is assumed to have a brickwall characteristics with a cut-off frequency at f_o, then

$$G_1(w) = \begin{cases} 1 , & \text{for} \quad |\omega| \leq \omega_0 \\ 0 , & \text{for} \quad |\omega| > \omega_0 \end{cases}$$

and $f(w). \ G_1(w) = f(w)$ in the pass band. As the receiver consists of only a low pass filter, $O(\omega). G_1(\omega) = \tilde{f}(\omega)$, and equation (B.13) can be written,

$$j\omega . E(\omega). G_1(\omega) = f(\omega) - \tilde{f}(\omega) \qquad \text{... (B.14).}$$
$$= N(\omega)$$

The modulator has again an abrupt non-linearity. The assumed error waveform is rectangular, and its amplitude ranges, between the peak limits of $\pm V_e$ (where V_e is height of the rectangular pulse)with equal probability. The power spectral density of this random waveform can be found by resolving it into a product of two components. One component is a regular rectangular waveform, which is multiplied by the other component of random amplitudes (with a mean zero value and peak limits of $\pm V_e$). The amplitude component varies randomly around zero with a probability defined by p(y). The correlation function of this oscillating function is $\overline{v^2}$, where $\overline{v^2}$ is the mean square amplitude of the pulses for the probability distribution assumed. The power density spectrum of this error waveform is the same as that given in eq.(B.3). Therefore, the noise power in the message band, from equation (B.14), is,

$$N = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} N_E(\omega).(j\omega)^2 d\omega$$

$$= \frac{\overline{v^2}\tau}{2\pi}\int_{-\omega_0}^{\omega_0} \omega^2 \left[\sin\frac{\omega\tau}{2}\Big/\frac{\omega\tau}{2}\right]^2 d\omega$$

As $\sin\frac{\omega\tau}{2}\Big/\frac{\omega\tau}{2}$ is almost equal to one, therefore,

$$\sqrt{N} = \left(\frac{8\pi^2}{3}.\ \overline{v^2}\right)^{1/2} . \frac{f_o^{3/2}}{f_r^{1/2}} \qquad \text{... (B.16).}$$

and the rms boise voltage

$$N_v = \left(\frac{8\pi^2}{3}.\ \overline{v^2}\right)^{1/2} .\ f_o^{3/2}\Big/f_r^{1/2} \qquad \text{... (B.17).}$$

If the pulse width of O(t) is equal to the sampling period, the dynamic range of the input signal is

$$2V_e/\tau = 2V_e \cdot f_r \qquad \text{... (B.18).}$$

Defining the ratio of peak to peak amplitude of the sinusoidal input signal to the dynamic range as M, the mean signal power is given by,

$$S = M^2 V_e^2 / 2\tau^2 \qquad \ldots (B.19).$$

or the rms signal voltage S_v is,

$$S_v = V_e M / \tau = V_e . M . f_r \qquad \ldots (B.20).$$

Therefore, the rms signal to noise ratio for the Δ-ΣM system is

$$\left[S_v / N_v \right] = \frac{V_e . M}{\sqrt{2}} . \frac{f_r^{3/2}}{f_o^{3/2}} . \frac{1}{\left(\frac{8\pi^2}{3} . \overline{\upsilon^2} \right)^{1/2}} \qquad \ldots (B.21).$$

If it is assumed that all error amplitudes are equally probable between the limits $\pm V_e$, the mean square amplitude $\overline{\upsilon^2}$ is found to be $\frac{V_e^2}{3}$. Substituting this value of $\overline{\upsilon^2}$ in the equation (B.21),

$$\left[S_v / N_v \right] = M . \frac{3}{4\pi} . \left[f_r / f_o \right]^{3/2} \qquad \ldots (B.22).$$

This is the same result as given by Inose et al. The SNR varies as $(f_r)^{3/2}$, similar to Δ-M system, but the SNR is seen to be independent of the input signal frequency.

