Abstract

In the past few years, most of the research work which has appeared in the literature on single server queue with Markovian arrival processes, i.e., MAP/G/1-type queues, depend heavily on matrix analytic method. But, there are real problems in applying these methods, especially when getting numerical results are concerned. There are constant efforts being made to develop an alternate methodology to solve such type of queues.

In this thesis, an unified methodology is presented for computing system-length and queueing/system-time distributions of MAP/G/1-type queues. It has been demonstrated that inversion of transforms using roots can be exploited to get the probability distribution of interest for queueing models with Markovian arrival processes. The proposed methodology is amenable to computation with ease and is alternative to the matrix analytic and spectral methods.

The thesis consists of six main chapters and a concluding chapter. Chapter 1 is the introductory chapter where a brief literature survey has been presented.

In Chapter 2, we consider MAP/R/1 queue, where R represents the subclass of general service-time distribution (G) whose Laplace-Stieltjes transforms are rational functions. In matrix analytic method, finding the unknown boundary vector is the fundamental problem. We have accomplished this task using the roots inside and on the unit circle of the characteristic equations of MAP/R/1 queue. Then, using the roots outside the unit circle we obtain the system-length distribution at various epochs. Again using the roots with the negative real parts of another characteristic equation, we evaluate virtual and actual queueing-time distribution. To accommodate the batch arrivals, we consider BMAP/R/1 queue in Chapter 3 and obtain the system-length distribution at various epochs, virtual queueing (system)-time distribution, queueing (system)-time distribution of first (arbitrary) customer in a batch. In Chapter 4, we consider MAP/R/1 queue with bulk service and present a simple closed-form analysis in terms of roots for evaluating queue-length distribution. In all the cases numerical results are presented for matrix-exponential (ME) service-time distribution which includes phase-type (PH) distribution.

Chapter 5 deals with MAP/D/1 queue, where service-time of each customer is constant. We evaluate the unknown boundary vector using the roots of the characteristic equation of MAP/D/1 queue with non-negative real parts. Then, by approximating the Laplace-Stieltjes transform of the service-time distribution by a rational function, using Padé approximation, we obtain the virtual queueing-time distribution using the roots with negative real parts. We also obtain system-length distribution by making use of the distributional Little's law.

In Chapter 6, we consider two models, M/D$_N$/1 and MAP/D$_N$/1 queue, where

service-time for each customer is constant and takes one of the values x_i with probability p_i $(i = 1, 2, ..., N)$. We develop necessary methodology to calculate queueing (system) time distribution of $M/D_N/1$ and $MAP/D_N/1$ queues.

Keywords: Batch Markovian arrival process, Distributional Little's law, Markovian arrival process, Matrix analytic method, Matrix-exponential distribution, Phase-type distribution, Queue-length, Queueing-time, Rational Laplace-Stieltjes transform, Roots, System-length, System-time.