ABSTRACT

The structure of directional data is somewhat different from that of the Euclidean data due to the disparate topologies between the Euclidean spaces and manifolds. In this thesis, we have considered various inference problems for parameters of some commonly used directional distributions. New point and interval procedures and new tests have been developed for these problems. A detailed study on comparative performance of these procedures has been carried out and applications to real data are given.

In **Chapter 1** we give a detailed overview of the literature on inference problems on parameters of directional distributions. A summary of results obtained in the thesis is also presented. **Chapter 2** provides some fundamental definitions and results for directional statistics which are useful in subsequent chapters.

Fisher-von Mises-Langevin (FvML) distribution is one of the most widely used directional distribution for modeling and analyzing directional data. **Chapter 3** deals with the testing for the homogeneity of the mean directions of several FvML populations when the concentration parameters are unknown and unequal. We consider several adaptive test procedures, viz., a parametric bootstrap based likelihood ratio test, a contrast based nonparametric bootstrap test and a permutation test. Algorithms for computation of critical points, size and power values are given. We present empirical size and power comparison results for different test procedures for various configurations of the concentration parameters, sample sizes and number of groups. These are also compared with the empirical size and power values of certain tests existing in literature. Robustness of the proposed tests is also investigated under contaminated FvML distributions. 'R' packages are developed for implementation of all procedures and these are further illustrated on a real data set.

In **Chapter 4** we develop inference procedures for the common mean direction of several Fisher-von Mises-Langevin (FvML) distributions. The concentration parameters are taken to be unknown and heterogeneous. An adaptive estimator is proposed and is seen to have substantially better risk performance than individual sample means in a simulation study. Further, a test based on this adaptive estimator is proposed, and nonparametric bootstrap and permutation resampling methods are developed for its implementation. Two more heuristic tests are proposed and their implementation is carried out using nonparametric bootstrap resampling. A detailed simulation study shows that

these test procedures achieve the nominal size and have good power performance. The robustness of these tests is also shown by implementing on a contaminated FvML distribution. 'R' packages are developed for the implementation of the tests. A real data set is considered for illustrating the procedures.

Chapter 5 focuses on the estimation and testing of a shape parameter (or concentration parameter κ) of a Fisher-von Mises-Langevin (FvML) distribution. It is noted that the estimators for κ using standard procedures such as Bayes, maximum likelihood, method of moments, unbiasedness etc., cannot be obtained in closed or explicit forms due to occurrence of Bessel functions in the density function. We derive asymptotic expressions up to first order for the bias and mean squared error (MSE) of the maximum likelihood estimator (MLE) of κ when FvML mean direction is unknown. Using these expressions in a selective manner, several nearly-unbiased estimators for κ are proposed. These new estimators are seen to offer substantial improvements in terms of bias and percentage risk improvement (PRI) over the earlier ones. Three parametric bootstrap tests are proposed for κ. The simulation study shows that these tests achieve size values quite close to their nominal values and have quite good power performances. We also propose nonparametric bootstrap confidence intervals for κ. Finally these estimators and test procedures are applied on a real data set. 'MATLAB' functions are developed for the practical implementation of all inference procedures.

The two parameter wrapped Cauchy distribution has been widely used for analyzing circular data. In **Chapter 6**, we consider point and interval estimation procedures for the mean direction of a wrapped Cauchy distribution. Three natural estimators are considered and their risk performance is compared numerically. Asymptotic distribution of an estimator is used to propose bootstrap tests for the mean direction. The asymptotic accuracy of bootstrap procedure is established. Further, a new method is developed to construct bootstrap confidence intervals for the mean direction. The coverage probabilities and the average lengths of these intervals are compared using simulations. 'R' package has been developed for implementation of these inference procedures and illustrated with the help of two data sets.

Keywords: Bias reduction technique, Bootstrap confidence interval, Directional data, Fisher-von Mises-Langevin distribution, Nonparametric bootstrap, Parametric bootstrap, Permutation test, wrapped Cauchy distribution.