CHAPTER I

Introduction

The concept of a group is probably the first instance of mathematical intuition shown by mankind ; the ornamentations of a period as far back as the Sumerian era show a keen appreciation of symmetry principles. It is, therefore, not surprising that attempts to understand the subject were made very early, though in the beginning they were mostly in the nature of isolated individual efforts. Once the scattered facts were organised, a rapid development followed resulting in a well established theory by the turn of this century.

In spite of its close connection with the symmetries of physical systems, the actual application of group theory to physics had to wait for a long time, partly due to the abstract nature of the theory and partly due to the initial reluctance of physicists to accept the theory as a natural mathematical tool. The first difficulty was surmounted to a great extent when the concept of matrix representations was introduced by Frobenious[1] and Schur[2]. The psychological reluctance, however, continued to prejudice the physicists in using alternative methods whenever possible and to employ group theory only as a last resort. It was after the spectacular successes in several fields where the conventional methods either failed to produce the desired results or were too cumbersome to work with, that the theory of groups was accepted as an essential mathematical tool.

The theory of groups enters into physics through considerations of invariance of physical systems under symmetry transformations. Whenever a physical system obeys a conservation law, the Hamiltonian displays certain symmetry and remains invariant under the corresponding group of transformations. This allows us to use the irreducible representations of the group for labelling the energy states of the system. The connection between the symmetry of a system and a conservation law in the Hamiltonian formalism is, however, not one-to-one, i.e., the invariance of the Hamiltonian under a group of transformations does not necessarily give rise to a conservation law. It has been shown[3] that all symmetry transformations of a quantum mechanical system can be chosen so as to have either unitary or anti-unitary operator representations. Though the anti-unitary transformations may have other experimentally detectable consequences, it is the unitary transformations which give rise to conservation laws.

The invariance of an interaction Hamiltonian under a symmetry transformation can provide us with useful information about the final states of a system. When a quantum mechanical system is characterized by the eigenvalues of conserved operators, invariance of the Hamiltonian leads to selection rules allowing transitions only to final states with the same values of quantum numbers as in the initial state. The conservation principles of energy, momenta, charge etc. are examples of such selection rules.

Another important application of group theory to quantum mechanical systems is through the group-lattices[4]. As indicated earlier, the irreducible representations of the group that commutes with the Hamiltonian of the system,and its subgroups provide us with a set of exact quantum numbers. The number of such exact quantum numbers is, however, not always sufficient to characterize the states of the system completely or to show correlation among states of related systems, and one has to find additional quantum numbers to complete the set. A convenient way of doing this is to omit a small part of the Hamiltonian so that the truncated Hamiltonian has the symmetry of a larger group. If the part of the Hamiltonian omitted is small enough to be treated as a perturbation, the irreducible representations of the larger group provide us with some additional quantum numbers. Continuing this process we can get a chain of groups which may give a complete set of quantum numbers to label the states. In the case of physically significant chains the part of the Hamiltonian omitted at each step is larger than the preceding part and one can use the perturbation technique. A collection of such chains of groups is known as the group-lattice. In the physically significant chains the head group, the group of the zero-order Hamiltonian, supplies us with the configurations and their quantum numbers. Considering the perturbation to be small, one can get good eigenvalues by diagonalising the perturbation Hamiltonian within the configurations. The quantum numbers of the zero-order states are now good and the set of all such good quantum numbers along the physically significant chain serves to characterize the states and show correlations among related states. Calculations can be further simplified by symmetry adaptation of the basis of the configurations to the group of the Hamiltonian.