Nuclear Physics, since its very inception, has been beset with what has come to be known as the 'Fundamental Problem' of nuclear forces. First there was the difficulty with regard to the constituents of nuclei (proton-electron hypothesis). Happily, this difficulty was solved with the discovery of the neutron by Chadwick (1932) and just within a few years of this discovery a beginning was made towards a solution of the fundamental problem by Yukawa's meson theory (1935). This theory looked very promising in the initial stages, especially after the discovery of the pions (Powell et al. 1947). However, the experimental discovery of heavier mesons and strange particles and multiple pion production seemed to have opened Pandora's box. And though physicists have devoted more man-hours to this problem than to any other question in the history of mankind (Bethe 1953), they have not been able to evolve a satisfactory fundamental theory of nuclear forces to-date. This failure notwithstanding, the physicists have not given up in despair. And highly desirable as it is to account for the main properties of nuclei by deriving them from the nucleon-nucleon interaction, they have adopted a different approach, the phenomenological approach, to circumvent the difficulties which the lack of a fundamental theory presents. In this approach, nucleon-nucleon potentials, which are known to have a short range, are derived by fitting the precisely known two-body data like the binding energy and the electric quadrupole moment of the deuteron, the phase shifts in nucleon-nucleon scattering and the two saturation properties of nuclear matter (saturation of binding energy and nuclear density). These considerations and consideration of the invariance properties lead to the following most general two-body potential of the form

$$
\begin{aligned}
V = & (V_C + W_C\ \vec{\tau}_1 . \vec{\tau}_2) + (V_{SC} + W_{SC}\ \vec{\tau}_1 . \vec{\tau}_2)(\vec{\sigma}_1 . \vec{\sigma}_2) \\
& + (V_T + W_T\ \vec{\tau}_1 . \vec{\tau}_2)\left[\frac{3(\vec{\sigma}_1 . \vec{r})(\vec{\sigma}_2 . \vec{r})}{r^2} - \vec{\sigma}_1 . \vec{\sigma}_2\right] \\
& + (V_{LS} + W_{LS}\ \vec{\tau}_1 . \vec{\tau}_2)(\vec{\sigma}_1 + \vec{\sigma}_2).(\vec{r} \times \vec{p}) \\
& + (V_Q + W_Q\ \vec{\tau}_1 . \vec{\tau}_2)\left[(\vec{\sigma}_1 + \vec{\sigma}_2).(\vec{r} \times \vec{p})\right]^2 \quad , \qquad ..(1.1)
\end{aligned}
$$

where the subscript C stands for the spin independent central potential, SC for the spin dependent central potential, T for the tensor potential, LS for the spin-orbit potential and Q for the so-called quadratic spin-orbit potential. In Eq. (1.1), V's and W's are functions of the internucleon distance r.

The two-body low energy data yield only four parameters relevant to the nucleon-nucleon interaction, namely, the singlet and triplet effective ranges and scattering lengths. The shape-dependent parameter that comes from the effective range expansion is poorly determined by these data. Hence to obtain further information regarding the nucleon-nucleon interaction, attempts were made to analyse the high-energy scattering data. The investigations in this energy region revealed that the nuclear force is weak in the odd parity states (Christian and Hart 1950). Also, the fact that the $^1S_0$ phase shift becomes negative around 250 MeV constitutes a strong argument against a potential that is attractive at all distances and has led Jastrow (1951) to postulate a hard core in the potential to explain the two-body scattering data at high energies. Jastrow's suggestion has been followed by many workers and a number of phenomenological potentials with hard core have been evolved. Well known and frequently used nucleon-nucleon potentials which contain hard core and reproduce the experimental two-body data comparatively well are, for example, the phenomenological potentials of Gammel and Thaler (1957), Hamada and Johnston (1962) and Reid (1968), and the Yale potential (Lassila et al.1962)

An alternative approach to describe the strong repulsion of the nucleons at small distances has been proposed by Peierls (1960) who has suggested the replacement of the hard core by a velocity-dependent term in the potential. This makes the nucleon-nucleon interaction more and more repulsive with increasing energy of the particles.

