

# Abstract

In recent years, dipolar quantum gas has attracted significant attention and broadened the scope of potential research directions in the field of ultracold quantum gases, featuring both long-range anisotropic dipolar and short-range contact interactions. In this doctoral thesis, we perform extensive numerical and theoretical investigation of a single-component and binary dipolar BEC.

Firstly, we examined the ground-state phase diagram and the non-equilibrium dynamics of a harmonically trapped  $^{164}\text{Dy}$  condensate under the influence of a rapidly rotating arbitrary angle polarized magnetic field. The polarizing angle of the magnetic field determines both the magnitude and sign of the dipolar interaction. Our investigations were based on the extended Gross-Pitaevskii equation, including quantum fluctuations to the first order. This allowed us to study the emergent phases from the delicate interplay of isotropic short-range and anisotropic long-range interactions in the quasi-1D and quasi-2D trapped geometries. The crossover from the superfluid to the supersolid phase and to single and droplet arrays is probed with particle number and dipolar interaction.

Then, we explore how misalignment between the rotation axis and the polarization direction of a magnetic field affects vortex nucleation in slowly rotating superfluid and supersolid states. Interestingly, the critical rotation frequency of vortex nucleation differs between superfluid and supersolid states and varies uniquely with the polarization direction. Moreover, we find that vortices in both superfluid and supersolid states are topologically robust against changes in the  $s$ -wave scattering length and polarizing angle. Utilizing these properties, we demonstrate dynamic protocols for the nucleation of vortices even when rotating an axially symmetric trap below its critical rotation frequency in both superfluid and supersolid phases.

Next, motivated by the recent experiments on binary dipolar BEC, we study ground-state phases and dynamics of a Dy-Dy mixture. We unravel the ground-state phase diagrams and characterize their different possible phases. The emergent phases include single droplet (SD), multiple droplets (MD), doubly supersolid (SS), and superfluid (SF) states in both miscible and immiscible phases. Intriguing mixed ground-states are observed for an imbalanced binary mixture, including a combination of SS-SF, SS-MD, and SS-SS phases. We also explore the dynamics across the phase boundaries by linear quenches of inter-species and intra-species scattering lengths. During these dynamical processes, we observed an abrupt change in phase which initially results in some instability in the system and form some metastable states in the intermediate time scale. However, the state produced in long-time evolution is similar to our predicted ground-state.

Finally, we explore the possibility of forming various intriguing ground-state phases in the recently realized dipolar mixture of Dy and Er atoms. The experimentally measured value of intra-species  $s$ -wave scattering length of  $^{166}\text{Er}$  condensate in a  $^{164}\text{Dy}$ - $^{166}\text{Er}$  mixture is larger than its intra-species dipolar length implies that the  $^{166}\text{Er}$  condensate itself will not be in a regime of dominated dipole-dipole interaction (DDI). However, we find that the presence of  $^{164}\text{Dy}$  atoms with high magnetic moment induces droplet nucleation and supersolidity in  $^{166}\text{Er}$  condensate via the long-range and anisotropic inter-species DDI. Remarkably, we find that the imbalance in the magnetic dipole moment combined with its strong anisotropic coupling led to the emergence of unique ground-state phases. The emerging phases include doubly superfluid states, a mixture of insulating droplets and supersolid states, binary supersolids with uniform and alternating domains and a combination of supersolid-superfluid mixed states. We also explore the dynamical evolution across these phase boundaries via a linear quench of inter-species scattering length.

**Keywords:** Dipolar Bose-Einstein condensate, Superfluid, Supersolid, Droplet, Vortex, Binary dipolar Bose-Einstein condensate