ABSTRACT

A number of graph colorings have their roots in communication problem known as *Channel Assignment Problem.* Radio k-colorings of graphs is one of them. For a simple connected graph G with diameter q, and an integer k, $1 \leqslant k \leqslant q$, a radio k-coloring of G is an assignment f of non-negative integers to the vertices of G such that $|f(u) - f(v)| \geqslant k + 1 - d(u, v)$ for each pair of distinct vertices u and v of G, where $d(u, v)$ is the distance between u and v in G. The *span* of a radio k-coloring f, $rc_k(f)$, is the maximum integer assigned by it to some vertex of G. The *radio k-chromatic number*, $rc_k(G)$ of G is $\min\{rc_k(f)\}$, where the minimum is taken over all possible radio k-colorings f of G. For $k = q$ and $k = q - 1$ the radio k-chromatic number of G is termed as the radio number ($rn(G)$) and antipodal number ($ac(G)$) of G respectively. Radio k-chromatic number is known for very limited families of graphs and specific values of k.

For an n-vertex graph G, we give a lower bound for $rc_k(G)$ depending on a parameter based on a Hamiltonian path in a specific weighted complete graph K_n. Using this we obtain lower bounds of $rc_k(C_n)$, $rc_k(P_m \Box P_n)$, $rc_k(C_m \Box P_n)$ and $rc_k(K_m \Box C_n)$. We obtain an upper bound of $rc_k(G)$ by giving a coloring scheme that works for general graph and depends on the partition of the vertices into two sets satisfying some conditions. Also, we illustrate this coloring scheme for n-dimensional hypercube Q_n and give an upper bound of $rc_k(Q_n)$ which is an improvement of the one known before. We investigate the radio k-chromatic number of power of cycles (C_n^r), Toroidal Grids $T_{m,n}$, hypercubes Q_n and some other classes of graphs. We give an upper bound of $rc_k(T_{m,n})$ when $mn \equiv 0 \pmod 2$ and a lower bound of the same for all values of m and n. From these bounds we determine the radio number $rn(T_{m,n})$ when $mn \equiv 0 \pmod 2$ and antipodal number $ac(T_{m,n})$ for some values of m and n. We obtain a lower and an upper bound of radio k-chromatic number of any power of cycles and we show that these bounds coincides with $rn(C_n^r)$, $ac(C_n^r)$ and $ac'(C_n^r)$ (nearly antipodal number of C_n^r) for some values of n and r. We give an upper and a

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lower bound of $rn(G^2)$ in terms of $rn(G)$ and $rc_{q+1}(G)$, where q is the diameter of G. These bounds are $\left\lceil \frac{rn(G)}{2} \right\rceil \leqslant rn(G^2) \leqslant \left\lfloor \frac{rn(G)+n-1}{2} \right\rfloor$ for even diameter q, and $\left\lceil \frac{rc_{q+1}(G)}{2} \right\rceil \leqslant rn(G^2) \leqslant \left\lfloor \frac{rc_{q+1}(G)+n-1}{2} \right\rfloor$ for odd diameter q. Also we determine the radio number for square of some class of graphs like hypercube, $C_m \square C_n$ when $mn \equiv 0 \pmod 2$, generalized prism graphs etc.. Further, we give an algorithm which generates a radio coloring of G^2 from that of G. For an n-vertex simple connected graph the time complexity of this algorithm is $O(n^3)$. Since finding radio k-chromatic number is NP-hard, we give algorithms for finding lower and upper bound of $rc_k(G)$. For an n-vertex simple connected graph the running time of the lower bound algorithm is $O(n^3)$ and that of the upper bound algorithm is $O(n^4)$. Applying these algorithms we get the exact values or bounds (lower and upper bound) for radio k-chromatic number of some graphs like $C_m \square P_2$, Circulant graph, $C_m \square K_n$ and $C_m \square C_n$.

Keywords : Channel assignment problem; Radio k-coloring; Span; Nearly antipodal coloring; Radio k-chromatic number; Radio number; Antipodal number; Nearly antipodal number; l-distance perfect matching; Power graph; Toroidal grid; Circulant graphs; generalized prism graph;

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