Abstract

This doctoral thesis presents a cohesive study on the estimation of two- and three-dimensional curvatures within the realm of digital geometry. While there are established mathematical analyses and formulas for curvature in continuous space, translating these concepts into the discrete or digital space poses significant challenges. This translation leads to the question of 'estimation of discrete curvature', which invariably involves some degree of 'error'. Consequently, this necessitates developing practical solutions tailored for the discrete or digital domain to address these inherent inaccuracies.

In recent decades, numerous works have focused on discrete curvature estimation in both 2D and 3D domains. However, the majority of existing techniques estimate curvature based on real points, such as discrete points on real curves, 3D point clouds, or vertices of triangle meshes. Few methods address curvature estimation at the pixel level for digital curves or voxel level for digital surfaces. The transition from real points to digital points presents challenges, as existing techniques often rely on computations involving floating-point operations. Due to this, they are not readily applicable to the digital domain without introducing errors. We aim to avoid such complexities and focus solely on integer operations. Additionally, digitization introduces constant jaggedness to curves and surfaces, further complicating curvature estimation in the digital domain. To counter this, we introduce the notion of k-neighborhood. In particular, we estimate the curvature at a pixel or a voxel by searching for a local neighborhood around it. We also recognize that pixels and voxels possess simple set-theoretic structures and lend themselves well to quick topological analysis, making them highly suitable for parallel computing environments. Yet, this potential remains underutilized in current practices. To account for this, all our contributions in the 3D domain are GPU-based algorithms.

We present a series of contributions in this thesis to bridge the above-mentioned existing gaps. Firstly, we introduce a novel technique for determining the inflection points in two-dimensional digital curves, combining discrete geometry with elementary concepts of word theory. Although it does not directly involve curvature, we demonstrate its use for efficient vectorization of digital curves using cubic B-splines. Building upon this foundation, we next present our

2D curvature estimation technique tailored for digital curves. This technique serves as the basis for proposing a curvature-adaptive control-point selection technique which expedites vectorization of digital curves. We then extend our 2D curvature estimation technique to a GPU-based version for 3D discrete curvature estimation on voxelized surfaces. This extension facilitates low-poly approximation of triangle meshes, where Centroidal Voronoi tessellation is used for remeshing purposes. Here, the curvature map is utilized to place seeds corresponding to each Voronoi cell in a curvature-adaptive manner, i.e., low-density seeds in low-curvature regions and high-density seeds in high-curvature regions. We also introduce a novel concept of mesh simplification based on discrete curvature in the voxel space. Here, edge lengths of triangles in the output mesh are either provided as input or computed automatically based on a specified Hausdorff error-bound. We present sufficient experiments to support our contributions and provide relevant findings and discussions afterward. We conclude this thesis with a summary of these findings and provide some noteworthy avenues for further exploration.