If a double integrator network is used in place of the single integrator, a similar analysis of the system will show that the SNR is now proportional to $(f_r)^{5/2}$, but still independent of f_m.

B.2.2. EXPERIMENTAL RESULTS.

The variation of SNR with input level variations of a sinusoidal signal is shown in Fig.B.9 for a PRF of 60 and 40 Kc/s. The best SNR is 29 db and 23 db, and the dynamic range of the input voltage for an SNR above 20 db is 15 and 9 db, for the PRFs of 60 and 40 Kc/s, respectively.

The variation of the output voltage with the input signal frequency is shown in Fig.B.10 (solid curve), and it is seen that there is no frequency distortion in Δ-ΣM . The variation of SNR with signal frequency is also shown in the Fig.B.10 (dotted curve) and is independent of the signal frequency. The variation of SNR with PRF is shown in Fig.B.11 and it is seen that the SNR increases by 9 db/octave change of PRF, as expected from eq.(B.22). The above results have been quoted for the single integrator only. The double-integrator- Δ-ΣM system shows poorer SNR than those of the single-integrator- Δ-ΣM system at PRFs lower than 200 Kc/s. As the improvement in SNR in the double integrator system is 15 db/octave change of PRF, the double integrator performs well at very high PRFs. The variation of SNR with PRF for the double integrator is also shown in the Fig.B.11. The linearity in the input-output voltage is quite good.

From the characteristics given above, it can be concluded that the Δ-ΣM system is very suitable for transmitting telemetring signals and will be quite adequate for speech transmission at 60-80 Kc/s PRF.

