Chapter 1

Introduction

1.1. The needs of Industrial Maintenance

Maintenance of assets is one of the most important activities in a continuous process industry to achieve the desired performance levels in terms of reliability, availability, safety and productivity. Having procured and installed the machinery, this remains the only option left to preserve and sustain the assets in a healthy condition. Therefore, various approaches for improving the effectiveness of maintenance of assets have evolved over the years. These include various maintenance philosophies, development and application of technology and IT based maintenance techniques. The key to effective deployment of these approaches, lies in a better understanding of the failure patterns, management of spare parts, allocation of human and other resources, and overall planning and controlling of the intended maintenance function. The development of mathematical models for maintained systems helps in carrying out the above functions in an objective manner.

The industrial environment today is much more complex than ever before, with large sophisticated and costly systems, that are required to be operated round the clock with as little unscheduled disruptions as possible. The consequences of breakdown/failure can be a simple maintenance loss to a loss of production with or without safety and environmental consequences causing ultimately a loss to the society where they operate.

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In order to effectively and efficiently operate such systems large maintenance organizations are employed either, in-house, off-loaded or mixed along with large budgets and resources. Typical costs of maintenance work out to approximately ten percent of the total expenditure of any large industrial organisation. If the maintenance activity is optimized it would enhance the organizations ability to be competitive and meet its stated objectives with as little loss as possible.

Managing maintenance in any organisation starts with elucidating and articulating the needs of the maintenance engineer / manager. The broader needs of maintenance are closely linked to the business goals of the organisation. The broad strategy for maintenance is framed based on these business goals. First the mission and vision of maintenance are decided and the broad objectives of maintenance are fixed. Next the policy / techniques / tasks / practices adopted for maintaining assets to achieve the above objectives are finalised.

These encompass activities such as scheduling of maintenance work, deployment of resources, planning for spares, creating and using maintenance systems employing the concepts of preventive maintenance, condition based maintenance and predictive maintenance. The goal of the maintenance engineers / managers is to carry out these activities in a productive and cost effective manner.

The greatest fear of the maintenance engineers / managers, in managing their twin goals, is the incidence of system breakdown or failure and their greatest challenge is to improve the system reliability. The questions faced by them in tackling these activities start with their eternal bugbear failure and revolve around the measures needed to protect the systems from its occurrence. Questions asked are;

1. What is the frequency of failure?
2. What is the trend of failures?
3. When will the next failure occur?

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4. How many failures will occur in the following specified time period?
5. What are the factors responsible for the failures?
6. Which factor is prominent in causing failure and needs to be tackled?
7. What are the consequences of the failures?
8. What are the measures to be taken to tackle these failures?
9. After taking these measures has the failure trend changed?
10. What should the maintenance policy be?
11. When to go for modifications or design changes?
12. How to estimate the effects of modifications / design changes?
13. How to distinguish the effects of various operating environments?
14. How to operate equipment to give the maximum reliability under the given conditions?
15. How many spares are to be kept?
16. How to decide on the resources for maintenance?
17. When to go for total replacement?
18. How to optimise maintenance decisions based on different criteria?
19. How to judge the effectiveness of the maintenance policy?
20. How to assess its contribution to business goals?

Finding answers to these questions will help them in arriving at suitable decisions as to what needs to be done to perform appropriate maintenance of their systems.

Today, the most common mode of assessing the information and making decisions based on them is by applying common sense to their engineering knowledge to arrive at heuristic decisions. This, more often than not, leads to

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incorrect decisions that detract heavily from the effectiveness and efficiency required in meeting their goals.

If the maintenance engineers / managers have at their beckoning tools that could provide them a means of objectively, scientifically and repeatedly arrive at correct decisions then their onerous burden would be lightened and their responsibility, in reducing the overall business risk, and enhancing their organisation's capability to be competitive and successful well met. The need of the hour is to develop such tools, to represent the concepts of industrial maintenance, to help in meeting their needs completely. This is achieved through correct mathematical modelling of the maintenance activities.

