CHAPTER 1

GENERAL INTRODUCTION

A method has been developed in this thesis for solving boundary value ~~of~~ problems of classical elasto-statics. This method essentially hinges on Somigliana integral [1885], which expresses the displacement in an elastic region in terms of tractions and displacements along the boundary of the region. Somigliana integral, known also as Betti's second identity, is indeed vector counterpart of Green's third identity of potential theory. Though this integral was derived as early as 1885, but its exploitation in solving boundary value problems started only during the last decade.

As mentioned above one may compute displacement field from the Somigliana integral if the boundary tractions and displacem ents are known. Unfortunately this is not the case in practice. In the boundary value problems of elasticity either boundary tractions (first boundary value problem~~s~~) or boundary displacements (second boundary value problem) or tractions along a part of the boundary and displacement on the remaining part (mixed boundary value problem) are known.

This appears to be the main reason for which people did not accept this integral as a tool for tackling the boundary value problems.

Furthermore, this integral when formulated in terms of boundary data reduces to integral equations, e.g. if the boundary tractions are known the integral reduces to integral equations in displacements. But the kernels in these integral equations are either singular or weakly singular, thereby creating difficulties even in obtaining numerical solutions. Lastly there were no existence and uniqueness theorems in connection with the solutions of these integral equations.

The difficulties mentioned in the last paragraph were overcome after the simultaneous publication of two works, one in the analytical side by Kupradze [1963] establishing existence and uniqueness theorems and the other in the numerical side by Jaswon and Ponter [1963] deriving the techniques of handling the singular kernels numerically. Jaswon and Ponter established the Green's boundary formula from the Green's third identity and then showed how the boundary value problems of potential theory and elasticity can be tackled with the help of this formula.

In his doctoral dissertation Rizzo [1964] first established the two-dimensional boundary Somigliana integral similar to Green's boundary formula following Jaswon and

Ponter and indicated that two-dimensional boundary value problems may be effectively solved with the help of his Somigliana boundary integral. Later he and his associates published a number of papers in this connection, see Rizzo [1967], Rizzo and Cruse [1968], Rizzo and Shippy [1968], Van Buren [1968], Cruse [1969] and Rizzo and Shippy [1970].

During the period under review some works have appeared in literature in the analytical side also. Mention may be made of the works of Williams [1966a, 1966b, 1966c], Kanwal [1969], Lawrence [1969, 1970] and Maiti and Makan [1972, 1973].

Though significant progress has been made with the Somigliana integral, ~~but~~ no boundary value problems of half-space have been solved so far with this integral. The main reason for this is that Somigliana integral is not known for a half-space; it is known only for a finite region with a closed boundary and an infinite region with an internal boundary.

We have established here for the first time the Somigliana integral for a half-space. Then with the help of the procedures of Maiti and Makan [1973] we have established the two dimensional 'modified Somigliana integral' for general regions and then for half-spaces, which expresses the displacement field in a region in terms of tractions and displacements along the boundary of the region and its complement. Indeed the modified Somigliana integral expresses that for a non-trivial displacement field there must be discontinuity either in traction or in displacement or in both across the boundary. This formulation is physically illuminating as it indicates that a displacement field inside a region is due to either point forces or dislocations or both distributed along the boundary, a fact so far accepted heuristically in solving boundary value problems of elasticity (admitted in private communication by two international authorities, Professors J.D. Eshelby and R.W. Lardner). Thus the modified Somigliana integral provides a mathematical justification for the formulations of boundary value problems in terms of point forces and alternatively in terms of dislocations. Another important feature of this formulation is that it provides analytical solutions in many cases, whereas the Somigliana integral itself fails to do so.

In the present work we have assumed the continuity in traction across the common boundary of the region and its complement or the common boundary to be equilibriated. This is exactly the case of a displacement field in a region in terms of dislocation layers along the boundary of the region. In dealing with a problem we have always considered an auxiliary stress field of the complementary region so that these two together gives the equilibriated common boundary.

Though our method is sufficiently general to cope with a wide variety of boundary value problems, we have restricted ourselves in the present thesis to the plane problems of isotropic elasticity.

The thesis contains nine chapters including the present one which gives the brief account of the present work along with other works in this connection.

In Chapter 2 we derive two basic results, one is the Navier-Cauchy equations and the other is complex variable representations of displacements in half-spaces.

Chapter 3 gives the Betti-Somigliana solutions of Navier-Cauchy equations.

Chapter 4 deals with modified Somigliana integrals for general regions and half-spaces as well.

Chapter 5 deals with a problem of a half-plane when the normal displacement and normal traction are specified along the boundary. The stresses and displacements have been computed in the upper half-plane.

Chapter 6 deals with a problem of a half-plane when the tangential displacement and shear traction are specified. The stresses and displacements have been computed in the upper half-plane.

Chapter 7 deals with a contact problem of a half-plane in the absence of friction.

Chapters 8 and 9 deal with crack problems under unsymmetrical loadings.

Thus the present approach has been effectively exploited in a variety of problems establishing its right as a method for solving two dimensional boundary value problems. We have chosen those problems which have been solved from dislocation considerations only to provide a mathematical justification of Lardner's approach [1972] to the formulations of two dimensional problems in terms of dislocation layers. We may finally add that the works reported in Chapters 3-8 have been accepted for publication in Journal of Elasticity.