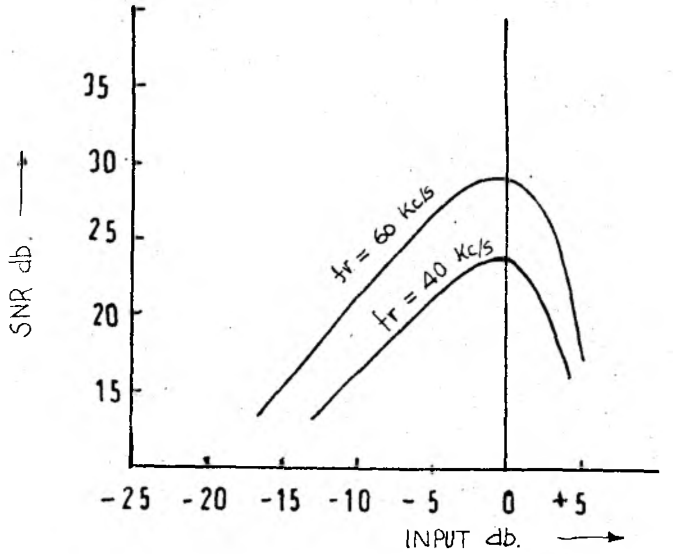


FIG. NO. B·9

VARIATION OF SNR WITH INPUT LEVEL

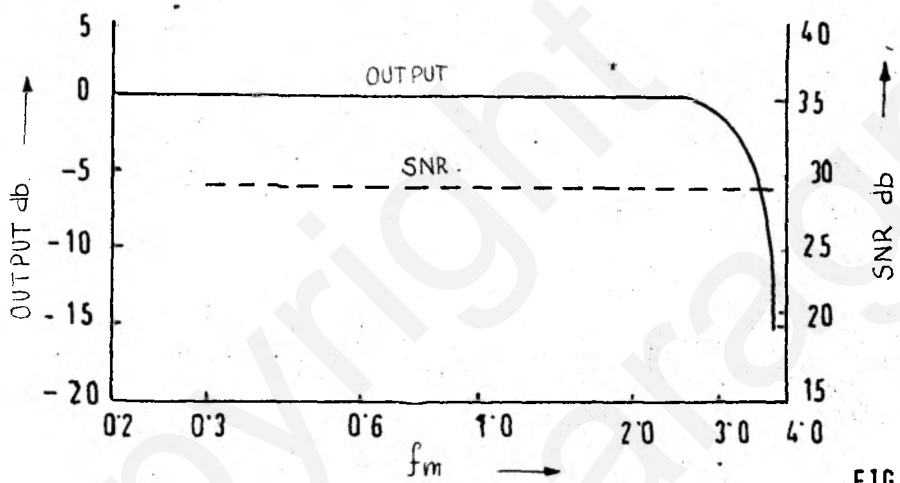


FIG. NO. B· 10

VARIATION OF OUTPUT AND SNR WITH fm

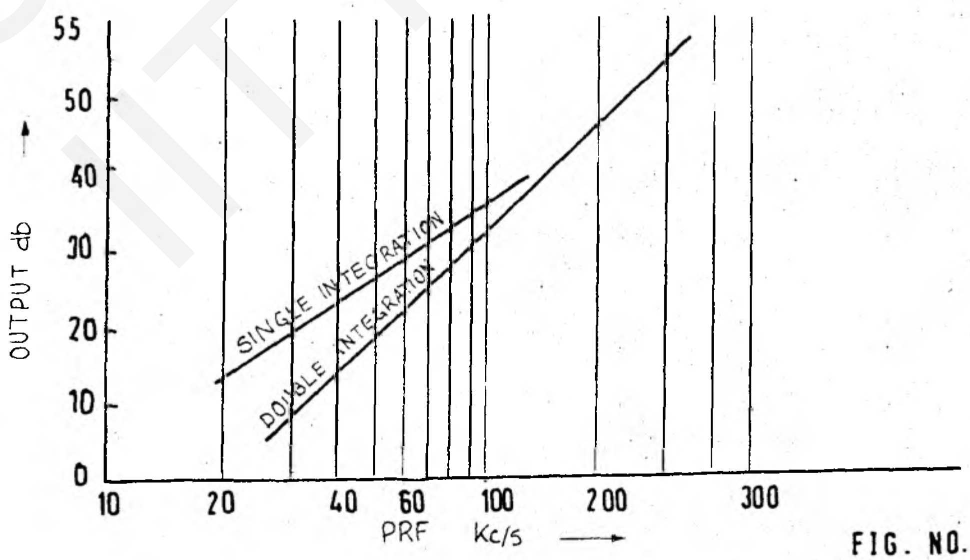


FIG. NO. B· 11

VARIATION OF SNR WITH PRF

B.3. DISCUSSION.

Both the Δ-M and Δ-ΣM systems are very simple in circuitry. The systems are comparable to AQ-PCM at lower PRFs, but as the improvement of SNR in AQ-PCM is very rapid with an increase of the equivalent PRF, the AQ-PCM system is better than these systems at higher PRFs. The Δ-M system has an overload characteristic which is proportional to the input signal frequency and the resultant SNR and output are poor at higher f_m. The use of Δ-M system for transmitting wideband signals is not possible if the frequency distortion normally obtained is above the allowable limit. However, the Δ-M system has been found adequate for speech transmission.

The Δ-ΣM system overcomes the defects of the frequency distortion and overload in the Δ-M system and has a comparable SNR. The biggest advantage of Δ-ΣM system lies in the practicability of d.c. transmission and hence, the choice of Δ-ΣM for the transmission of telemetry signals. The lower SNR in the Δ-ΣM has been obtained, possibly because the equalisation of the variation of the SNR with f_m has been done at the SNR obtained at the highest signal frequency. A definite improvement in the system is possible if the equalisation is made for the SNR at the centre of the frequency band.

De Jager has shown that a double integration network (with prediction) in the feedback loop gives a better SNR than the system with a single integrator only. Inose, on the other hand, has used a simple double integrator (no prediction), and his results are poorer than the single integrator results, at least at PRFs below 200 Kc/s. The time constants of the integrating networks, and the consequent 12 db/octave slope of their frequency response, has to be decreased to avoid oscillations. This may be the reason for the poorer results with the double integrator as reported by Inose. Both these systems have, however, been reported to be suitable for transmission of Television signals, where the advantages of a simple circuit are very marked.

