Synopsis

General Relativity (GR), the most successful theory of gravity, is a four-dimensional classical theory. It provides an understanding of gravity as manifest in the geometrical structure of spacetime. GR is a highly non-linear theory and, therefore, exact solutions of the Einstein field equations are hard to obtain. Over the years, it has been noticed, that in fewer than four dimensions the mathematical technicalities of GR become vastly simpler, though some of the general reasoning, bearing connections with the actual four dimensional theory, remain, to some extent, similar. Thus, gravitational theories in lower dimensions provide theorists with a platform to explore the basic foundations of the theory of gravity without encountering the technical complexities that arise in four dimensions.

GR is in remarkable agreement with observations from typical solar system scales to the fraction of a millimeter. It has also been applied to explain galactic and extra galactic physics. Despite these successes, there still exist many situations – such as the understanding of physics at very high energies or very short distances – where classical GR fails. The origin of the universe or the quantum physics of black holes are well known examples where an understanding requires us to go beyond classical GR. It is hoped that the domain where the classical theory fails, can be understood by quantizing gravity. Moreover, the Standard Model of elementary particles based on quantum field theory, describes, with great success, three of the four fundamental interactions in nature: electromagnetic, weak and strong. Unfortunately, gravity does not seem to fit into this framework. Hence, a new quantum theory, with a consistent classical limit, which unifies gravity with all other interactions, is required to explain phenomena at all length and energy scales.

The leading candidate for such a quantum theory is String Theory. In order to construct a consistent String Theory, it turns out that one needs the spacetime dimensions to be greater than four. Thus, the failure of GR to explain physics at very small length scales and the long time search for a 'theory of everything' leads to the fact that we must learn to understand physics in dimensions more than four. In addition, the introduction of the so-called extra dimensions, in order to address well-known four dimensional problems may also shed light on the fundamental question as to why our world is four dimensional.

The above mentioned facts provide us with motivations for studying physics in lower as well as in higher spacetime dimensions. The work reported in this thesis can broadly be divided into two parts – (i) selected aspects of geometry and gravity in dimensions lower than four [*Chapters 2 and 3*] and (ii) gravity and effective field theory in dimensions greater than four [*Chapters 4, 5 and 6*].

Chapter 1, begins with motivational aspects of why we are interested in physics in dimensions not equal to four. Thereafter, we qualitatively review some of the basic features of gravity in two and three spacetime dimensions. Some of the known higher dimensional models are also summarized here, briefly.

Chapter 2 first discusses the standard two dimensional dilaton-Maxwell theory and then reviews a well known 2D black hole spacetime due to Mandal, Sengupta, Wadia and Witten. For this two dimensional geometry, we then obtain exact analytical results on geodesics, geodesic deviation and also solve the Raychaudhuri equation for the expansion. Our detailed investigation on timelike geodesic motion includes an understanding of the effective potentials involved as well as explicit solutions of the geodesic equation. In the context of geodesic deviation, we obtain the deviation vector exactly and provide geometrical interpretations. Finally, the Raychaudhuri equation for the expansion is also solved to arrive at the focusing theorem. The results in this chapter, though pedagogical, to some extent, provides an exactly solvable example in the context of a spacetime metric of reasonable current interest.

Chapter 3 is devoted to the study of three dimensional gravity. After discussing the basic features of general relativity in three dimensions we review several examples of three dimensional black holes. In some of these cases we obtain exact timelike geodesics and interpret the solutions. Finally, we show that the 3D BTZ black hole spacetime can be viewed as a two dimensional brane (asymmetrically warped) embedded in a three dimensional anti de-Sitter (AdS) space – a fact which provides a link with the work in the rest of the thesis.

Chapter 4 deals with braneworld models in five dimensions. A brief review of the warped braneworld models, due to Randall-Sundrum (RS), is followed by the introduction of new braneworld models that arise from the coupling of gravity with different types of bulk scalars. In each example, we obtain exact analytical solutions and analyze the features of the corresponding spacetime geometry.

In the first case (Model-I), the bulk scalar action is inspired by the proposed low energy effective action around the tachyon vacuum. Warped geometries representing solutions of this Einstein-scalar system for a specific potential is found. The solution obtained provides an alternative to the above mentioned RS model. We also address the hierarchy problem in a way similar to RS using this type of warping. The second example (Model-II) uses scalar kinks propagating along the bulk in warped spacetimes to obtain a 'thick' brane realization of the braneworld. In this class of models, the brane is realized as a domain wall in the bulk. Exact solutions of the full Einstein-scalar system with a bulk sine-Gordon potential and a negative cosmological constant are obtained. Finally, in the third example (Model-III), a bulk phantom scalar field (with negative kinetic energy) in a sine-Gordon type potential is used to generate an exact thick brane solution with an increasing warp factor (in contrast with the other two models where we have decreasing warp factors).

Chapter 5 treats the issue of localization of spin half fields and gravity on the brane. After a brief overview of the generic aspects of the field localization scenario in a warped background, we concentrate on the new braneworld models introduced in *Chapter 4*. In this context, we first mention that the generic aspects of the localization problem are mathematically similar to ordinary nonrelativistic quantum mechanics in a volcano potential. We have discussed quantum mechanics in such a potential in detail in the *Appendix B*.

In case of Model-I, the fermionic zero modes with left chirality are found to be localized around the brane in the presence of a Yukawa coupling between the scalar and spinor field. The right chiral modes are found to be localized on the brane for an opposite value of the Yukawa coupling parameter.

In the background of the kink and the corresponding warped geometry in Model-II, we discuss the localization of spin half fermions (with emphasis on massive ones) on the brane in the presence of different types of kink-fermion Yukawa couplings. We analyze the possibility of quasi-bound states for large values of the Yukawa coupling parameter. In particular, the spectrum of the low-lying states and their lifetimes are obtained. Our results indicate quantitatively, within this model, that it is possible to tune the nature of warping and the strength and form of the Yukawa interaction to obtain trapped massive fermion states on the brane, which, however, do have a finite (but very small) probability of escaping into the bulk.

For Model-III, we show that the growing nature of the warp factor allows the localization of massive as well as massless spin-half fermions on the brane even without any additional non-gravitational interactions. The exact solutions for the localized massive fermionic modes are presented and discussed. The inclusion of a fermion–scalar Yukawa coupling appears to change the mass spectrum and wave functions of the localized fermion though it does not play the crucial role it did in the case of a decreasing warp factor.

Additionally, the graviton zero mode confinement on the brane is considered in all the three models. The massive modes are studied for the Model-II.

Chapter 6 focuses on six dimensional bulk spacetimes with 3 and 4-branes. These are constructed using non–standard bulk scalars as sources. In particular, we investigate the consequences of having the phantom and the Brans–Dicke scalar in the bulk while obtaining such solutions. We find 4-branes without a conical singularity but with a compact on–brane dimension which may be assumed to be small in order to realize a 3–braneworld at low energies. The purely 3–brane solutions, on the other hand, do have the usual conical singularity, characteristic of codimension two branes. Furthermore, we look into the issue of localization of matter fields on these 3 and 4-branes and conclude with our comments on these six dimensional models.

Each chapter individually summarizes the results discussed in it. Some of our failures as well as open issues relevant to the subject are highlighted in the final chapter.

The thesis ends with a set of appendices where we discuss (i) the Schrödinger equation in a warped space and (ii) quantum mechanics in a family of volcano potentials.