Preface

Our main objective in this thesis is to do some work in prime and semiprime rings satisfying some functional identity. A functional identity(FI) on R is, roughly speaking, an identity for arbitrary elements in R, which involves maps of R. A simple example is the identity [f(x), x] = 0 for all $x \in R$, where the Lie product of x, y is defined by [x, y] = xy - yx. One can view that the concept of a functional identity as a generalization of the concept of polynomial identity. In this thesis, we study some engel type functional identities in a prime and semiprime ring R and then determine the structure of the ring R and the associated function. The functions some times considered here are derivations, centralizer maps and some times just any additive mappings.

A map $f: R \to R$ is called commuting (centralizing) on R if [f(x), x] = 0 (respectively $[f(x), x] \in Z(R)$) for all $x \in R$. The study of centralizing and commuting mappings is initiated by Posner [56] in 1957. Posner's result states that the existence of a nonzero centralizing derivation on a prime ring R implies that R is commutative. Since then the result was generalized in many directions by a number of authors. In chapter 2 and chapter 4, we study this type of identity in case when f is a derivation. In chapter 4, we study the annihilator condition of the identity also. Our method of approaching the problem is to use the results of differential identity due to Kharchenko [40] to show that derivation involved is inner, and then to conclude that R satisfies a generalized polynomial identity (GPI). Then by Martindale [51], R embeds nicely in a primitive ring with nonzero socle and by extending the base field we argue that one can assume that R is a matrix ring over a field. At this point matrix computations yield desired results.

In chapter 3, we study functional identity in case of Banach algebras. The main object

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here is to show that any continuous derivation maps the Banach algebra into its radical. This work is initiated by Singer and Wermer [60] in commutative Banach algebras and then by Yood [66], Brešar and Vukman [20] in noncommutative Banach algebras. Our main technique here is to convert the problem in case of prime rings. Direct computations lead to the results.

Chapter 5 deals with some functional identities on rings with involution containing multi-additive mappings. Here it is considered that the center of R contains at least one nonzero symmetric element. Also the functional identities are studied in matrix rings.

A map $f: R \to R$ is called skew-commuting (skew-centralizing) on R if (f(x), x) = 0(respectively $(f(x), x) \in Z(R)$) for all $x \in R$, where (x, y) denotes the Jordon product of x, y and is defined by (x, y) = xy + yx. Chapter 6 discusses skew-commuting and skew-centralizing mappings on rings with left identity. In this chapter, the rings need not be prime or semiprime.

In chapter 7, our results are with respect to a new product introduced in a prime ring, which might be of some independent interest. Lastly in chapter 8, we aim to show that any two left centralizer maps are C-dependent under certain conditions.