APPENDIX C.

TRUNCATED NORMAL DISTRIBUTION.

The probability density of a normal distribution with mean zero and standard deviation σ , truncated at $\pm K\sigma$, is given by

$$\left[\left\{\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-x^2/2\sigma^2}\right\} \Big/ \int_{-K\sigma}^{+K\sigma} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-x^2/2\sigma^2} . dx\right] = f(x\,; K\sigma), \text{ say,}$$

Hence, the Mean Square Deviation for this distribution is

$$\sigma_{tr}^2 = \int_{-K\sigma}^{K\sigma} x^2 . \ f\,(x\,;\ K\sigma)\, dx$$

Making the substitution $x = \mu\sigma$, the value of σ_{tr}^2 is obtained as

$$\sigma_{tr}^2 = \sigma^2 \frac{\int_0^K \mu^2 \frac{1}{\sqrt{2\pi}} e^{-\mu^2/2} . d\mu}{\int_0^K \frac{1}{\sqrt{2\pi}} e^{-\mu^2/2} . d\mu}$$

$$= \sigma^2 \frac{2\int_0^{K^2/2} e^{-v}\, v^{3/2-1}\, dv}{\int_0^{K^2/2} e^{-v}\, v^{1/2-1}\, dv}$$

$$= 2\sigma^2 \frac{\Gamma(3/2) . \gamma\left(\frac{K^2}{2}, \frac{3}{2}\right)}{\Gamma(1/2) . \gamma\left(\frac{K^2}{2}, \frac{1}{2}\right)}$$

$$\text{or} \quad \sigma_{tr}^2 = \sigma^2 \left[\frac{\gamma(k^2/2\ ,\ 3/2)}{\gamma(k^2/2\ ,\ 1/2)} \right]$$

where $\gamma(x, n)$ is the reduced incomplete γ integral

$$\left\{ \int_0^x e^{-x}\ x^{n-1}\ dx \right\} \Big/ \left\{ \int_0^\infty e^{-x}\ x^{n-1} \cdot dx \right\}$$

Standardising the truncation interval by taking $K\sigma = 1$, the value of $K = 1/\sigma$, and

$$\sigma_{tr}^2 = \sigma^2 \frac{\gamma\left(\frac{1}{2\sigma^2}\ ,\ \frac{3}{2}\right)}{\gamma\left(\frac{1}{2\sigma^2}\ ,\ \frac{1}{2}\right)}.$$

The table No.1[45] gives $F(k)$;

$$F(k) = \int_{-\infty}^{K} \frac{1}{\sqrt{2\pi}}\ e^{-x^2/2} dx$$

for different K, and table 1 and 2[46] gives

$$I(U, p) = \gamma\left(Z\sqrt{p+1}\ ,\ p+1\right)$$

Hence,

$$\sigma_{tr}^2 = \sigma^2 \left[\frac{I\left(U = \frac{1/\sigma^2}{\sqrt{6}}\ ;\ p = 0.5\right)}{I\left(U = \frac{1/\sigma^2}{\sqrt{2}}\ ;\ p = -0.5\right)} \right]$$

The Table I below shows the computations for σ_{tr}^2 corresponding to different amounts of truncation 2α .

α	K [where $F(K) = (1-2\alpha)$] (Ref 45)	K^2	$\sigma^2 = \frac{1}{K^2}$	$\frac{K^2}{\sqrt{6}}$	$\frac{K^2}{\sqrt{2}}$	$I\left(\frac{K^2}{\sqrt{6}}, 0.5\right)$ (Ref 46)	$I\left(\frac{K^2}{\sqrt{2}}, -0.5\right)$ (Ref 46)	$\frac{I\left(\frac{K^2}{\sqrt{6}}, .5\right)}{I\left(\frac{K^2}{\sqrt{2}}, -.5\right)}$	$\sigma_{t_r}^2$
1	2	3	4	5	6	7	8	9	10
0.025	1.96	3.8416	0.2603	1.568	2.717	0.72111	0.95014	0.759	0.1976
0.0025	2.81	7.8961	0.1266	3.223	5.584	0.95156	0.99503	0.9563	0.121111
0.0005	3.29	10.8241	0.09239	4.418	7.655	0.98727	0.99899	0.98827	0.0913
0.0005	3.90	15.21	0.06576	6.208	10.757	0.9983508	.9999039	0.998447	0.0656

