

Abstract

In several branches of analysis, for instance, the structural theory of topological vector spaces, Schauder basis theory, Summability theory and the theory of functions, the study of sequence spaces occupies a prominent position. The development of scalar and vector valued sequence spaces has been taken place in various ways by earlier authors namely, Simons (1965), Maddox (1967), Nakano (1953), Ruckle (1973), Connor (1989), Kizmaz (1981), Et and Colak (1995) and others.

Later on, the development of the study of sequence spaces got momentum after the introduction of new convergence methods and theories in the process, as well as a requirement. Some of them are statistical convergence, λ -statistical convergence, Lacunary statistical convergence etc. After introducing the concept of statistical convergence by Fast and Schoenberg independently, Fridy (1985), Salat (1980), Connor (1989) and others generalized and developed this concept further.

The concept of fuzzy sets and fuzzy set operations are first introduced by Zadeh. Later on, Matloka (1986), Nanda (1989), Nuray and Savas (1995), Mursaleen and Basarir (2003), Savas (2000), Talo and Basar (2010), Ayhan and Mehmet (2011) and several others studied the sequence spaces by using fuzzy numbers in an analogous way as Simons (1965), Maddox (1967), Kizmaz (1981) and others studied for scalar valued sequence spaces. Similarly Savas (2000), Aytar and Pehlivan (2006), Bilgin (2003), Colak and Mursaleen (2011), Kwon and Shim (2001) and several others studied the concept of statistical convergence and lacunary statistical convergence of sequences of fuzzy numbers in an analogous way as Fridy (1985), Salat (1980), Freedman et al. (1978) and others studied for scalar valued sequence spaces.

The present work is aimed to introduce and study new scalar and normed valued sequence spaces of fuzzy numbers by using modulus functions with suitable topologies.

The thesis consists of seven chapters including introductory chapter.

Chapter 1 consists of two parts. Part A deals with a brief survey of the work done by earlier workers on scalar valued sequence spaces including the results on fuzzy statistical convergence, fuzzy lacunary statistical convergence, fuzzy statistically pre-Cauchy sequences etc. Part B of this Chapter deals with definitions, lemmas and known results, which are used in the sequel. In Chapter 2, the scalar valued sequence space $\mathcal{N}^F[A, f, p]$ using infinite matrices $A = (a_{nk})_{n,k=1,2,3,\dots}$ and modulus function f is introduced. For suitable choices of \mathcal{N} , f , A and (p_k) , the sequence space $\mathcal{N}^F[A, f, p]$ includes many of the known scalar valued sequence spaces of fuzzy numbers. Chapter 3 deals with the scalar valued sequence space $\mathcal{N}_p^F[\mathcal{F}, \Delta^m, s]$ using m^{th} order difference operator Δ^m and modulus function f . For suitable choices of \mathcal{N}_p , f , s , m and (p_k) , the sequence space $\mathcal{N}_p^F[\mathcal{F}, \Delta^m, s]$ includes many of the known scalar valued sequence spaces of fuzzy numbers. In Chapter 4, we study the concept of Δ^m -summable sequence of fuzzy numbers by using a modulus function and Δ^m -statistical convergence of sequences of fuzzy numbers. We have also studied the relation between Δ^m -statistical convergence and Δ^m -statistical pre-Cauchy sequences of fuzzy numbers. In Chapter 5, we have studied the space of lacunary strongly Δ_p^m -summable sequences of fuzzy numbers and discussed the relationship between Δ^m -statistically convergent sequence and lacunary Δ^m -statistically convergent sequences of fuzzy numbers. We have also studied inclusion relations using different lacunary sequences. In Chapter 6, we have introduced a Banach valued sequence space $\mathcal{N}_F[E_k]$. This is done by combining a normal sequence space \mathcal{N}_F of fuzzy numbers and a sequence of Banach spaces E_k , $k = 1, 2, \dots$ of fuzzy numbers over \mathbb{R} . It is shown that $\mathcal{N}_F[E_k]$ is a complete paranormed space under a suitable topology. Various algebraic and topological properties for this space are also studied.