Abstract

The present thesis deals with specific initial and boundary value problems arising in certain multi-dimensional hyperbolic systems of conservation laws. Our aim in this thesis is to develop the existence and regularity results of the solutions to these problems. We tend to use the generalized characteristic analysis method, constructing solutions via some implicit functions based on initial discontinuity or by using well received characteristic decomposition method.

First, we develop a three-dimensional thin film flow model for an anti-surfactant solution and obtain a reduced hyperbolic model in two-dimensions by assuming that the solute is perfectly soluble and diffusivity effects are negligible. We consider a four-quadrant Riemann problem for it and develop six topologically distinct structures of solutions in the self-similar plane using the generalized characteristic analysis method. Next, we consider the same system with a non self-similar Riemann data where the initial data is separated by a smooth curve and self-similarity can not be applied. However, using the implicit functions of the initial discontinuity curve, we develop solutions for five different cases for this initial value problem based on different choices of initial data.

Next, we consider a gas expansion problem through a sharp corner into the vacuum for isentropic Euler equations with a general equation of state. This problem can be considered as the interaction of a centered simple wave with a planar rarefaction wave. In order to obtain the global existence of a solution up to the vacuum boundary of the corresponding two-dimensional Riemann problem, we consider several Goursat-type boundary value problems for 2-D self-similar Euler equations and use the ideas of characteristic decomposition and bootstrap method. Further, we formulate two-dimensional modified shallow water equations newly and solve a dam-break type problem for them as an application of this work. Next, for the same system, we are concerned with developing the existence and regularity of a semi-hyperbolic patch problem with a van der Waals gas. We use the idea of characteristic decomposition and the bootstrap method to prove the existence of a global smooth solution which is uniformly $C^{1,\frac{1}{2}}$ continuous up to the sonic curve. We also prove that the sonic curve is $C^{1,\frac{1}{2}}$ continuous. Further, we show the formation of shock as an envelope for positive characteristics before reaching their sonic points.

Lastly, we prove the existence and regularity of a smooth solution for a supersonic-sonic patch arising in a modified Frankl problem in the study of three-dimensional axisymmetric steady isentropic relativistic transonic flows over a symmetric airfoil. We consider a general convex equation of state which makes this problem complicated as well as interesting in the context of the general theory for transonic flows. Here the main difficulty is the coupling of nonhomogeneous terms due to axisymmetry and the sonic degeneracy for the relativistic flow. However, using the well-received characteristic decompositions of angle variables and a partial hodograph transformation we prove the existence and regularity of solution in the partial hodograph plane first. Further, by using an inverse transformation we construct a smooth solution in the physical plane and discuss the uniform regularity of solutions up to the associated sonic curve. Finally, we also discuss the uniform regularity of the sonic curve.

Keywords: Two-dimensional Riemann problem; Elementary waves; Delta shock wave; Isentropic Euler equations; Isentropic relativistic Euler equations; Self-similar solutions; Characteristic decomposition; Thin-film flows; Goursat problems; Gas expansion; Semi-hyperbolic patch; Sonic-supersonic patch