A comparison of entries in columns 4 and 10 shows that, for a truncation error $\alpha = 5 \times 10^{-5}$, σ^2 and σ_{tr}^2 are practically equal, the ratio σ^2/σ_{tr}^2 being 1.0024. This corresponds to a crest factor of 3.90. Making the crest factor equal to 4 only improves the agreement and hence the conclusion: If in the Normal distribution of amplitudes, a restriction is imposed on the maximum amplitude such that it does not exceed a particular value of $\pm K$ for than, say, 3 in 10^5, this truncated Normal Distribution will have approximately the same rms value as the Normal distribution.

R E F E R E N C E S.

1. Bennett, W.R. and Feldman, C.B. - Bandwidth and Transmission Performance. Bell.Sys.Tech.Jour., Vol.28, p.490 (1949).

2. Das, J. - Pulse Slope Modulation - a New Method of Modulating Video Pulses and its Possible Application on Line Circuits. Ind.Jour.Physics, Vol.28, p.449, (1954).

3. Goodall, W.M. - Telephony by Pulse Code Modulation. Bell.Sys.Tech.Jour. Vol.26, p.395 (1947).

4. De Jager, F. - Deltamodulation - A Method of PCM Transmission using the 1-Unit Code. Phillips Res.Repts.Vol.7, p.442,1952

5. Inose H. et al. - A Unity Bit Coding Method by Negative Feedback. Proc.IEEE, Vol.51, p.1524, (1963).

6. Alexander, W. - The Ternary Computer. Jour.IEEE, Vol.10, p.36, (1964).

7. Saunders, R.W. - Communication Efficiency Comparison of Several Communication Systems. Proc.IRE, Vol.48, p.575, (April 1960

8. Shannon, C.E. - A Mathematical Theory of Communication Bell.Sys.Tech.Jour.Vol.27, p.379 (1948).

9. Meacham, L.A. and Peterson, E. - An Experimental Multichannel P.C.M. System of Toll Quality. Bell.Sys.Tech. Jour.Vol.27, p.1, (1948).

10. Feldman, C.B. - A 96-Channel Pulse-Code Modulation System. Bell.Lab.Rec. Vol.26, p.364 (1948).

11. Zetterberg, L.H. - A Comparison between Delta and Pulse Code Modulation. Ericsson Technics, Vol.11, p.95, 1955.

12. Faruqui, M.N. and Das, J. - On a Slope-Quantized Ternary Pulse Code Modulation. Trans.IEEE, PG Communication System (June 1964).

13. Faruqui, M.N. - A Slope-Quantized Binary Pulse Code Modulation. Accepted for publication in Electronic Engineering, U.K.

14. Bose, A.G. - A Two-State Modulation System. Western Electronic Show and Convention/San Francisco, August 20-23 (1963).

15. Oliver, Pierce, Shannon. - The Philosophy of PCM. Proc.IRE, Vol.36, p.1324, (1948).

16. Gabor, D. - Communication Theory and Cybernatics. Trans.IRE, PGCT,Vol.CT-1, p.19 (1954).

17. Bennett, W.R. - Spectra of Quantized Signals. Bell.Sys.

18. Solodonikov, V.V. - Introduction to the Statistical Dynamics of Automatic Control Systems. Dover Publications, 1960, p.108.

19. Rice, S.O. - Mathematical Analysis of Random Noise. Bell.Sys.Tech.Jour. Vol.23, p.282, (1944) and Vol.24. p.706, (1945).

