1.1 INTRODUCTION

Classical optimization approaches evaluate a decision with its performance on a single criterion, e.g., maximization of profits or minimization of total distance travelled. A single criterion model is usually the simplified form of the reality which sometimes leads to a wrong decision. For instance, in water resources development the design of projects and programmes is based traditionally on the estimation of national benefits and costs. A more realistic analysis would include environmental, social and regional objectives as well. In complex systems, a good decision is possible only when the multiplicity of criteria are taken into account. Such decision problems are referred to as the Multiple Criteria Decision Making (MCDM) problems.

Multiobjective programming is a powerful mathematical procedure in the area of Operations Research in making good decisions. It has awakened widespread interest and acceptance among the researchers, decision makers, management scientists and others for its broad applicability to real world decision making problems.

In the recent past it has become more and more obvious that comparing different ways of action as to their desirability, determining optimal solutions in decision problems in many cases can not be done by using a single objective function. Thus multiobjective analysis has led to numerous evaluation schemes.

Multiobjective problems arise in the design, modelling and planning of many complex resource allocation systems in the areas of industrial production, urban transportation, health care, layout and landscaping of new cities, energy production and distribution, wildlife management, operation and control of a firm, portfolio theory, agricultural and live stock production and local government administration to mention a few. We now briefly indicate the nature of these problems.

Research organizations are often faced with the task of evaluating a large number of proposals that compete with each other for the allocation of limited resources. The management is interested in the application of methodologies of multiobjective programming useful in the selection of a choice set of projects.

Multiobjective problem often arises in the allocation of vehicles for the delivery of finished products to destination points. An excellent review and application of multiobjective methods to the evaluation of transportation plans is given by Hill [7].

Public investment is another area to which multiobjective programming is applicable. Managerial decision problems regarding water resource systems, urban transportation planning, educational establishments, city corporations are just a few examples of the problems in this area. In the evaluation of urban transportation plans we can consider the objectives as the reduction of air pollution, noise, accidents and fiscal efficiency.

In multiobjective programming the objectives are normally conflicting in nature. For example, the most direct route for an urban highway will usually maximize accessibility and fiscal efficiency but gives a high level of air pollution and noise impacts. On the other hand, a circular route will have less air pollution and noise but requires longer travel time and more cost of travel.

In a MCDM problem, the objectives are inherently competitive among themselves. Unlike an unique optimal solution in the case of a single objective function, we get a set of 'efficient' or 'non-dominated' solutions in the case of a multiobjective problem. The problem then is usually to select one of these 'efficient' solutions according to some physical or theoretical extra information inputs. There are broadly three types of solution procedures to select the preferred solution out of a set of 'efficient' solutions for a multiobjective programming problem. They are known in the literature as :

i) Global modellings,

ii) Interactive procedures,

iii) Outranking methods.

In global modelling methods one constructs an overall decision function corresponding to the objectives and optimizes the decision function within the constraints of the problem. In general this methodology finally leads to a linear programming or a non-linear programming problem.

In the interactive procedures, one first generates a set of 'non-dominated' solutions and offers them for evaluation by the decision maker. Having the reaction inputs from the decision maker in the form of trade-offs among the objective values, one generates another set of 'efficient' solutions or a single 'efficient' solution. If the decision maker is satisfied with one of these solutions the process is terminated. However, if the decision maker further interacts with these new solutions and offers trade-offs between the objective values the process is repeated.

The conflict among K criteria involved in a MCDM problem reflects K different orderings of the set of alternatives. In global modelling the decision maker aggregates these orderings into a single one using an overall criterion. However, in social choice area where the criteria are interpreted as voters, such an aggregation is not realistic. The problem then is how to construct a relation which best reflects the individual orderings. The investigations of outranking relations (fuzzy or non-fuzzy) are aimed at solving the above problem.

In an outranking method, one constructs an outranking relation and then exploits this relation in ranking the alternatives. We may mention that the outranking procedures are of use even in some of the real world problems of multicriteria decision making apart from the problems of the social choice area.

1.2 TERMINOLOGY

In the MCDM literature, the most often used words and phrases are objectives, goals, criteria, non-dominated solutions and compromise solutions. These terminologies together with some other relevant concepts are introduced which are necessary for the development of the theory involved.

