w&mmrnmtvi m~. in .wmBvwmmBmmmmwvmt

A THESIS

ON

"COMPRESSIBLE FLOWS WITH HEAT TRANSFER AND SOME ASTROPHYSICAL APPLICATIONS".

BY

A. S. GUPTA.

Department of Mathematics,
Indian Institute of Technology,
Kharagpur.

Submitted to the Indian Institute of Technology, Kharagpur for the award of the degree of Doctor of Philosophy (Mathematics).

1958.

Copyright IIT Kharagpur

INTRODUCTION

The thesis consists of two parts. In the first part, which contains five chapters, a question has been posed. Attempts to answer this question have been put in the first four chapters. A technique has been elaborated in Chapter I. Apart from the question raised the technique helps to investigate certain types of motion of compressible fluids with heat transfer. In particular certain types of radial motion of a star with radiative heat transfer has been handled in Chapters I and II with the help of this technique. Chapter V has been put in the first part because the method of separation (on which further elaboration is made in Chapter I) has been used in the Chapter though this has no direct relation to the main question under discussion. The second part of the thesis consists of Chapters VI to VIII. They are concerned mainly with laminar boundary layers in compressible flows over flat plates, heat transfer being taken into account.

The question which concerns the first four chapters may be introduced as follows: Suppose the motion of a gas mass as worked out from the theory tallies with the observed motion and let the theory be based on the equations of continuity, momentum and the relation

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^n, \qquad (A)$$

(where p, ρ are pressure and density respectively and the

Copyright IIT Kharagpur

subscript Zero refers to some standard state). The question arises how far it is possible to conclude that the motion is such that $n = c_p/c_v$ (no separate experiment being done to find c_p and c_v) and there is neither any generation of energy within or heat exchange between parts of the gas ? It obviously suggests that the above conclusion is not inevitable for it may so happen that $n \neq c_p/c_v$ and the generation of energy and heat transfer (or heat transfer alone, generation of energy being absent) so conspire that eventually relation (A) holds. It is then a polytropic and not necessarily an adiabatic relation. Further it may also happen that $n = c_p/c_v$, being caused by the generation of energy and heat transfer in each element balancing each other so that eventually an adiabatic relation holds. Our question then simply is under what circumstances a relation like (A) can hold, where n is not necessarily equal to c_p/c_v . It may be noted, however, that often one uses the relation

$$p = A\rho^n , \qquad (B)$$

A being a constant. This is only a special case of (A) with $p_0 = A\rho_0^n$ holding between the initial values or with the initial values p_0 and ρ_0 constants.

The facts which have led us to raise the question are as follows: The relations (A) or (B) assumed in the literature of gas dynamics or stellar structures, are often taken as mathematical simplification having its physical justification in low conductivity or very quick motion so that there is little time for heat exchange to take place.* This is in vogue since Laplace's famous investigations on sound waves. Our question is that apart from the usual justification just statedit may very well happen that a polytropic (and not necessarily adiabatic) relation may subsist despite heat exchange under certain circumstances. If a polytropic relation holds (despite heat transfer) then the motion determined from dynamical equation, equation of continuity and a polytropic relation can run in the usual lines with n written for γ . We shall find that such cases do occur.

When no shocks are present adiabatic condition is synonymous with isentropic conditions. In case of a shock, however, the constant A of equation (B) undergoes sudden change. We are not concerned with this as we have dealt with continuous motions only.

These considerations form the subject matter of the first four chapters. The enquiry is essentially theoretical.

In Chapter I, the question of the possibility of a polytropic relation of type (A) for radial motion of a gravitating gas sphere with continuous subatomic energy generation and heat exchange through radiation has been investigated. Partly due to the importance of homology in astrophysics and partly due to the fact that it makes the problem simpler, an additional restriction of homology has been introduced. We should, however, like to state that our interest in the main is not astrophysical. Still a few conclusions of astrophysical significance have been

* The argument does not hold for high temperatures e.g., in stellar bodies.

Copyright IIT Kharagpur

pointed out.

The procedure in handling the equations of this Chapter is directed by the works of previous workers. A review of the work done by previous authors is discussed in Art.3 of this Chapter. The essentials of the procedure adopted are explained in Art. 4.

In Chapter II, the radial motion of a gas sphere with a point source has been considered.

Both Chapters I and II incidentally arrive at certain astrophysical conclusions which may be summarised as follows :

(i) Under suitable laws of energy generation and opacity a vibration of a homogeneous gas sphere is possible which is exactly of the type obtained by Bhatnagar [1] following the Darwin Lecture by Rosseland [2] in 1943.

(ii) Under suitable circumstances it is found that a configuration resembling Eddington (Equilibrium) model at any instant can contract or expand with a constant velocity.

In Chapter III condition under which a polytropic relation holds in a flow similar to that of the expansion region of Prandtl-Meyer for a compressible heat-conducting fluid has been investigated. It turns out that an adiabatic relation is the only polytropic relation in such flows in the expansion region provided the thermal conductivity is a suitable function of the temperature.

In Chapter IV a linearised one-dimensional flow of a compressible heat conducting fluid with a polytropic relation between pressure and density has been considered.

The indications obtained from the four chapters have been summarised in Art.5 of Chapter IV.

In Chapter V, heat transfer through conduction in a spherically symmetric flow of an inviscid compressible fluid has been investigated.

In the second part of the thesis some problems of steady and unsteady boundary layer motions of compressible fluids over flat plates and in a particular case over a curved body have been studied. In the investigation viscosity and conductivity have been taken into account. In some cases the effect of suction has also been included. The elasto-viscous flow of a compressible as well as incompressible heat-conducting fluid has been incidentally investigated.

In Chapter VI we have considered the problem of the advancement of a compressible fluid over a flat plate with heat transfer. Integration of the boundary layer equations leads to the following results :

(i) When the surface of the plate is kept insulated, the wall temperature gradually increases with Prandtl number. Further, the temperature at the wall for a given Prandtl number increases with the increase of the free stream Mach number.

(ii) The Drag coefficient is found to be inversely proportional to the square root of the local Reynold's number $u_1 (u_1 t - x)/\nu_1$ where u_1 is the free stream velocity, t, the time, x the distance along the plate and ν_1, the coefficient of Kinematic Viscosity.

Copyright IIT Kharagpur

Chapter VII deals with the velocity and temperature distribution in the steady laminar boundary layer over a curved surface as well as a semi-infinite flat plate by using an approach similar to that of Kármán - Pohlhausen. The surface of the body is assumed to be thermally insulated and suction is applied to it. The investigation shows that the amount of fluid which has got to be sucked in to prevent separation of the boundary layer increases with increase in compressibility. This is to be expected physically since the effect of compressibility is to thicken the boundary layer and more amount of fluid has to be sucked to preserve a uniform thickness and prevent separation. Conditions for similar velocity profiles have also been investigated.

In Chapter VIII we have examined the flow of an elasto-viscous compressible heat-conducting fluid past an infinite porous flat plate. The incompressible case has also been considered. It is found that in the compressible fluid skin-friction at the plate decreases with increase of elastic constant. Incidentally an interesting result of incompressible case turns out viz., the horizontal velocity, the drag and the rate of heat transfer from the wall decrease with increase of elastic constant.

REFERENCES

[1] Bhatnagar, P.L., - Bull. Cal. Math. Soc., 38, 34(1946).

[2] Rosseland, S - M.N.R.A.S., 103, 233(1953).

Copyright IIT Kharagpur

C O N T E N T S

-*-

Copyright IIT Kharagpur