Abstract

In this thesis, we consider the numerical approximation of nonlinear Fredholm-Hammerstein as well as system of linear and nonlinear Fredholm and Volterra integral equations using projection and multi-projection methods based on piecewise and Jacobi polynomials for both smooth as well as weakly singular kernels. We develop the Newton-iteration scheme based on Galerkin and multi-Galerkin operators to solve the nonlinear Fredholm Hammerstein integral equations for both smooth and weakly singular algebraic and logarithmic type kernels and derive the superconvergence rates. Considering the approximating subspace as the space of piecewise polynomials, we show that the superconvergence rates are increasing in every step of iteration. We also establish the framework for Galerkin method and its iterated version for solving the system of linear Fredholm and nonlinear Fredholm Hammerstein integral equations of the second kind for both smooth and weakly singular algebraic and logarithmic type kernels using piecewise polynomial basis functions. We obtain the superconvergence results similar to that of single linear and nonlinear Fredholm integral equation of the second kind. We show that the iterated Galerkin approximation yields better convergence rates than Galerkin approximate solution. Further we enhance these results by using multi-Galerkin and iterated-multi-Galerkin methods. To get better accuracy in piecewise polynomial based methods, the number of partition points needs to be increased that results a large system of algebraic equations to solve. This motivates us to use global polynomials as basis functions instead of piecewise polynomials. We apply Jacobi spectral Galerkin, multi-Galerkin methods and their iterated versions to approximate the system of Volterra integral equations for smooth as well as weakly singular kernels. For this, first, we develop the regularity properties of the solutions of the system of linear Volterra integral equations and then obtain the superconvergence rates in both infinity and weighted-L^2 norms. We show that the Jacobi spectral iterated multi-Galerkin approximation yields improved superconvergence rates over Jacobi spectral iterated Galerkin and multi-Galerkin approximations. Numerical results are provided to justify the theoretical results.

Keywords: Fredholm-Hammerstein integral equations, System of Fredholm integral equations, System of Volterra integral equations, Projection methods, multi-Galerkin methods, Piecewise polynomials, Jacobi polynomials, Smooth kernels, Weakly singular kernels, Superconvergence rates.