Objectives :

An objective is the reflection of the desire of the decision maker and generally indicates the direction in which he should strive to achieve his goal. For example, the objectives of the government in devising the acceptable plan would be to maximize the national welfare, to minimize dependence on foreign aid etc.

Aspiration Level :

An aspiration level is a specific numerical value assigned to an objective according to the desire of the decision maker. It is expressed in terms of a measure of the achievement of an objective.

Goal :

An objective specified with an aspiration level is termed as a goal. An objective indicates the desired direction whereas a goal gives an aspired level of achievement in the desired direction.

Criteria :

An aspect of the problem acting as one of the standards of judgment to test the acceptability of an action or decision is called a criterion. Criteria usually emerge as objectives in the actual problem setting.

Criterion Space :

Let $x_1, x_2, \ldots, x_n$ be the n number of decision variables and let $Z_1, Z_2, \ldots, Z_k$ be the K number of objectives which are functions of the decision variables. We can represent the decision variables $(x_1, x_2, \ldots, x_n)$ and the objectives $(Z_1, Z_2, \ldots, Z_k)$ as a vector in E^n and E^k respectively where E^n and E^k are n-dimensional and k-dimensional Euclidean spaces. For each point $\underline{x} \in X$ in the decision space, i.e., in E^n, there corresponds a k-tuple $(Z_1(\underline{x}), Z_2(\underline{x}), \ldots, Z_k(\underline{x}))$. The set of such k-tuples obtained by mapping the points of the decision space to the k-dimensional space is known as the criterion space or objective space.

Non-dominated or Efficient Solution :

A non-dominated solution is one for which none of the objective functions can be improved without a simultaneous adverse effect on at least one of the other objectives. That is, a point $\underline{x}^*$ is said to be non-dominated if there exists no other point $\underline{x} \in X$, such that

$$Z_i(\underline{x}^*) \leq Z_i(\underline{x}) \; \forall \; i, \; i = 1,2,\ldots, k,$$

$$Z_j(\underline{x}^*) < Z_j(\underline{x}) \quad \text{for at least one } j.$$

The non-dominated solution is also known as pareto-optimal solution, non-inferior solution, or efficient solution.

Efficient Frontier :

The set of all non-dominated solutions constitutes a frontier known as the efficient frontier in the feasible criterion space. Figures 1.1 and 1.2 depict continuous and discrete efficient frontiers for a MCDM problem involving only two objectives $Z_1(\underline{x})$ and $Z_2(\underline{x})$. $\underline{Z}^1 = (Z_1^1, Z_2^1)$ and $\underline{Z}^2 = (Z_1^2, Z_2^2)$ are two non-dominated points on the efficient frontier.

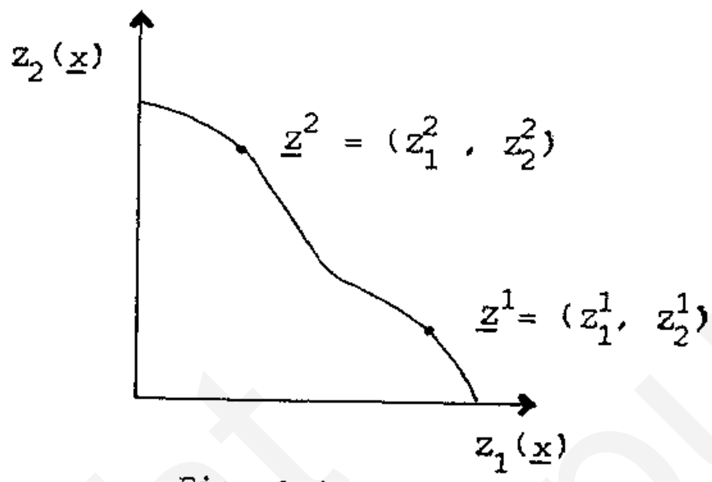

Fig. 1.1

(Efficient Frontier - Continuous Case)