1.2. Mathematical modelling of maintenance using Point Processes

The occurrence of failure or breakdown of a system is a negation of all the efforts of maintenance. A proper and better understanding of the failure process will lead to improved maintenance practices for reducing the incidence of failure. Hence, a study of the maintenance activities connected with any system begins with a study of its failure process.

Two approaches have been employed to model and understand the failure process of any system. The first is the Physics of failure approach or the white box approach, where the mechanism of failures is modelled based on the established theories of component failures. The second is the Empirical approach or the black box approach, where the mechanism of failure is not properly understood and the failure process is modelled based on the failure data. In the Empirical approach, the failure process of a maintained system has been traditionally modelled with stochastic processes called point processes. Point process models, generally used for maintained systems, are reviewed below.

As the time of occurrence of any failure of the system is uncertain, it's time to failure from the time at which a system is put into operation is represented mathematically as a random variable.

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The system is put back into operation on maintenance and continues to function till its next failure. Thus, the failure process of a maintained system consists of a series of failures occurring sequentially at different points in time. This increasing sequence of times to failures can be represented mathematically by a sequence of random variables. A collection, family or sequence of random variables, forms a random (uncertain) or stochastic process.

The sequence of failures represented by the times to failure of a system treated as a sequence of point events (the failure events) occurring at specified points in time lead to their modelling by stochastic processes called Point processes. A point process is, roughly speaking, a countable random collection of points on the real line.

Let $T_1, T_2, T_3, \ldots, T_n$ be the times at which failure events occur on the time axis. Each of these times to failure is represented mathematically by a random variable, and the collection of these random variables, forms a point process.

Let $N(t)$ be the number of points/events in $[0,t]$ where the real line is the time axis. The process N can be said to count the events of the point process, and hence is called a (univariate) counting process. In the case of a failure process the counting process $N(t)$ counts the failure events.

Both the processes are equivalent to each other and are governed by the relation;

$$Pr\,(T_n \leq t) \equiv Pr\,(N(t) \geq n) \tag{1.1}$$

A study of the failure process can be made by studying either of the stochastic processes, the point process $T_1, T_2, T_3, \ldots, T_n$ or the counting process $N_1, N_2, N_3, \ldots, N_n$.

The stochastic process called the intensity process of the counting process can be used to study the points/events $N(t)$.

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Let $\lambda\,(t \mid H_{t^-})$ = Conditional event intensity or conditional rate of occurrence of events

$$= \lim_{dt \to 0} Pr\,(\,An\ Event\ occurs\ in\,[t, t+dt) \mid H_{t^-}\,)\,/\,dt$$

$$= \lim_{dt \to 0} Pr\,(\,N(t+dt)^{-}\ ) - N(t)^{-}\,) = 1 \mid H_{t^-}\ /\ dt$$

$$= \lim_{dt \to 0} Pr\,(dN(t) = 1 \mid H_{t^-}\,)\,/\,dt \qquad (1.2)$$

where H_{t^-} represents the available data just prior to time t or the history of the process, the collection of all events in this case, the failures observed on *[0,t]* for an orderly point processes i.e., a point process for which no simultaneous failures occur at any given instant of time.

Since the development in time of a counting process *N(t)* is governed by its intensity process, we can specify a counting process model for failure history data by giving a specification for the intensity processes. Since the study of the counting process *N(t)* is equivalent to a study of the point process T_n, the intensity process of a counting process can be used to study the point process of failure events T_n.

The conditional intensity process $\lambda\,(t \mid H_{t^-})$ provides a general framework for the modelling of the failure event processes of a maintained system.

