Abstract

Spatial discrete solitons (DSs) are stable self-localized beam-like structures in nonlinear waveguide arrays (WAs) formed through the balance between discrete diffraction and nonlinear self-focusing. These solitary waves show unique properties distinct from waves in homogeneous media and allow for the incorporation of different phenomena due to the underlying system in which they are sustained. We begin by introducing a linear modulation in the inter-waveguide separation in the WA and demonstrate the self-acceleration of DSs accompanied by the formation of a dynamically evolving diffractive resonant radiation, a spatial analog of phase-matched dispersive wave generation by a temporal soliton. Based on the evolution behavior of the soliton in space and wavenumber, we propose the formation of an effective linear potential arising out of the linear modulation. Adopting the perturbative variational technique under a linear potential approximation, we obtain the analytical expression for the evolution of the soliton, which allows us to determine the relationship between the effective potential and the linear chirp strengths. Furthermore, these results allow us to establish a phase-matching relationship that effectively predicts the evolution of the dynamic diffractive radiation while considering the shift in the soliton wavenumber. This modified phase-matching further establishes the formation of dual radiations under the effect of this WA modulation. For symmetrically linear chirped waveguide array we also observe an unique oscillatory dynamics of a DS. Next, we extended our study to the interaction of two DSs inside a WA. In the case of identical solitons, the interaction between them results in an elastic collision or formation of a breather depending on the initial soliton parameters. They even form a stable, bound state. We map the interaction domain by proposing a phase diagram. The complex collision dynamics between two DSs are investigated analytically by exploring the variational technique. It is also found that two-soliton collision leads to additional radiation in the Fourier domain. Finally, in this thesis, we explore the emerging field of $\mathcal{PT}$-symmetric waveguide arrays in the context of the generation of diffractive radiation. A $\mathcal{PT}$-symmetric WA is realized by alternating gain-loss waveguide channels with a neutral waveguide as a defect at the center. We examine the dynamics of a DS in unbroken and broken $\mathcal{PT}$ regime for normal incidence. With the introduction of a phase gradient, a DS emits standard diffractive radiation, accompanied by another as a result of the interaction with the $\mathcal{PT}$ elements. We establish the phase matching equation that predicts the emission of a $\mathcal{PT}$-symmetry-assisted diffractive radiation. The results of this thesis should be useful for future applications and experiments.

Keywords: Waveguide Arrays; Discrete Nolinear Schrödinger equation; Discrete Spatial Soliton; Phase-Matched Radiation; Diffractive Resonant Radiation; Two Soliton Interaction; $\mathcal{PT}$-Symmetric Waveguide Array.