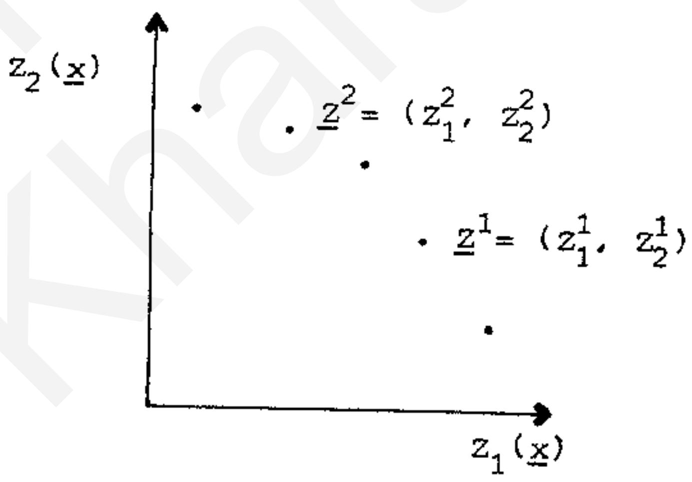

Fig. 1.2

(Efficient Frontier - Discrete Case)

1.3 METHODS OF SOLUTION

The criteria involved in a MCDM problem are often conflicting and non-commensurable in nature. Conflict among the criteria does not allow a solution to be optimal with respect to all criteria. That is, an improvement in the achievement level of one is not possible without adversely affecting that of another. Consequently, there exists a multiplicity of alternative solutions called as non-dominated or non-inferior feasible solutions. In such cases, we do not have an optimal solution, rather we can choose a good compromise solution. The first step, therefore, is to obtain the complete set of non-dominated solutions for the problem. The choice of the best among them, suited for the decision maker will be known after prescribing some additional criteria. Detailed descriptions of multiobjective analysis and its solution procedures for obtaining an efficient compromise solution is discussed by Cohon [3], Hwang and Masud [8] and Goicoechea et al. [4]. There are several methods available to generate a set of non-dominated solutions. We discuss some of these methods which have been used in the later chapters of the thesis.

The Weighting Method :

A multiobjective problem may be transformed into a single optimization problem by assigning weights to the various objective functions for which solution methods exist. Generating a non-dominated set of solutions by parametrically varying the weights was first proposed by Zadeh [25].

Using weighting method, the multiobjective problem :

$$\text{Max } Z(x_1, x_2, \ldots, x_n) = [\, Z_1(x_1, x_2, \ldots, x_n), Z_2(x_1, x_2, \ldots, x_n), \ldots, Z_k(x_1, x_2, \ldots, x_n)]$$

$$\text{subject to } (x_1, x_2, \ldots, x_n) \in X$$

is transformed into the single optimization problem :

$$\text{Max } Z(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{k} W_i Z_i(x_1, x_2, \ldots, x_n)$$

$$\text{subject to } (x_1, x_2, \ldots, x_n) \in X.$$

The co-efficient W_i operating on the i^{th} objective function $Z_i(x_1, x_2, \ldots x_n)$ is called a weight and can be interpreted as the relative importance of that objective when compared to the other objectives. If the weights of the various objectives are interpreted so as to represent the relative preferences of the decision maker, then this solution is equivalent to the best compromise solution. This solution is a non-dominated solution provided all the weights are positive. Several weight sets can generate the same non-dominated point. The solution obtained by the method depends only on the choice of W_i. We may mention that this weighting method is a global modelling technique.

The Lexicographic Method :

In many decision problems some of the goals are so important that unless these are achieved, the decision maker can not consider the achievement of the other goals. This method requires that the objectives be ranked in order of importance by the decision maker.

Given the multiobjective problem with k objectives,

$$\text{Max } Z(\underline{x}) = [Z_1(\underline{x}), Z_2(\underline{x}), \ldots, Z_k(\underline{x})]$$

$$\text{subject to } \underline{x} \in X$$

the Lexicographic method can be described as follows. Assume that the objective function $Z_1(\underline{x})$ is the most important and the objective functions have been ranked according to the priority list $Z_1(\underline{x}), Z_2(\underline{x}), \ldots, Z_k(\underline{x})$. Hence, first one solves the specified problem

$$\text{Max } Z_1(\underline{x})$$

$$\text{subject to } \underline{x} \in X.$$

If α_1 denotes the optimal value of $Z_1(\underline{x})$, then in the second step one solves the problem

$$\text{Max } Z_2(\underline{x})$$

$$\text{subject to } Z_1(\underline{x}) = \alpha_1,$$

$$\underline{x} \in X.$$

If α_2 denotes the optimal value of $Z_2(\underline{x})$, then in the third step the problem

$$\text{Max } Z_3(\underline{x})$$
$$\text{subject to } Z_i(\underline{x}) = \alpha_i \quad (i = 1,2) ,$$
$$\underline{x} \in X$$

is solved and so on. In general one solves the following problem at the m^{th} step :

$$\text{Max } Z_m(\underline{x}) \quad , \quad (m \leq k)$$
$$\text{subject to } Z_i(\underline{x}) = \alpha_i \quad (i = 1,2,\ldots,m-1),$$
$$\underline{x} \in X.$$