The approach involves the specification of a baseline intensity function $\lambda(t)$, which we can think of as being the hazard function of the time to first failure. This function will then be used in describing the various models by means of their conditional intensity function $\lambda\,(t \mid H_{t^-})$

1.2.1. Maximal Repair

Systems which undergo major repairs and after repair the system can be termed as a good as a new system are modelled by means of renewal processes. For these

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processes, the failure intensity starts anew from zero after each repair as if a new system has been put into operation at this point of time and thus the process represents maximal repair and the system is termed same as new (SAN).

The word renewal owes its genesis to the fact that the system after repair is considered to be the same as being replaced with a new one and thus is considered to be renewed.

The inter failure times of a renewal process are independent of each other and identically distributed. Thus, a renewal process is formed by a sequence of independent and identically distributed (i.i.d) random variables.

For a renewal process the conditional intensity function is a function of the backward recurrent time i.e., the time since the last failure;

$$\lambda(t \mid H_{t^-}) = \lambda(t - T_{N(t^-)}) \tag{1.3}$$

A renewal process can be formed with the inter failure times coming from any probability distribution. The most common ones used for maintained systems are the exponential, the gamma, the log normal and the weibull.

The exponentially distributed inter failure times form the homogeneous poisson process (HPP) whose conditional intensity function is a constant;

$$\lambda(t \mid H_{t^-}) = \lambda \tag{1.4}$$

These processes have been widely used in literature for modelling maintained systems and also by industrial engineers for optimizing cost, reliability and availability of maintained systems. A vast literature has developed in this field in the sixties to eighties and innumerable books have been written based on these two processes.

Renewal processes have been comprehensively depicted by Cox (1962). The application of these processes to maintained systems has been dealt thoroughly in Ascher and Feingold (1984). The HPP is treated comprehensively in Rigdon and Basu (2000).

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1.2.2. Minimal Repair

Systems which undergo minor repairs and the system failure intensity remains the same after repair as before repair i.e., the repair process brings the system to a functioning state but does not change its failure intensity with the system remaining same as old (SAO) are modelled by non homogeneous point processes. The most common non homogeneous point process used for maintained systems is the non homogeneous poisson process (NHPP). For an NHPP the conditional intensity function is a function of the chronological age of the system;

$$\lambda(t \mid H_{t^-}) = \lambda(t) \tag{1.5}$$

Two NHPPs are commonly used for modelling maintained systems the power law process and the log linear process. For a power law process the conditional intensity function is given by;

$$\lambda(t \mid H_{t^-}) = \lambda(t) = \alpha\beta t^{\beta-1} \tag{1.6}$$

For a log linear process the conditional intensity function is given by;

$$\lambda(t \mid H_{t^-}) = \lambda(t) = e^{\alpha+\beta t} \tag{1.7}$$

The power law process has also been widely applied to maintained systems. Rigdon and Basu (2000) give a comprehensive account of its use for both single and multiple systems.

The log linear process has been introduced by Cox (1972). This has been applied to maintained systems in Majumdar (1995).

Berman (1982) introduced the non homogeneous gamma process and provided its characteristics.

The use of other non homogeneous point processes has been discussed in Krivstov (2007). He introduced the non homogeneous log normal process and applied it to a maintained systems data set.

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A generalisation of the renewal process and the NHPP, the trend renewal process (TRP) i.e., a renewal process with a trend has been proposed by Lindqvist (1993) where the conditional intensity function is given by,

$$\lambda(t \mid H_{t^-}) = z(\Lambda(t) - \Lambda(T_{N(t^-)}))\, \lambda(t) \tag{1.8}$$

Elvebakk, Lindqvist and Heggland (2003) provide methods for parametric inference of the trend renewal process for observations from both single and multiple systems, and also with a possible unobserved heterogeneity between the systems.

1.2.3. Imperfect Repair

Usually most maintained systems after repair lie somewhere in between these two extremes.

A repair after a failure need not be always either completely effective leading to perfect or maximal repair or completely ineffective leading to minimal repair. The quality of repairs performed generally lies in between these two extremes resulting in some loss of effectiveness leading to imperfect repair. A partial or imperfect repair leads to a failure intensity which improves from what it was before repair but does not achieve the state of perfect repair leading to a failure intensity lying in between the minimal and the maximal.

