SYNOPSIS

In the present era of high altitude flights, the study of rarefied gases has been recognised to be of immense importance. At high altitudes the density of the gas becomes so low that the mean free path is no longer negligible as compared to a characteristic dimension of the flow field. In such cases the ordinary continuum approach may fail to yield satisfactory results. However, when the gas is only slightly rarefied, the effects of rarefaction can be studied by the continuum approach based on the usual Navier-Stokes equations of gas dynamics provided the no-slip boundary conditions are replaced by conditions allowing for a relative velocity and a difference in temperatures between the solid surface and the adjoining layer of fluid. It is now well accepted that the velocity slip and the temperature jump at the solid surface can be taken proportional to the normal gradients of velocity and temperature respectively at the surface - the proportionality factors depending upon the mean free path which in turn depends upon the temperature of the gas. The objective of the present thesis is to investigate the gas rarefaction effects on some of the basic flow problems of fluid mechanics by means of the aforementioned continuum approach of slip flow regime. The thesis has been divided into six chapters which contain besides the introduction the following five problems of slip flow:

1. Rayleigh problem
2. Rayleigh problem with modified initial condition
3. Flow due to the fluctuating motion of a porous infinite flat plate
4. Steady Couette flow
5. Leading edge problem in Rarefied Hypersonic flow.

Chapter I contains introduction to the subject and Chapter II deals with the flow due to the impulsive motion of an infinite flat plate in an infinite fluid at rest, referred to as the Rayleigh problem, under slip conditions. This problem has been first investigated by Schaaf, but his analysis is restricted to small temperature variations since he has taken the proportionality factor in the slip boundary conditions to be constant. In the present analysis of the problem the non-linearity of the slip boundary conditions, due to the variation of the mean free path with temperature, has been taken into account. An exact solution of the problem in series of fractional powers of time has been obtained. The solution has been further generalised for the case of the plate moving with general unsteady velocity and temperature both expressible in power series of time. Asymptotic solutions of the problem valid for large times after the start of the motion have also been obtained. Numerical calculations for velocity slip, temperature jump, skin friction etc., have been carried out and results discussed for Rayleigh problem and a particular case of its generalization.

In the classical no-slip solution of the Rayleigh problem the shear stress at the wall at the initial instant becomes infinite. The slip flow analysis of the problem predicts a finite value for the shear stress at the surface at the start of the motion and is thus an improvement over the no-slip case. But this value is about twice the correct value predicted by the free molecule flow theory when the reflection of the molecules from the surface is mostly diffusive. This discrepancy could be accounted for the fact that according to the free molecule flow the fluid at the wall acquires almost instantaneously about half the velocity of the wall as the plate starts moving impulsively and this fact could not be represented by the existing initial condition of the above continuum approach.

The aim of Chapter III is to imporve further the slip flow analysis of the Rayleigh problem to obtain satisfactory results even for short times after the start of the motion. The correct picture immediately after the start of the motion of the plate could be obtained by reckoning the motion of the fluid from the instant it acquires the non-zero velocity as predicted by the free molecule flow. Thus in Chapter III a rough modification in the initial condition of the Rayleigh problem is proposed. An exponentially varying velocity distribution of the fluid has been assumed at the start of the motion in such a way that the correct values for the shearing stress and slip velocity at the wall at the initial instant are assured. For the sake of simplicity constant property gas is considered and exact solution of the problem has been obtained in closed form with the help of Laplace transforms. The significant result of this simple analysis is that it yields correct shear stress and slip velocity at the wall both for short times and long times after the start of the motion, in agreement with some of the kinetic theory analyses of the problem.

The effects of velocity slip and temperature jump vis-a-vis the no-slip effects have been studied in Chapter IV in the problem of flow due to the fluctuating motion of a porous infinite plate about a uniform mean velocity. For the incompressible case the problem is equivalent to the fluid oscillating about a mean velocity and the plate stationary and comparison with Stuart's no-slip problem brings out some interesting results. For the compressible case it has been possible to obtain a simple exact solution for low-Mach number flows for which the viscous dissipation is negligible. In general the slip boundary conditions exhibit a subduing influence over the response of the flow near the wall to the fluctuations of the plate.

Steady Couette flow of a rarefied gas is the subject of study in Chapter V. For obtaining a solution that would be valid for arbitrary Mach numbers and arbitrary temperature ratios we consider the compressible Navier-Stokes equations and non-linear slip boundary conditions. The viscosity temperature relation is assumed to be governed by the general Sutherland's law. The problem is finally reduced to solving three simultaneous non-linear equations - two algebraic and one integral. These equations have been solved on a digital computer for different values of the physical parameters involved and the coefficients of skin friction, heat trnasfer etc. have been computed. Several graphs and tables are given for studying the different aspects of the problem. Velocity profiles and temperature profiles have been compared with those obtained by Liu and Lees who used the two-stream Maxwellian method and the agreement is excellent in the low Knudsen number regime.

In Chapter VI an exact solution for the two-dimensional compressible boundary layer equations with arbitrary pressure gradient together with the non-linear slip boundary conditions has been obtained in power series of the coordinate along the body. The series solution obtained is particularly suitable for studying the rarefied hypersonic flow phenomenon in the leading edge region. The shock wave and the boundary layer are assumed to coincide in this region and the tangent-wedge approximation is used for studying the shock wave-boundary layer pressure interaction for the case of a semi-infinite flat plate. The usual strong interaction theory of the problem gives rise to infinite interaction pressure at the leading edge. In the present study, by introducing the slip boundary conditions, this anomaly has been removed and a finite interaction pressure is predicted. The interaction pressure rises sharply in the region just downstream of the leading edge.