The idea of a velocity-dependent potential is, of course, not completely new. For example, in Classical Electrodynamics (Panofsky and Phillips 1955), a potential of the form

$$\frac{1}{4\pi\epsilon_0}\left( V - \frac{u^2}{c^2} V \right) , \qquad ..(1.2)$$

where V is the Coulomb potential, is used to calculate the force exerted by an electron on another electron moving with a velocity $\vec{u}$ parallel to that of the first electron.

The velocity-dependent nucleon-nucleon interactions commonly found in literature (Razavy et al. 1962, Rojo and Simmons 1962, Werner 1962, Green 1962, Bhaduri and Preston 1964, Davies et al. 1966, Nestor et al. 1968, Appel 1969a) are of the forms

$$V_1(r,p) = V_{static} + \frac{\lambda}{M} \vec{p}\cdot\omega(r)\vec{p} \quad , \qquad ..(1.3)$$

$$V_2(r,p) = V_{static} + \frac{\lambda}{2M}\left[p^2\omega(r) + \omega(r)p^2\right] , \qquad ..(1.4)$$

in which $V_{static}$ is the static part of the potential and the rest is the velocity-dependent part. Both the potentials $V_1$ and $V_2$ satisfy the following conditions (Eisenbud and Wigner 1941, Okubo and Marshak 1958) :

1. The potential can depend, apart from the two spin vectors $\vec{\sigma}_1$ and $\vec{\sigma}_2$, only on the relative separation r and the relative momentum $\vec{p}$ (Invariance under displacement and Galilean invariance).
2. The potential should be invariant under space rotations (Conservation of angular momentum).
3. The potential should be invariant under space reflections (Conservation of parity).
4. The potential should be symmetric under the interchange of two identical particles (Conservation of statistics).

5. The potential should be invariant under rotation in isotopic spin space about the z-axis (Conservation of electric charge).

6. The potential should be Hermitian.

7. The potential should be invariant under time reversal (Conservation of temporal parity). Since the Hamiltonian is the generator for the time evolution operator, this presupposes Hermiticity of the potential.

Though both the potentials (1.3) and (1.4) are equivalent so far as these conditions are concerned, it has been shown by Appel (1969b) that the form (1.4) is more suitable for giving the useful details of the nucleon-nucleon interaction.

A velocity-dependent potential of a type somewhat different from the potentials of the above forms has been used by Tabakin and Davies (1966). This potential contains an exponential velocity-dependence.

Works of various authors (Razavy et al. 1962, Rojo and Simmons 1962, Werner 1962, Green 1962, Bhaduri and Preston 1964, Tabakin and Davies 1966, Nestor et al. 1968, Appel 1969a) show that the velocity-dependent potentials and hard-core potentials are equivalent, because they are capable of giving equally good fits to the relevant two-body data. (A theoretical justification for this aspect of equivalence has been provided by Bell (1961) and Baker (1962), while Finkler and Valk (1970) have shown the two types of potentials to be equivalent in the two-body photo-effect sum rule calculations). Nonetheless, a velocity-dependent potential has decidely got certain advantages to commend itself. Since a velocity-dependent potential is well-behaved, it is easier to work with and has the additional advantage that the conventional perturbation treatment can be used successfully in solving the many-body problem. While the second order terms are by no means negligible, they are reasonably small and there is a fair hope that the perturbation series will converge (Green 1962, Bhaduri and Preston 1964, Davies et al. 1966, Nestor et al. 1968). On the other hand, a hard-core potential on account of its singular behaviour gives infinite matrix elements when used in the perturbation theory calculations. Although this problem can be overcome (Brueckner and Gammel 1958, Eden 1958, Moszkowski and Scott 1960, Bethe 1965, Day 1967, Rajaraman and Bethe 1967), computational difficulties exist inherently in such theories. It is worth mentioning that soft core (Reid 1968) and even super-soft core (Srivastava et al. 1970) local static potentials are unsuitable for perturbation theory calculations. However, it should be noted that the velocity-dependent potentials do not constitute the only alternative to the hard-core potentials so far as the difficulty associated with hard-core potentials in the perturbation theory is concerned. Another widely adopted alternative is the use of nonlocal separable potentials (Yamaguchi 1954, Mitra and Narasimham 1960, Mitra and Naqvi 1961, Naqvi 1964, 1967, Tabakin 1964, Muthukrishnan and Baranger 1965, Mongan 1968, Hodgson 1969).

