ABSTRACT

In the presence of a strong magnetic field at very low temperatures, confining the motion of electrons in two dimensions, the occurrence of Hall resistance at quantized values has emerged as a crowning achievement in the realm of condensed matter systems. When the quantization occurs at fractional multiplets, this gives rise to the fractional quantum Hall effect (FQHE). Prediction of Majorana particles for the half-filled state in the second Landau level (SLL) has driven active research in this field, as non-Abelian anyon excitations like Majorana particles find a profound application in fault-tolerant topological quantum computation. Besides this enigmatic half-filled FQHE state, several other states have been experimentally observed in the SLL, and many theories have been proposed predicting the Abelian and non-Abelian nature of vivid states in the SLL, but their experimental relevances are not yet confirmed. Moreover, there is no theory so far that unifies all the observed states in the SLL. Unlike the lowest Landau level, even the composite fermion series is not seen in the SLL.

The states in the SLL are observed at a comparatively lower magnetic field, $B \sim 5$ Tesla for typical GaAs-AlGaAs heterostructures, which amounts to Landau level mixing strength, $\kappa \sim 1$. However, earlier numerical studies for proposed model wavefunctions are performed for a very small range of $\kappa \lesssim 1$. Incorporating the Landau level mixing effect at a moderate strength, we find a reentrant quantized anomalous phase ($\mathcal{A}$ phase) for all the experimentally observed states in the SLL in GaAs systems. We, thereby, report that the generically found $\mathcal{A}$ phase belongs to the spin-polarized filling factor sequence, $\nu = n/(nm-1)$ and it's particle-hole conjugate sequence, $1 - n/(nm-1)$, $(n \geqslant 1, m \geqslant 3)$. Thereafter, we propose a generic trial wavefunction for all the states falling in the above sequence having remarkably high overlap with the corresponding exact ground state in the $\mathcal{A}$ phase. These wavefunctions are argued to support non-Abelian quasiparticle excitation with charge $e/\left[2(nm-1)\right]$. For all the FQHE states ν, we predict experimentally verifiable unique thermal Hall conductance 2.5 $\pi^2 k_B^2 T/3h$ governed by two bosonic downstream edge modes for two

filled Landau levels and one downstream Majorana edge mode. Additionally, for the particle-hole conjugate sequence, i.e. $1-\nu$, the same thermal Hall conductance is predicted but followed by three bosonic downstream edge modes for three filled Landau levels and one upstream Majorana edge mode. Following our proposed series, we predict a few more FQHE states in the SLL, which, if experimentally found, will bolster our theory even further. We believe the $\mathcal{A}$ phase and the corresponding proposed wavefunction will possibly corroborate with the experimental observation.

The observation of $\nu = 6/13$ in the SLL is quite surprising as no state has been observed with lower numerator 4 or 5. This 6/13 state, found to be exceptional in our earlier prescribed series, is rather mentioned to be an intermittent topological state as found in between two prominent states at $\nu = 1/2$ and $\nu = 2/5$ of our proposed series. Furthermore, in between 2/5 and 3/8 states, another intermittent FQHE state is predicted at $\nu = 5/13$. Our proposed wavefunctions for both states have excellent overlap with their exact ground state in the $\mathcal{A}$ phase, and from these wavefunctions, we also extract experimentally verifiable topological properties.

Key words: Fractional quantum Hall effect, Second Landau level, Landau-level-mixing, Anomalous phase, Non-Abelian anyon, Majorana particles, Edge modes, Thermal Hall conductance.