Chapter 1

Introduction and A Review of the Literature

1.1 Introduction

In Reliability, Economics, Queues, Inventory, Actuarial Science, Stochastic Process and related fields, it is often required to compare the lifetimes of products of different brands and take decision about those products. In Reliability and Survival Analysis, the remaining lifetimes of a component at different time of its life span need to be compared to take decision about the aging of that component with time. A repairable system can improve or deteriorate with the passage of time according as the interarrival times of failure tend to get larger or smaller in some sense. So the comparison of interarrival times of a system at different time is also required to take decision about the system improvement or deterioration. In Econometrics, different income distributions are to be compared in terms of the corresponding income inequalities and the various random prospects need comparison so that better one can be chosen. In Biological Sciences, the lifetimes of control group of living organisms need to be compared

with group receiving drug, in order to conclude about the effectiveness of a particular drug.

Let us consider one simple example. Suppose we want to compare the lifetime of scooters produced by Bajaj and that by LML company. Let X be the lifetime of the scooters produced by Bajaj and Y be that by LML company. Here our aim is to take some decision whether X is greater (or smaller) than Y in some sense. Now, as X and Y are two random variables, and not real numbers, it is not possible to compare these random variables in traditional way, that is, the way we compare real numbers. For any two random variables X and Y, one way of comparing is by comparison of there means, *i.e.*, X is said to be larger than Y if $E(X) \geq E(Y)$ (called expectation ordering in the literature and we write $X \geq_E Y$). Now, this type of comparison has the main drawback that it compares only two points from two distributions. If someone is interested in comparing two random variables having same mean, then this type of comparison fails. Again, one can think of comparing two random variables in terms of their dispersions, which can be done by comparing their respective standard deviations. As this type of comparison is based on two single real numbers, this has the same drawback as the comparison through means. To overcome this drawback, possibly the best way of comparing two random variables is by means of stochastic order relations.

Throughout, the words increasing (decreasing) and nondecreasing (nonincreasing) are used interchangeably.