20. Fry, T.C. - Probability and its Engineering Uses. D.Van Nostrand 1928.

21. Spang, H.A.,III - Reduction of Quantizing Noise by Use of Feedback. IRE Trans.on Communication systems, Vol.CS-10, p.373 (1962).

22. Smith, B. - Instantaneous Companding of Quantized Signals, Beal.Sys.Tech.Jour. Vol.36, p.653, (1957).

23. Lawton, J. - Compandors for Reducing Noise and Cros talk. A.T.E.Journal. Vol.9 p.109 (1953

24. Mallinckrodt, C.O. - Instantaneous Compandors. Bell.Sys. Tech.Jour.Vol.30, p.706, (1951).

25. Lozier, J.C. - Instantaneous Compandors on Narrow Ban Speech Channels. Bell.Sys.Tech.Jour. Vol.30, p.1214, (1951).

26. Davenport, W.D. - An Experimental Study of Speech-Wave Probability Distributions. J.Acoust. Soc.of A. Vol.24, p.390 (1952).

27. Aaron, M.R. - PCM Transmission in Exchange Plant. Bell.Sys.Tech.Jour.Vol.41, p.109 (1962)

28. Clavier, A.G., Panter, P.F., and Dite, W. Signal-to-Noise-Ratio Improvement in PCM System, Vol.37, p.355, (1949).

29. Weaver, C.S. - A Comparison of Several types of Modulation. Trans.IRE, PG.Comm.Syst. Vol.CS-10, p.96 (1962).

30. Cahn, C.R. - Performance of Digital Phase Modulation Communication Systems. Trans.IRE. PG. Comm.Syst.Vol.CS-7, p.3 (1959).

31. Goldman, S. - Frequency Analysis, Modulation and Noise McGraw Hill Book Company Inc. (1948).

32. Clark, M.A. - Considerations in the Choice of Optimum Data Transmission Systems for Use over Telephone Circuits. Jour.Brit.IRE, Vol.23, p.331, (1962).

33. Wright, W.L., Jolliffe, S.A.W. - Optimum System Engineering for Satellite Communication Links with Special Reference to the Choice of Modulation Method. Jour.Brit.IRE, Vol.23, p.381, (1962).

34. Debart, H. - A Study of Delta Modulation System from the point of view of Information Theory. Cables et. Transmission, Vol.15 p.198 (1961).

35. Cornet, F, Chatelon, A, and Le Corre, J. - Application of PCM to Integrated Telephone Network. Electrical Communication, Vol.38, p.23 (1963).

36. Mann, H, Staube, H.M. and Villars C.F. - A Companded Coder for an Experimental PCM Terminal. Bell.Sys.Tech.Jour. Vol.41, p.173, (1962).

37. Shennum, R.H. and Gray, J.R. - Performance Limitations of a Practical PCM Terminal. Bell.Sys.Tech.Jour. Vol.41, p.25 (1962).

38. Davis, C.G. - An Experimental Pulse Code Modulation System for Short-Hand Trunks. Bell. Sys.Tech.Jour.Vol.41, p.1 (1962).

39. Lender, A and Kozuch, M. - Single Bit Delta Modulating Systems. Electronics, Vol.34, p.125, (1961).

40. Beranek, L.L. - Acoustic Measurements, John Wiley and Sons, N.Y. (1949).

41. Watson, R.B. and Hudson, O.K. - Transmitting Systems Using Delta Modulation. Electronics Vol.29, p.164 (1956) Oct.no.10.

42. Balder, J.C. and Kramer, C. - Video Transmission by Delta Modulation Proc.IRE, Vol.50, p.428 (1962).

43. Williams, B.E. - A Delta Modulation Using Junction Transistors, Electronics Engineering Vol.31, p.674, (1959).

44. Varsos, S.G. and Norvell, B. - Datec-Digital Adaptive Technique for Efficient Communications. IEEE International Convention Record Part 6, p.435, (March 1964).