Once a phenomenological potential has been obtained by fitting the relevant two-body data and the saturation properties of nuclear matter, it can be used in calculating the properties of finite nuclei as well as various other properties of nuclear matter. Though comparatively new to enter the field, the velocity-dependent potentials have been employed with considerable success for this purpose (Herndon et al. 1963, Heywood 1964, Folk and Bonnem 1965, Jibuti et al. 1965, Raghavan and Srivastava 1971, Dohnert and Rojo 1964, Mckellar and Naqvi 1965, Bhaduri and Van Leuven 1966, Macke et al. 1966).

One purpose of the present investigation is to determine the wave function (or structure) of the $\alpha$-particle using the realistic velocity-dependent potential of Nestor et al. (1968) in a variational calculation of the binding energy. We employ this wave function to calculate the negative muon capture rate in the $\alpha$-particle and also its charge form factor.

A testing ground for the suitability of ground state wave functions of light nuclei is provided by the experiments on the negative muon capture rate by these nuclei. The negative muon ($\mu^-$) may be captured in a manner similar to orbital electron capture, the basic capture processes in the two cases being

$$e^- + p \rightarrow n + \nu_e, \qquad ..(1.5)$$

$$\mu^- + p \rightarrow n + \nu_\mu. \qquad ..(1.6)$$

However, the $\mu^-$-capture process differs from the orbital electron capture in that it is energetically possible even in free space. A muon is captured by a nucleus in the following sequence of events. First, the negative muon whose energy has been degraded to a few KeV can be captured into atomic orbits of high principal quantum number ( $n \sim 14$ ). Then it cascades down first through Auger processes and then through X-ray transitions. Ultimately, it reaches the 1s atomic orbit since the overall time to fall into lowest orbit is between $10^{-9}$ to $10^{-12}$ sec which is much shorter than the average life time of the muon,

$$\tau_{av} = (2.1983 \pm 0.0008) \times 10^{-6} \text{ sec.} \qquad ..(1.7)$$

Once in the lowest (1s) orbit, the muon may either decay or be captured into the nucleus according to (1.6). If the muon and the free proton in the reaction (1.6) are initially at rest, conservation of energy and momentum gives $E_\nu \approx 100$ MeV and $E_n \approx 5$ MeV; that is, most of the energy is carried off by the neutrino. Inside a heavy nucleus, the other nucleons can also carry off some of the momentum, and, in addition, the protons are not at rest; they form a Fermi gas. As a result, more energy is given to the nucleus, the average excitation energy is now of the order of 15 to 20 MeV.

Let us assume that the capture interaction is a mixture of V and A and the two-component neutrino theory holds. Writing the interaction nonrelativistically and assuming that the muon also moves nonrelativistically, it is easy to show (Wu and Moszkowski