If problem is unbounded, then no solution is said to exist. The procedure is terminated as and when a unique solution is obtained at any of the m stages. This solution will be the preferred solution to the entire problem.

1.4 METHODS FOR PRIOR ARTICULATION OF PREFERENCE GIVEN :

Once the set of non-dominated solutions for a multiobjective problem has been identified using any of the methods described in the last section, a decision maker may be able to select one of those non-dominated solutions as his final choice. However, the situation may arise where the decision maker is unable to choose one of those solutions made available to him and would like to introduce his preference regarding the various objective functions in the search for that choice solution. This section presents the methods where the decision maker is asked to articulate his preference structure and these preferences are then built into the formulation of the mathematical model for the multiobjective problem.

The preferences are assigned by various weighting schemes, constraints, goals and utility functions, etc. All of these methods have a common feature of identifying a non-dominated solution as a best compromise solution.

We now give a brief account of some of the existing methods.

<u>Utility Function Method</u> :

In all of the utility function methods the multi-objective problem is converted to

$$\text{Max } U[Z_1(\underline{x}),\ Z_2(\underline{x}),\ldots,Z_k(\underline{x})] = U(\underline{Z})$$

$$\text{subject to } \underline{x} \in X$$

where $U(\underline{Z})$ is the utility function of the multiple objectives. Thus the methods require that $U(\underline{Z})$ be known prior to solving multiobjective problem.

Normally, a utility function is defined over a set of attributes with values in the set of real numbers. For multiobjective problems utility function can order completely the set of non-dominated solutions. A utility can be associated with each non-dominated solution and the solution with the highest utility is referred to as the best compromise solution.

The major advantage of utility function methods is that if $U(\underline{Z})$ has been correctly assessed and used, it will ensure the most satisfactory solution to the decision maker. The solution will be a point at which the non-dominated set of solutions and the indifference curves of the decision maker are tangent to each other (Keeney and Raiffa [12]). The indifference curves are the set of curves which have the same utility along each.

Goal Programming :

A decision situation is generally characterized by multiple objectives. Some of these objectives may be complementary, while others may be conflicting in nature. Goal programming allows the decision maker to specify a target or aspiration level for each objective function. A preferred solution is defined as the one that minimizes the sum of the deviations from the prescribed set of aspiration levels.

The goal programming was originally proposed by Charnes and Cooper [2]. It has been further developed by Ijiri [11], Lee [13] and Ignizio [10].

Formulation of a simple goal programming problem is given by

$$\text{Min } \bar{a} = \sum_{i=1}^{k} p_i \, [d_i^- + d_i^+]$$

$$\text{subject to } \quad Z_i(\underline{x}) + d_i^- - d_i^+ = b_i \;;\; i = 1,2,\ldots,k,$$

$$\underline{x},\; d_i^-,\; d_i^+ \geq 0,$$

where b_i - the target for goal i ,

d_i^- - the underachievement of goal i ,

d_i^+ - the overachievement of goal i ,

p_i - priority level for the i^{th} goal ,

$\bar{a}$ - the function of deviational variables known as achievement function.

Though goal programming is one of the most widely used techniques for solving many real world managerial multi-criteria decision making problems, it has got some drawbacks. The weak points of the methodology are that the assessment of aspiration levels and priorities are subjective. Moreover, it does not guarantee a non-dominated solution.

1.5 USE OF FUZZY SET THEORETIC CONCEPTS IN MCDM

In some decision problems the criteria themselves are defined qualitatively rather than quantitatively. For example, in hiring a new faculty member in the Industrial Engineering Department, the department sets the following set of goals :

(i) The candidate should be young ,

(ii) The candidate should be educated in a famous college ,

(iii) The candidate should be experienced in industry.

