CHAPTER - I

INTRODUCTION AND BASIC EQUATIONS

1.1 Introduction

The present work is concerned mainly with computation of plane steady inviscid transonic flow past thin profiles at zero angle of incidence. A flow field where both subsonic and supersonic regions are present and are significant in determining the overall character of the flow field is known as a transonic flow field. The subsonic and supersonic regions are separated by sonic lines (in case of two dimensional flow) or by sonic surfaces, where the flow velocity is equal to the local speed of sound. It is frequently observed that in a transonic flow field the acceleration from subsonic flow to supersonic flow is smooth, that is continuous, whereas during deceleration from supersonic to subsonic flow, a shock discontinuity appears ~~at the sonic line~~.

Transonic flow fields appear in a large variety of aerodynamic problems like flow in nozzles, around blunt bodies flying supersonically and near airplane wings flying close to Mach number unity. Study of such flow fields has become particularly important with the development of modern high speed aircrafts. It is one of the most efficient flight regimes, and such studies have become important for speeding up the performance of subsonic aircrafts. Moreover, the problem of drag reduction in flight at transonic speed range is of much interest.

The complexity of the high speed vehicles requires thousands of costly wind-tunnel testing to study the complete flow field for their development and design. Even then it is not always possible to match correctly the wind-tunnel results to real flight conditions. With the advent of electronic digital computers, several numerical techniques give economical alternatives for the experimental data and in some cases it is possible to eleminate completely the need for experimental testing (Kutler (1975)).

Under the assumptions of small perturbation theory, the basic gasdynamic equation for the velocity potential may be linearised in the cases of purely subsonic or purely supersonic flows. But in case of transonic flow, it remains non-linear, strictly speaking quasi-linear, even under the assumption of small perturbation. Further, the equation is of mixed elliptic-hyperbolic type, being of elliptic type in subsonic flow and of hyperbolic type in supersonic flow. Their line of demarcation, viz. the sonic line is not known a priori and is to be found out as a part of the solution. This leads to considerable mathematical difficulty. For subsonic flow, under the assumption of small perturbation, the linearised form of the gasdynamic equation with linearised boundary conditions may be solved by Prandtl-Glauert rule (Oswatitsch (1956)) and the solution obtained is of first-order accuracy. Solutions with higher order accuracy may also be obtained as presented by Van Dyke (1954) and Gretler (1965). In case of purely supersonic flow, solution, with desired accuracy can be obtained by the method of characteristics (Oswatitsch (1956)) even for the quasi-linear equation. The methods for solving quasi-linear equations of mixed type are only of recent origin.

There are generally two types of transonic aerodynamic problems : (i) the direct problem in which the free-stream Mach number and the airfoil shape are prescribed and the resulting flow field is to be determined and (ii) the design problem belonging to the category of indirect problems where the body shape is to be determined for prescribed pressure distribution at the airfoil and the free-stream Mach number.

For flow past a profile, as the free-stream Mach number increases, the value of the maximum velocity at the profile also increases. When $M_\infty < (M_\infty)_{crit.}$, the flow is purely subsonic. Here $(M_\infty)_{crit.}$ is the value of M_∞ for which the maximum value of the velocity at the profile