1966) that the capture rate per atom per second in this simple treatment is given by

$$\Lambda^{(\mu)} = \frac{G^2}{2\pi^2}\left(\frac{m_e c^2}{\hbar}\right)\left(\frac{\alpha m_\mu Z}{m_e}\right)^3\left(\frac{E_\nu}{m_e c^2}\right)^2 |M|^2$$

$$= 270\ Z^3\ \sec^{-1}. \qquad ..(1.8)$$

This expression yields the capture rate for hydrogen, $\Lambda^{(\mu)}({}_1H^1) = 270\ \sec^{-1}$ which is less than 0.1% of the spontaneous decay rate (life time, $\tau_{av} \approx 2.20 \times 10^{-6}$ sec), so that this fundamental capture process is rather difficult to study experimentally. However, the experimental study of $\mu^-$ capture by heavier nuclei is comparatively easy for the following reasons :

1. The capture rate is increased by a factor Z relative to the hydrogen case, because there are Z protons in each nucleus.
2. There is a factor $Z^3$ arising from the large probability of the bound muon wave function at the nucleus (for hydrogenic wave functions, $|\psi(0)|^2 \sim Z^3$).
3. Recoil effects are important, for the neutron produced in the capture reaction can recoil. The recoil of the nucleus as a whole has very little effect on the capture rate. Correction for nucleon recoil reduces the capture rate.

4. The muon spends an appreciable part of the time inside the nucleus itself. By Gauss's theorem, the effective charge felt is less than the actual nuclear charge and equals the charge inside the orbit. Thus we should replace Z by $Z_{eff}$. For lightest nuclei, $Z_{eff} \sim Z$. With increasing A, the inequality $Z_{eff} \ll Z$ is progressively satisfied.

5. The muon capture rate is further reduced by the effect produced by Pauli's exclusion principle, simply because the process cannot occur if the resulting neutron finds itself in a state already occupied by another neutron.

When these five factors are taken into consideration, the total negative muon capture rate (1.8) becomes (Wu and Moszkowski 1966

$$\Lambda^{(\mu)} = 270 \times Z^4_{eff} \times C_{Pauli} \times C_{recoil} \quad . \qquad ..(1.9)$$

For the heaviest nuclei, the capture rate is approximately $10^7$ $sec^{-1}$, which is much larger than the spontaneous decay rate.

Primakoff (1959) has made an extensive theoretical study o the negative muon capture rate in nuclei in the closure approximation using the Fujii-Primakoff Hamiltonian (Fujii and Primakof 1959). This is a Hamiltonian effective in muon capture and corresponds, in a nonrelativistic approximation for the muon and for the nucleons, to the most general Lorentz covariant transiti matrix element for the reaction (1.6) in a theory where the lepton-bare nucleon coupling is V and A , and where neutrinos are emitted with unit negative helicity. In this Hamiltonian, 'many-body' terms arising from the exchange of virtual pions, kayons, etc. among the nucleons are neglected. However, all first-order nucleon recoil corrections $\sim \nu/m_p$ , where $\nu$ is the neutrino momentum, are included. This inclusion results in the modification of muon-dressed nucleon coupling constants $g_\xi^{(\mu)}$ ($\xi$ = A,V,P ) to 'effective' coupling constants denoted by $G_\xi^{(\mu)}$ . The two sets are related by

$$G_V^{(\mu)} \equiv g_V^{(\mu)} \left(1 + \frac{\nu}{2m_p}\right) ,$$

$$G_A^{(\mu)} \equiv g_A^{(\mu)} - g_V^{(\mu)} (1 + \mu_p - \mu_n) \frac{\nu}{2m_p} , \qquad ..(1.10)$$

$$G_P^{(\mu)} \equiv \left[ g_P^{(\mu)} - g_A^{(\mu)} - g_V^{(\mu)} (1 + \mu_p - \mu_n) \right] \frac{\nu}{2m_p} .$$

The appearance of the nucleon magnetic moments $\mu_p$ and $\mu_n$ in the expressions for $G^{(\mu)}$ is a consequence of the assumption of conserved vector current (Feynman and Gell-Mann 1958) which implies the possibility of muon capture via the process :

$$\mu^- + p \rightarrow \mu^- + \pi^+ + n \rightarrow \nu + \pi^0 + n \rightarrow \nu + n.$$

It is interesting to note that, though the lepton-bare nucleon coupling is V and A , an effective pseudoscalar coupling $G_P^{(\mu)}$ related to $g_P^{(\mu)}$ occurs in the expression for the Hamiltonian and other equations for the muon capture rate. This implies the possibility of muon capture via the process :

$\mu^- + p \rightarrow \mu^- + \pi^+ + n \rightarrow \mu^- + p + \bar{n} + n \rightarrow n + \nu$ .