In these goals the underlined terms are responsible for the fuzziness which has been expressed qualitatively instead of quantitatively. Moreover, in realistic situations, it is not always rational to assume rigid limits to the existing resources. That is, the constraints involved in a decision problem are not generally strictly binding. In view of these reasons, Bellman and Zadeh [1] have suggested that a MCDM problem is better modelled using fuzzy set theoretic approach, where objectives and constraints are expressed as fuzzy goals.

Zadeh [26] laid the initial foundations of fuzzy set theory in the year 1965. This has induced an increasing interest among the researchers because of its remarkable usefulness in dealing with inherent vagueness or ambiguity involved in various realistic systems.

Decision making in the real world of human interaction and in socio technological design, planning and management processes with humanistic intervention is very often vague and imprecise. It is equally imprecise in legal, medical and environmental contexts. Conventional precise mathematics has not helped us in the understanding of human decision processes in such tasks. An underlying philosophy of the theory of fuzzy sets is to provide a strict mathematical framework, where these imprecise conceptual phenomena in decision making may be precisely and rigorously studied. For very highly precise complicated and detailed models, one needs equally an elaborate system or measurements. In many industrial processes, this is difficult to achieve. Supervisory personnel with years of experience can express their control processes effectively in linguistic terms, but not so effectively in mathematical terms. For an example, 'the possible amount of annual profit of a given firm should be substantially large'. This type of expression is generally ill defined and can be well formulated by the use of fuzzy sets. Thus fuzziness is inherent in human attempts to conceptualize, categorize, classify and relate all perceived phenomena. Fuzziness is not synonymous with probability. Probability deals with uncertainty concerning membership or non-membership of an object in a non-fuzzy set. On the otherhand, fuzziness has to do with classes in which there may be grades of membership intermediate between full membership and non-membership.

Before discussing some aspects of decision making in fuzzy environments we will review some of the basic concepts of fuzzy sets and their operations as it applies to MCDM problem.

<u>Fuzzy Sets</u> :

Informally, a fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. A more precise definition may be stated as follow.

<u>Definition</u> :

Let X be a set, denumberable or not, and let x be an element of X. Then a fuzzy subset A of X is a set of order pairs

$$A = \{x, \mu_A(x)\} \quad \forall\, x \in X$$

where $\mu_A(x)$ is the grade or degree of membership of x in A. Thus, if $\mu_A(x)$ takes its values in a set M, called the membership set, one may say that x takes its value in M through the function $\mu_A(x)$. This function will be called the membership function. When M contains only the two points 0 and 1, A is non-fuzzy and $\mu_A(x)$ is identical to the characteristic function of a non-fuzzy set.

<u>Example</u> :

Let $x = \{5,6,7\}$ be possible heights (in feet) of some people. Then the fuzzy set A of 'class of tall people' may be defined by a certain individual as

$$A = \{5/0.7,\ 6/0.79,\ 7/0.85\}.$$

Next we will refer to the following basic concepts.

Equality :

Two fuzzy sets A and B are said to be equal (denoted A = B) iff

$$\mu_A(x) = \mu_B(x) \ \forall \ x \in X.$$

Containment :

A fuzzy set A is contained in or is a subset of a fuzzy set B, written as $A \subset B$, iff $\mu_A(x) \leq \mu_B(x)$.

Complementation :

A fuzzy set B is said to be the complement of a fuzzy set A iff

$$\mu_B(x) = 1 - \mu_A(x) \ \forall \ x \in X.$$

Support :

The support of a fuzzy set A is a set S(A) such that $x \in S(A) \iff \mu_A(x) > 0$. The support of a fuzzy set A is a ordinary subset of X.

Normality :

A fuzzy set A is normal iff $\sup_x \mu_A(x) = 1$. A non-empty fuzzy set can always be normalized by dividing $\mu_A(x)$ by $\sup_x \mu_A(x)$.

