THEORETICAL AND NUMERICAL ASPECTS OF THE POPULATION BALANCE EQUATIONS

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ABSTRACT

In the motion of particles in a dynamical structure, the spontaneous interactions between them result in the change of size, shape, porosity, volume, etc., of the particles. Therefore, the number distribution or density of particles differs with time. Thus, a mathematical model known as the population balance equation (PBE) is necessitated to represent this variation. This thesis deals with the PBEs corresponding to the coagulation and fragmentation processes. The PBE representing pure fragmentation is classified into two classes: linear and non-linear processes, whereas the PBE representing the coagulation process is purely non-linear. So, in this thesis, we aim to prove the mathematical existence theory involving a considerable class of practically oriented kernels for the non-linear fragmentation process.

Meanwhile, establishing the existence result, it is noticed that the exact solution can only be derived for very simple problems. In this part of the study, we develop two new and efficient numerical schemes based on finite volume methods to approximate the non-linear fragmentation equation. The development of the weighted schemes is concluded by demonstrating a detailed convergence and consistency analysis. Moreover, we extend the one-dimensional models to solve multivariate problems. Several numerical examples are presented to validate the proficiency and accuracy of the developed schemes.

Population balance models incorporating spatially inhomogeneous coagulation and condensation processes play a significant role in modeling many real-life applications. It is necessary to mention that the creation of dust particles (or particles with nearly zero size/volume) takes place in several industrial sectors. So, the system is set up with coagulation kernels having singularity at the coordinate axes. In this context, we prove the mathematical existence-uniqueness of a continuous solution involving the singular coagulation kernel.

Finally, the existence-uniqueness of a steady-state solution to the coagulation equation with source and efflux terms is addressed. The coagulation kernel considered in this work has singularity on both coordinate axes. Further, an explicit form of the equilibrium solutions to the problems with sources and effluxes for the constant and product kernels have been derived.

Keywords: Population balance equations; Coagulation; Fragmentation; Existence of solutions; Uniqueness of solutions; Finite volume scheme; Volume conservation; Number preservation; Consistency; Convergence; Multi-dimensional model.