The presence of strong interactions in this process generates additional terms in the S-matrix and one of the terms behaves as a pseudoscalar coupling. This term is quite small in beta decay, but large in the muon capture rate because of the large muon rest mass and the consequent recoil of the residual nucleus. This induced pseudoscalar coupling $g_P^{(\mu)}$ is about eight times larger than the axial vector component $g_A^{(\mu)}$ $(g_P^{(\mu)}/g_A^{(\mu)} \approx 8)$ which generates it (Goldberger and Treiman 1958, Wolfenstein 1958). Also the rate of muon capture with simultaneous emission of radiation depends fairly sensitively on the ratio $g_P^{(\mu)}/g_A^{(\mu)}$ . A study of this process in $^{40}$Ca gives $g_P^{(\mu)}/g_A^{(\mu)} \approx 12$ .

The universality of weak interaction should mean equality between $g_V^{(\mu)}$ and $g_V^{(\beta)}$ and also between $g_A^{(\mu)}$ and $g_A^{(\beta)}$ . The actual relations are

$$g_V^{(\mu)} \approx g_V^{(\beta)} \times 0.972 \ , \ g_A^{(\mu)} \approx g_A^{(\beta)} \times 0.999 \ . \qquad ..(1.11)$$

The slight difference is due to differing nucleon four-momentum transfers in the muon capture and in the beta decay.

Application of Fermi's golden rule leads to the capture probability for any nucleus in the form

$$\Lambda^{(\mu)} \sim \sum \nu_f^2 \int \left| \langle f \,|\, H_{eff}^{(\mu)} ( \vec{\nu}_f ) \,|\, i \rangle \right|^2 d\Omega_\nu \ , \qquad ..(1.12)$$

where the summation is over the possible final nuclear states f, $\nu_f$ is the neutrino momentum for a final state f, and the integration is over the directions of neutrino momentum. Primakoff(1959) uses the Fujii-Primakoff Hamiltonian for $H_{eff}^{(\mu)}$ in Eq. (1.12). Primakoff's approach to the evaluation of this expression using closure is based on the following approximations :

1. Replace the neutrino momentum by suitably chosen average values, since the neutrino spectrum is known to be sharply peaked near the maximum value of the neutrino energy.
2. Extend the summation over all possible states to a summation over all states of the nuclear Hamiltonian on the ground that the contribution from states that are energetically inaccessible is small.
3. Extend the summation to states of the Hamiltonian of any other symmetry (this introduces no further errors, because $H_{eff}$ is a symmetric operator).

These approximations enable closure to be applied to Eq. (1.12). On invoking closure, the final expression is found to be (Primakoff 1959)

$$\Lambda^{(\mu)}\ (Z,A) = (Z_{eff})^4 (\ \langle \eta \rangle_a\ )^2 (272\ \sec^{-1})\, \mathcal{R}\, (1 - \delta_a) \quad ..(1.13)$$

in which 'a' denotes the ground state of the nucleus and

$$\mathcal{R} \equiv \left[ (G_V^{(\mu)})^2 + 3\ (\Gamma_A^{(\mu)\,2}) \right] \Big/ \left[ (g_V^{(\beta)})^2 + 3\ (g_A^{(\beta)})^2 \right] \qquad ..(1.14)$$

with

$$\left[\Gamma_A^{(\mu)}\right]^2 = (G_A^{(\mu)})^2 + \frac{1}{3}\left[(G_P^{(\mu)})^2 - 2G_A^{(\mu)}\, G_P^{(\mu)}\right]. \qquad ..(1.15)$$

In Eq.(1.13), $\langle\nu\rangle_a$ is a quantity related to the average neutrino momentum which also takes into account the recoil of the nucleus. Further, the Pauli exclusion principle manifests itself through the term, $\mathcal{J}_a$ called the exclusion principle inhibition factor, which depends on the ground state wave function of the nucleus.

