SYNOPSIS

With the rapid advances of computer technology in the last forty years, computer-based mathematical modeling has become the most important method of experimentation in many different fields. In particular, during the last two decades, scientific computing has become a hot topic within the computational mathematics.

Recently many researchers identified that the Krylov subspace methods are the most powerful and efficient iterative based methods for solving huge sparse system of linear equations, nonlinear systems and also for solving standard and generalized eigenvalue problems.

At the outset the thesis focused on two aspects of study :

(1) Lanczos-Krylov method for finding eigenvalues and the corresponding eigenvectors of the standard eigenvalue problems and the generalized eigenvalue problems.
(2) Krylov methods in solving system of linear and nonlinear equations which are arising out of numerical heat transfer problems.

In the former case we have introduced a New Recursive Partitioning Technique along with the Krylov Lanczos algorithm to extract the extreme eigenvalues and the corresponding eigenvectors. The recursive partitioning algorithm is being proposed as an alternative for the existing Lanczos algorithm with Sturm sequence bisection method. The comparative results for the standard eigenvalue problem have been discussed in chapter 2 and a similar comparative study for the generalized eigenvalue problem has been presented in Chapter 3.

In the latter case we have taken into consideration some of the powerful preconditioning technique and our study proposes a possible selection of Krylov methods with appropriate preconditioning techniques to handle the system of linear equations and system of nonlinear equations. We have presented our comparative results in chapter 4(nonsymmetric linear system), chapter 5(symmetric linear system) and chapter 6(system of nonlinear equations).

We now briefly touch upon the salient features of each of the ensuing chapters:

Chapter 2 proposes a Lanczos algorithm with a new recursive partitioning technique for extracting i^{th} extreme eigenvalue, in a given specified interval of a standard eigenvalue problems. Comparisons are made in respect of numerical results as well as the CPU-time between the Lanczos algorithm with a recursive partitioning technique and Lanczos algorithm with Sturm sequence-bisection method. Our study clearly indicates that the recursive partitioning technique takes relatively less computing time than the Sturm sequence-bisection method. The Lanczos algorithm with a recursive partitioning technique is an alternative procedure to the Lanczos algorithm with Sturm sequence-bisection method.

Chapter 3 explores the application of the Lanczos algorithm with the recursive partitioning technique introduced in the previous chapter to the vibration and stability problems which are in general considered as generalized eigenvalue problems. For this purpose we have used an inverse iteration in the Lanczos procedure, to extract the smallest eigenvalues with suitable choice of the shift, and it is required to solve a linear system of equations in each of the Lanczos iteration. We compute the first four mode shapes corresponding to the four smallest eigenvalue in 'beam-buckling problem' and the first three smallest frequencies and the associated mode of vibration of a 'ten-story reinforced concrete building vibration problem'. We also present a comparison of the numerical results and the CPU-time between the Lanczos algorithm with recursive partitioning method and the Lanczos algorithm with Sturm sequence-bisection method. This study indicates that the former method takes relatively less computing time than the latter method.

Chapter 4 deals with a comparative study of the Lanczos solver without preconditioning and CGS solver with preconditioning for solving numerical heat transfer problems. The CGS method has been discussed with incomplete Cholesky factorization(ILU) and polynomial preconditioning techniques. The Lanczos nonsymmetric algorithm has also been carried out without implementing preconditioning. Our study on comparative simulation for the heat transfer problems(both Convergence and CPU-time) indicates that the Lanczos algorithm without preconditioning is faster enough than the CGS solver, where as the CGS solver with preconditioning is faster than the Lanczos algorithm. The linear relationship between the number of iterations and the number of equations shows that the Lanczos and CGS methods are good competent and stable for large-scale computing.

Chapter 5 presents comparison of Krylov methods with preconditioning techniques for solving Boundary Value Problems. The CG method and symmetric Lanczos algorithm have been described with appropriate preconditioning techniques such as ILU, polynomial preconditioning(P(5)). The numerical results show that the convergence speed of the Lanczos algorithm and CG method are equivalent. But while applying preconditioning to the Lanczos algorithm and CG method, the speed of convergence of the Lanczos algorithm is faster than the CG method. At an error level of(1×10^{-5}), a 4-fold reduction in number of iterations of Conjugate Gradient polynomial preconditioning(CG-PP(5)) and a 3-fold reduction in number of iterations of Conjugate Gradient incomplete Cholesky factorization(CG-ILU) are achieved. The Lanczos and CG based computation exhibit a linear relationship between the number of iterations and the number of equations, which indicates that the rate of increase in the number of iterations is lower than that of the number of equations.

Chapter 6 deals with Krylov subspace linear solvers with appropriate preconditioners at every Newton step of Newton's scheme for solving the system of non-linear equations. The comparative results show that the Newton-Krylov solvers with appropriate preconditioning techniques are the fast converging procedures in solving system of non-linear equations with respect to the other Newton-SOR iterative method. As the Jacobian is the coefficient matrix and it has to be computed at every Newton step, we have directly implemented a finite difference discretization for the directional derivative, i.e $\nabla\bar{\mathbf{F}}.\bar{\mathbf{y}}$, and applied Krylov subspace linear solver to find $\bar{y}$ from $\nabla\bar{\mathbf{F}}.\bar{\mathbf{y}} = -\bar{\mathbf{F}}\bar{\mathbf{X}}^{(old)}$ so that the newton iterate $\bar{\mathbf{X}}^{(new)}$ at every Newton step is computed from $\bar{\mathbf{X}}^{(new)} = \bar{\mathbf{X}}^{(old)} + \bar{\mathbf{y}}$.