α-Level Set :

The α-level set of a fuzzy subset A of X is a non-fuzzy subset of X denoted by A_α and is defined by

$$A_\alpha = (x \mid \mu_A(x) \geq \alpha, \ x \in X).$$

Union :

The union of A and B, denoted as $A \cup B$, is defined as the smallest set containing both A and B. The membership function of $A \cup B$ is given by

$$\mu_{A \cup B}(x) = \text{Max } (\mu_A(x), \ \mu_B(x)) \ \forall x \in X.$$

Intersection :

The intersection of A and B is denoted by $A \cap B$ and is defined as the largest fuzzy set contained in both A and B. The membership function of $A \cap B$ is given by

$$\mu_{A \cap B}(x) = \text{Min } (\mu_A(x), \ \mu_B(x)) \ \forall x \in X.$$

1.6 FUZZY MATHEMATICAL PROGRAMMING

The basic problem in decision making is how to choose a course of action when multiple decision criteria need to be taken into account. There is no optimal solution in which all the objectives are simultaneously maximized. The objectives are frequently incompatible and a decision maker must find a compromise solution. Moreover, in most of the real world MCDM problems goals and/or constraints which constitute the environment of the decision process are usually not precise. For example, the targets of production, profit, etc., usually can not be ascertained precisely. These type of situations are better modelled with the help of fuzzy set theory. Approaches which solve the vectormaximum problem (VMP) efficiently and at the same time model a decision problem properly are still in demand. Fuzzy mathematical programming can be regarded as an example of such a method. It can model problems which can be described by either crisp or fuzzy relations and it can solve multiobjective models with reasonable effort.

In a multiple criteria decision problem the difficulty occurs because of :

(a) the objectives and the constraints are not precise and

(b) many objectives are involved in the decision problem.

The difficulty due to the imprecise nature of the system is handled by the use of fuzzy set theory as mentioned above. The difficulty encountered because of the number of objectives involved can be eradicated by developing a decision function which is nothing but a representation function of all the objectives. Based upon this decision function a decision maker can select one alternative from the space of alternatives which is best suitable for the decision problem. More precisely if we have a set of alternatives, the decision function associates with each alternative a real number which indicates the measure of satisfaction of that alternative over the decision criteria involved. The usual procedure is to select the alternative with the highest value produced by the decision function.

Now the main problem is to obtain a suitable decision function by aggregating all the objectives. Aggregation of objectives with different operators on fuzzy sets results in different decision functions reflecting different concepts in modelling preferences (Bellman and Zadeh [1], Zimmermann [27,28,29], Yager [21], Zimmermann and Zysno [30], Luhandjula [14]).

In the next section we discuss about some of these operators which help in aggregating the objectives.

1.7 MODELLING DECISION FUNCTION USING VARIOUS OPERATORS

It is intended to design an operator for the aggregation of fuzzy sets. The different forms of decision function using various operators are discussed below.

Minimum Operator :

A decision in a fuzzy environment has been defined by Bellman and Zadeh [1] as the intersection of fuzzy sets representing either objectives and/or constraints. The grade of membership of an objective in the intersection of the two fuzzy sets, i.e., the fuzzy set decision is determined by the use of either the min-operator or the product operator.

Assume that we have k objectives $Z_1(\underline{x}), Z_2(\underline{x}),\ldots, Z_k(\underline{x})$ and m constraints $g_1(\underline{x}), g_2(\underline{x}),\ldots, g_m(\underline{x})$ defined over a decision space X. Let $\mu_{Z_1}(\underline{x}), \mu_{Z_2}(\underline{x}),\ldots, \mu_{Z_k}(\underline{x})$; $\mu_{g_1}(\underline{x}), \mu_{g_2}(\underline{x}),\ldots, \mu_{g_m}(\underline{x})$ be the respective membership functions of the objectives and the constraints. Then according to Bellman and Zadeh [1] the fuzzy membership function of decision D is defined as

$$\mu_D(\underline{x}) = \left[\mu_{Z_1}(\underline{x}), \mu_{Z_2}(\underline{x}),\ldots, \mu_{Z_k}(\underline{x})\right] \wedge \left[\mu_{g_1}(\underline{x}), \mu_{g_2}(\underline{x}),\ldots, \mu_{g_k}(\underline{x})\right]$$

where the symbol $\wedge$ stands for infrimum.