For heavy nuclei ($Z > 6$, $A > 12$), neglecting terms of the order of $1/Z$, Primakoff (1959) has shown that the inhibition factor

$$\mathcal{J}_a = \frac{A - Z}{2A}\,\delta_a\ , \qquad ..(1.16)$$

where $\delta_a$ is known as the nucleon-nucleon correlation parameter. A not very rigorous treatment (Primakoff 1959) shows that for heavy nuclei, $\delta_a \approx 3$ and $\mathcal{J}_a$ is, therefore, model independent. However, for lightest nuclei (nuclei in the 1s shell) the dependence of $\mathcal{J}_a$ on the ground state wave function is quite explicit and pronounced (Primakoff 1959).

Goulard et al. (1964) have modified Primakoff's theory by taking into account relativistic corrections. These corrections, in the closure approximation, affect only the exclusion principle inhibition factor $\mathcal{J}_a$, and are important for nuclei for which $\mathcal{J}_a$ is large (e.g. $\alpha$-particle, medium-heavy and heavy nuclei). Taking into consideration relativistic corrections $x_a$, the muon capture rate expression (1.13) becomes

$$\Lambda^{(\mu)}(Z,A) = (Z_{eff})^4 (\langle\nu\rangle_a)^2 (272\ \sec^{-1})\, \mathcal{R}\, (1 - \mathcal{J}_a)(1 - x_a). \qquad ..(1.17)$$

It is obvious that if we describe the ground state of a nucleus in the 1s shell by a plausible wave function and use this wave function to calculate the muon capture rate by the nucleus in the closure approximation using Eq. (1.17), then the agreement or disagreement of the calculated value with experiments will give information about the suitability or otherwise of the wave function.

Experiments on elastic scattering of high energy electrons by a nucleus provide another valuable tool for studying the structure (or ground state wave function) of the nucleus concerned. Experimental data are analysed in terms of electric charge and magnetic form factors and these may be identified with the Fourier transforms of the spatial distributions of the electric charge density and the magnetic moment density of the nucleus. As the spin and magnetic moment of the α-particle are zero, the elastic scattering of electrons is due to its charge alone. We, therefore, study only the charge form factor of the α-particle with our variational wave function and compare with the charge form factor obtained from the electron scattering experiments. We also compute the charge distribution in $^4$He.

The format of our study is as follows :

In Chapter II of the present work, we make a variational calculation of the binding energy of the α-particle with the realistic velocity-dependent potential of Nestor et al. (1968). In Chapter III, we employ this variational wave function to compute $\Lambda^{(\mu)}(\,^4He\,)$, and Chapter IV is devoted to the study of the charge form factor of the α-particle. Throughout we compare with experiments to find out how satisfactory our model for the α-particle is. And with a view to bringing out clearly the effect of velocity-dependent forces, we compare the results of our calculations with similar calculations using potentials without hard core or velocity-dependence. Further, comparison with calculations using hard-core potentials tells us about the equivalence of hard-core and velocity-dependent forces.

In Chapter V, we make a variational calculation of the binding energy of $^6Li$. As our aim is just to find out how the binding energy and size of $^6Li$ are affected by velocity-dependent forces - calculations without repulsive core or velocity-dependence give too large a binding energy and too small a size (Irving and Schonland 1955) - we employ the central velocity-dependent potential of Davies et al. (1966) (and not a realistic potential like that of Nestor et al. (1968) ) in our calculation.