

Abstract

Dissipative solitons (DSs) are stable, self-sustained, pulse-like structures of light forming inside a nonlinear dissipative system due to the delicate balance between the medium's nonlinear gain and loss and dispersion or diffraction. In contrast to conventional solitons in Hamiltonian systems, DSs typically exist in nonintegrable systems as isolated fixed-point solutions in the parameter space. Based on the localization dimensions, the large family of DSs is broadly categorized into three subclasses: spatial, temporal, and spatiotemporal. In this thesis, we take a step towards unraveling the fundamental aspects of the nonlinear dynamics of spatial, temporal, and spatiotemporal DSs in a graphene-based active random metamaterial (ARM) composed of randomly dispersed graphene nanoflakes (GNFs) embedded within an externally pumped gain medium. The SDS dynamics is modeled through a generalized nonlinear Schrödinger equation (NLSE). First, we investigate the dynamics of spatial dissipative soliton (SDS) inside the graphene-based ARM by solving the NLSE numerically. We observe that graphene-saturable nonlinearity produces a subcritical bifurcation of nonlinear modes, enabling self-organization of the emitted radiation into several dissipative soliton structures with distinct topological charges. We systematically investigate the existence domains of such nonlinear waves and their spatial dynamics, finding that soliton vortices are unstable, thus enabling self-organization into single dissipative structures with vanishing topological charge independently of the shape of the graphene nanoflakes. Next, we extend our study to temporal dissipative solitons (TDSs) dynamics. We theoretically demonstrate the existence of robust temporal dissipative soliton molecules (TDSMs) in a graphene-based active disordered metamaterial. We observe that localized pulses with single and multiple peaks can coexist inside this metamaterial for a wide range of gain parameter, exhibiting optical bistability. We further investigate the stability of such bound states under perturbation by analyzing their interaction force, finding stable tri-soliton molecule states with a periodically oscillating relative phase and a stable two-soliton molecule. Adopting the perturbative variational technique, we obtain

the analytical expression for the evolution of the TDS under a shock-like perturbation. Finally, we explore the emerging field of spatiotemporal solitons,, also known as dissipative light bullets (LBs) and spatiotemporal optical vortices (STOVs). By numerically solving the governing NLSE, we establish their domain of existence and interaction dynamics. We examine the stability of LBs and STOVs by adopting a semi-analytical linear stability analysis. Our results offer a paradigm for mode-locking in active random metamaterials and pave the way for versatile applications such as multilevel encoding in optical communications, future designs of low-cost cavity-free lasers, and optical amplifiers.

Keywords: Dissipative Soliton; Optical vortex; Soliton Molecule; Light Bullet.