SYNOPSIS

Axisymmetric stress distribution problems occupy an important position in the mathematical theory of elasticity. Such problems occur very frequently in engineering, technology and designing. For example in the designing of pressure vessels a knowledge of stresses under different internal pressures or temperatures is a prerequisite. In the case of isotropic materials Love's biharmonic stress function is generally used to solve such problems while for different types of aeolotropic media various stress function or displacement function approaches may be used. In the present work some torsionless axisymmetric stress distribution problems for transversely isotropic and isotropic media have been solved with the help of stress functions given by Elliott in the former case and Love in the second case. A new stress function approach has been developed to solve corresponding thermoelastic problems for transversely isotropic media.

The work is divided into four Chapters.

The first chapter deals with the problem of stress distribution within transversely isotropic bodies of revolution bounded by one or two cones due to axisymmetric normal or tangential loading and as examples the problems of stress distribution within such bodies due to gravity and due to rotation have also been solved. Similarity solutions have been used to solve the basic differential equations satisfied by the stress functions given by Elliott. This method is applicable only when the external loadings are expressed as an arbitrary real number power of the distance from the common apex of the two bounding concentric cones. The external loadings have therefore been assumed in this form. By various choices of the semi-vertical angles of the two bounding cones stress distributions within various types of bodies can be obtained as particular cases of the above problems such as circular plate of linearly varying thickness and infinite radius, semi-infinite conical shell and semi-infinite solid cone. The case of a solid cone has been discussed in some detail and it is seen that when the external loadings are constant, the stresses are also constant everywhere and do not depend upon the elastic constants of the material. In the case of stress distribution due to rotation about symmetry axis and that due to gravity acting in the direction of symmetry axis which is vertical the boundaries of the cone have been taken to be stress free. In all the above cases the stresses and displacements are expressed in terms of Legendre functions of certain orders depending upon various boundary conditions.

In this chapter the whole surface bounding the material was supposed to be loaded and the similarity method was found applicable but it may be that part of the surface is loaded and the rest is free or any other type of discontinuous or piecewise continuous or mixed boundary conditions may be imposed. In such cases the similarity solutions method connot be applied but a suitable integral transform method has to be used.

In second chapter such an axially-symmetric problem for the transversely isotropic material bounded by one or two cones has been solved with the help of Mellin transform. The piecewise continuous external loadings have been assumed to be Mellin transformable and a system of spherical polar coordinates used. The solution of the complete problem has been presented again in terms of Legendre functions in a formal manner. The special case of a semi-infinite medium loaded normally and uniformly over a circular area around the origin is considered in some detail. For the sake of comparison one of the stress components σ_{zz} is found on the symmetry axis and this result when specialised to isotropic case agrees with a known result. In the case of transverse isotropy it has been seen that this stress component depends upon the elastic constants also whereas such is not the case in isotropy. It has been noted that a single modification from isotropy to transverse isotropy presents great difficulties to the solution of the problems considered.

In third chapter some thermoelastic problems for isotropic material have been tackled. The material is again supposed to be bounded by one or two cones and the special case of semi-infinite medium again draws the main attention partly because of the comparison which can be had with knowm results and partly because of its importance in practice. The solution of the problem depends upon two stress functions and the temperature, all satisfying second order partial differential equations. These differential equations have been transformed to spherical polar coordinates because the case of piecewise continuous external temperature or loadings has only been considered. The Mellin transform has again been applied to present the solution in a formal manner. A few special cases have been considered, for example, the stress distribution due to a constant temperature over a circular area on the bounding surface of the semi-infinite medium and zero temperature outside with the boundary stress-free and also the case of constant normal loading over the circular area with no thermal strains present. In all the above cases results are identified to already known results.

In chapter four a stress function approach to solve the axisymmetric steady state thermoelastic problem for transversely isotropic material in the presence of body forces has been presented. It has been found that all the components of the stress tensor and of the displacement vector are expressible in terms of either of the two pairs of stress functions and certain parameters which satisfy quadratic equations involving elastic constants. After the derivation the use of the method has been illustrated by solving a mixed boundary value problem dealing with semi-infinite medium, viz., the problem of stress distribution within transversely isotropic semi-space, in a steady-state temperature field the bounding plane of which is stress free and is subjected to a constant temperature over a circular area, the exterior of the circle being thermally insulated. It is seen that Hankel transform in suitable to solve this problem. All the stresses and displacements are therefore expressed in terms of integrals involving Bessel functions. These integrals have, in turn, been evaluated in terms of suitably chosen systems of oblate spheroidal coordinates and the stresses, displacements and temperature are therefore expressed in closed forms. In the particular case of Magnesium, which is transversely isotropic, the stresses have been evaluated numerically and the results, presented in tabular and graphical form, have been compared with the corresponding results in the isotropic case and considerable diviations are noted which are up to 12 per cent in some cases. Magnesium itself is not a very strongly transversely isotropic material; that is why the deviation is not very high but, nevertheless, it is quite considerable. Other strongly transversely isotropic materials, such as zinc, are expected to reveal a very appreciable deviation from the isotropic case, as regards stresses, displacements and temperature etc.