45. Pearson, E.S. and Hartley, H.O. - Biometrika Tables for Statisticians, published by office of Biometrika, Cambridge ,University Press (1954).

46. Pearson, C. - Table of Incomplete Gamma Function. Office of Biometrika, Cambridge University Press (1951).

S U M M A R Y.

The thesis gives an account of the experimental and theoretical investigations carried out on some pulse-modulation systems resulting in new Slope-quantized Pulse Code Modulation Systems using ternary and binary codes. The slopes of the signal waveform are quantized into groups of Ternary 3 level or Binary 2 level pulses. The principal characteristics viz., performance with different types of input signals, like sinusoidal, FM, noise and speech, have been investigated and the transmission characteristics of the coded signals through RF and Video channels have been analysed. The overall performance of the systems developed here have been compared with similar other systems, and the superior characteristics of these systems for direct applications in analogue to digital conversions and digital transmissions established. The results of these investigations are reported in the seven chapters and the appendices.

Chapter I starts with a brief survey of the principal characteristics of the existing coded-pulse-modulation systems and brings out their limitations. Ternary coding has been suggested to have some advantages over binary coding.

Chapter II shows the advantages of the ternary code over the binary code and discusses the principle of operation of the Ternary SQ-PCM system (without and with feedback) and the

Binary SQ-PCM system. The essential difference between the Binary SQ-PCM and Delta Modulation is also pointed out. A theoretical estimate of the quantizing-noise and hence, the signal-to-noise ratio for the two systems have been made. The performances of syllabic and instantaneous compandors have been compared, and Detail theoretical calculations of the dynamic range and the SNR have been made in the case of instantaneous compandors.

Chapter III deals with the characteristics of the Ternary SQ-PCM system with and without feedback. Complete circuit diagrams of the systems are given. The results of tests with different input signals, like sinusoidal, FM, noise and speech, have been given. The results include the SNR, the dynamic range, the gain vs. frequency characteristics, and input-output linearity of these two systems. The power spectrum of the Ternary pulses has been shown. The spectrum at the input of the filter is also given.

Chapter IV gives the circuit diagram and the gain vs. frequency response of the feedback network used in the Binary SQ-PCM. The performance characteristics of the system has been evaluated for the same four types of input signals. A comparison of the Binary and Ternary system has been made.

Chapter V discusses the transmission characteristics of the SQ-PCM signals. The effect of channel noise disturbance on the Video, AM, and FSK transmission schemes have been analysed. In the light of the transmitter power requirements, several wideband

RF modulation schemes, like AM, FSK and PM, have been discussed. Coding efficiency of the Binary and Ternary systems has been evaluated. The power and communication efficiencies have been defined and evaluated for the different SQ-PCM-RF modulated schemes. Two schemes of TDM of SQ-PCM have been indicated.

Chapter VI compares the performance characteristics i.e., coding and transmission of the Ternary and Binary SQ-PCM with other digital systems like AQ-PCM, Δ-M, and Δ-ΣM . It is shown that the Ternary SQ-PCM at 40 Kc/s PRF is equivalent to the 7-digit AQ-PCM system, and Binary SQ-PCM at 40 Kc/s PRF is equivalent to the 6-digit AQ-PCM but superior to Δ-M and Δ-ΣM systems at 60 Kc/s PRF.

Chapter VII gives the general conclusions drawn from the results of the previous Chapters. The overall characteristics of the SQ-PCM systems are given in a tabular form together with the similar characteristics of the other systems. Some applications of SQ-PCM systems and future scope of work have been indicated.

Appendix A gives a brief review of the AQ-PCM systems. It deals with the qualitative description of the AQ-PCM together with its coding and transmission characteristics.

Appendix B gives a brief review of the Δ-M and Δ-ΣM systems and mainly deals with their coding characteristics.

Appendix C shows the error produced in truncating a Normal distribution curve at a point approximately four times its standard deviation.