The quality of repair action can be incorporated into the failure process by a means of a random variable, called the degree of repair attached to each failure event or failure age, taking values between 0 and 1. Both extremes of the degree of repair indicate the maximal and minimal Repair states, while any value in between represents imperfect Repair.

The degree of repair can be treated using the concept of virtual age which has been introduced to model the concept of age recovery or age loss. The model is given by;

$$\lambda(t \mid H_{t^-}) = \lambda(\varepsilon(t)) \tag{1.9}$$

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where, $\varepsilon(t)$ is the effective age of the unit

This model was introduced by Kijima (1989) as model I and model II. Non parametric estimation for the model was developed in Dorado, Hollander and Sethuraman (1997). Stadje & Zuckerman (1991) generalised the improvement factor of Malik (1979), Finkelstein (1993) extended Kijima's model II while Makis and Jardine (1993) extended Kijima's model I. Guo and Love (1995) proposed a unified model.

The above models have been represented by the conditional distributions of successive inter failure times. Doyen and Gaudoin (2004) defined these models in a more convenient way by using the conditional failure intensity arithmetic reduction of age (ARA) or arithmetic reduction of intensity (ARI) models,

An ARA model is given by,

$$\lambda(t \mid H_{t^-}) = \lambda\left(t - \sum_{i=1}^{n} s(i, T_1, \ldots. T_n)\right) \tag{1.10}$$

An ARI model is given by,

$$\lambda(t \mid H_{t^-}) = \lambda(t) - \sum_{i=1}^{n} s(i, T_1, \ldots., T_n) \tag{1.11}$$

where s (i, $T_1, \ldots, T_n$) = constants reflecting age or intensity reduction factors which form the marks on the failure process.

For a power law intensity the ARI model with n failures and memory one ARI_1 i.e., only the previous failure is involved in the failure intensity process, is given by;

$$\lambda(t \mid H_{t^-}) = \sum_{j=1}^{n} \alpha\beta t^{\beta-1} - \rho\alpha\beta(t_{j-1})^{\beta-1} I(t_{j-1} < t < t_j)$$

$$= \sum_{j=1}^{n} \alpha\beta(t^{\beta-1} - \rho(t_{j-1})^{\beta-1}) I(t_{j-1} < t < t_j)$$

(1.12)

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where j=1,2,3..n is the number of failures, I is the indicator function and $t_0=0$

These models can be extended to models with memories m and ∞ i.e., the previous m failures and all previous failures respectively are involved in the failure process.

Doyen and Gaudoin (2004) also suggested the possible use of geometric reduction of age (GRA) or geometric reduction of intensity (GRI) models in place of the above.

1.2.4. Proportional Intensity models

As stated by Kumar (1995) in actual maintenance practice the failure times of a maintained system may be dependent on many factors, viz., the operating environment (e.g., temperature, pressure, humidity dust), the operating history of a machine (e.g., overhauls, the effect of repairs, preventive or opportunistic maintenance), or the type of design or material.

Considering some of these factors as covariates, explanatory or concomitant variables, their influence is modelled by means of a regression model based on the Cox regression model, Cox (1972) with time varying covariates given by;

$$\lambda(t|H_{t^-}) = \lambda(t)f(\gamma' Z(t)) \tag{1.13}$$

where $Z(t)$ = Time varying covariates and

γ = Regression coefficients of the time varying covariates

A model with time invariant covariates is given by;

$$\lambda(t|H_{t^-}) = \lambda(t)f(\lambda' Z) \tag{1.14}$$

where, $\lambda(t)$ is the baseline intensity.

These models are known as the proportional intensities models. The effect of the covariates is to modify the baseline intensity function up or down depending on the nature and strength of the covariates.

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