The optimal decision can be obtained as

$$\underset{\underline{x} \in X.}{\text{Max}} \ \mu_D(\underline{x})$$

Zimmermann [28] has introduced this concept in vectormaximum problems. He has defined the membership function of the decision D for a VMP as

$$\mu_D(\underline{x}) = \underset{i}{\text{Min}} \ (\mu_{Z_i}(\underline{x})) \ , \quad i = 1,2,\ldots, k.$$

Now the optimal decision is obtained by maximizing the decision over the feasible space, i.e., by determining

$$\underset{\underline{x} \in X}{\text{Max}} \ \underset{i}{\text{Min}} \ (\mu_{Z_i}(\underline{x}))$$

The above maximum can be found by solving the following linear programming problem,

$$\text{Max}\ \underset{i}{\text{Min}}\ (\mu_{Z_i}(\underline{x})) = \text{Max}\ \lambda$$

$$\begin{aligned} \text{subject to}\ & \lambda \le \mu_{Z_i}(\underline{x}),\ i = 1,2,\ldots,\ k \\ & g_j(\underline{x}) \le b_j \quad j = 1,2,\ldots,\ m \\ & \underline{x} \ge 0\ . \qquad \text{(original constraints)} \end{aligned}$$

Following Zimmermann [28], the membership function of the objective $Z_i(\underline{x})$ is defined as follows :

$$\mu_{Z_i}(\underline{x}) = \begin{cases} 1 & Z_i(\underline{x}) \ge Z_i(x_i^*) \\ \dfrac{Z_i(\underline{x}) - Z_i^{min}}{Z_i(x_i^*) - Z_i^{min}} & Z_i^{min} \le Z_i(\underline{x}) \le Z_i(x_i^*) \\ 0 & Z_i(\underline{x}) \le Z_i^{min} \end{cases}$$

where $\underset{\underline{x}}{\text{Max}}\ Z_i(\underline{x}) = Z_i(x_i^*)$, ($i = 1,2,\ldots,\ k$),

$$Z_i^{min} = \underset{j}{\text{Min}}\ Z_i(x_j^*),$$

and x_i^* is the optimal solution for the objective $Z_i(\underline{x})$ when solved individually as if the other objectives are not there.

This is a MCDM model in maximin sense. This model ensures a minimum degree of satisfaction of each criterion and is known as the competitive model. This reflects equal importance of each criterion. Also the optimal solution obtained by using this operator is an efficient solution. Tanaka et al. [20], Hannan [5,6] have discussed fuzzy mathematical programming based on this operator.

Maximum Operator :

A multiobjective programming problem modelled with max-operator is given by

$$\text{Max}\ \underset{i}{\text{Max}}\ (\mu_{Z_i}(\underline{x})) = \text{Max}\ \delta$$

subject to

$$\delta \geq \mu_{Z_i}(\underline{x}), \qquad i = 1,2,\ldots, k$$

$$g_j(\underline{x}) \leq b_j, \qquad j = 1,2,\ldots, m$$

$$\underline{x} \geq 0.$$

This model is called fully compensatory in the sense that it achieves the full satisfaction of a single goal. The full satisfaction of one goal is considered as compensation for lower degree of satisfaction of the other goals (Yager [22]). This model has rarely been used to solve any MCDM problem because of its stress on the optimization of a single criterion.

Compensatory Operator :

Several operators have been proposed for the aggregation of fuzzy sets. However, they do not seem to be very suitable for modelling the real world problems.

All the operators suggested for the intersection, result in membership grades between zero and minimum and those suggested for the union, between maximum and one, respectively.

Empirical studies of Zimmermann and Zysno [30] on the suitability of various aggregating operators to define an appropriate decision function reveal that subjective aggregation processes in the framework of human decision always show some degree of compensation which does not correspond to the logical connectives 'and' and 'or' (i.e., fully competitive or fully compensatory). Therefore, the authors have suggested a class of hybrid operators called compensatory operator with the help of a suitable parameter of compensation. The formulation of decision function using compensatory operator is given by

$$(1-\gamma)\ \mathrm{Min}\ (\mu_{Z_i}(\underline{x})) + \gamma\ \mathrm{Max}(\mu_{Z_i}(\underline{x})),\ 0 \leq \gamma \leq 1.$$

They have, however, not given any method for determining the value of γ.

Recently Rao et al. [16] have developed a method to determine the value of γ dependening on the inherent compensation in the realistic human decision situations. In this method the personal utility function is suggested for obtaining the risk attitude or compensatory behaviour of the decision maker.